Progress In Electromagnetics Research Letters, Vol. 44, 15–21, 2014

Phase Shifting Holography for THz Near-field/Far-field Prediction

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Abstract—With a view to extending techniques for THz antenna near-field/far-field prediction, this communication derives general analytic expressions for calibrated phase shifting holography (PSH) and introduces a new 120° three-step PSH method that avoids switching off the reference field and has symmetrical performance over the entire complex plane, providing spurious free far-field prediction. Numerical tests with simulated near-field patterns at 372 GHz confirm the convenience of the method and give an indication of the precision required for the phase shifts.

1. INTRODUCTION

Near field intensity holograms carried out with arrays of power detector elements could be a viable technology for far-field radiation pattern prediction of electrically large THz antennas. Holography has a number of attractive features, for example, waves from both the reference and the antenna-under-test (AUT) travel similar paths, so errors related to source instability in frequency and phase as well as propagation effects can be reduced. Moreover, for prediction angles within about ±30° holography is insensitive to scanner planarity errors, provided of course that the reference wave incident angle is also within this range. Off-axis near-field holography requiring the entire hologram pattern was experimentally demonstrated at 94 GHz in [1]. The method was adapted from the well known Fourier transform version of holography by Leith & Upatnieks [2], but requires relatively large scan distances. Additionally, sampling requirements are greater than those of conventional near-field methods. The goals of near-field holography therefore suggest the use of in-line holograms, originally proposed by Gabor [3].

In 1966, Gabor & Goss [4] presented a practical solution for in-line holography that has over the last decade evolved into Phase Shifting Holography (PSH) for digital in-line optical microscopy [5]. This is a point by point in-line holographic method that typically requires three intensity measurements; two intensity measurements with the reference in quadrature phase states, and additionally the intensity of the AUT (or object) near-field with the reference switched off. PSH can be performed with holograms having any arbitrary reference wave phase shift not close to integer multiples of π. We shall refer to this two-step method as A2H to recall that one AUT plus two hologram measurements are required.

This communication extends the analysis of PSH in order to facilitate general calibration and error budget analysis for phase-shifters having arbitrary S_{21}. It is shown that when the reference wave is sufficiently strong a condition C is met [6], and then only two holograms are required (C2H). However, in this case an entire scan of the AUT near-field intensity is needed to determine the minimum reference level. More importantly, the sensitivity of the two-step complex field solution to phase-step calibration errors is highly asymmetric, leading to spurious far-field grating effects. To avoid this, a new three-step hologram method (3H) is proposed here, employing phase shifts of 0°, 120° and 240° to achieve optimum solution symmetry. No switching-off the reference wave is required, and any level of reference wave is valid. This choice has the following added advantages: (a) avoids difficulties in parts of the complex plane when noise is present; (b) reduces the solution sensitivity to errors in the value of the phase shift;

Received 30 September 2013, Accepted 12 November 2013, Scheduled 19 November 2013
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(c) provides three complex field solutions that can be averaged to recover signal-to-noise ratio. The proposed hologram recording configuration shown in Fig. 1 is quasi on-axis, given that the reference horn must be positioned to avoid illuminating the AUT.

2. THEORETICAL DEVELOPMENT

2.1. General Formulation of Two Step PSH (A2H)

Consider two voltage waves S and R entering a power detector, where S is the complex AUT field and R is a complex reference wave. For convenience divide S by $e^{j\phi_R} = R/|R|$ so that $|S+R|$ can be written as $|A+|R||$ where $A = A_r + jA_i = Se^{-j\phi_R}$. Hologram intensity is the power detected from the interference between AUT and reference fields. Two holograms are recorded, the first is H_1 given by

$$H_1 = |A + |\tau_1 R||^2 = I_o + 2\,|R|\,A_r \tag{1}$$

where we have chosen $\tau_1 = 1$. The sum of the separate power terms is

$$I_o = |A|^2 + |R|^2 \tag{2}$$

and the associated 'difference hologram' [6] is

$$I_1 = H_1 - I_o = 2\,|R|\,A_r \tag{3}$$

The second hologram intensity H_2 is formed by phase shifting the reference wave by a complex factor $\tau_2 = \tau_{2r} + j\tau_{2i} = |\tau_2|e^{ja_{\tau 2}}$ where $a_{\tau 2} = \arg(\tau_2)$ can have any value not close to an integer multiple of π. The magnitude $|\tau_2|$ is left arbitrary to model the variation of S_{21} of a real phase shifter when the phase state is changed.

$$H_2 = \left|Ae^{-ja_{\tau 2}} + |\tau_2 R|\right|^2 = |A|^2 + |\tau_2 R|^2 + 2\,|\tau_2 R|\,\mathrm{Re}(Ae^{-ja_{\tau 2}}) \tag{4}$$

and the corresponding difference hologram is

$$I_2 = H_2 - \left(I_o + |R|^2\left(|\tau_2|^2 - 1\right)\right) = 2\,|\tau_2 R|\,\mathrm{Re}(Ae^{-ja_{\tau 2}}) \tag{5}$$

Combine I_1 with I_2 weighted by a complex factor α, and defining γ as follows

$$I_1 + \alpha I_2 = 2\,|R|\,(A_r + \alpha\,(A_r\tau_{2r} + A_i\tau_{2i})) \stackrel{\text{def}}{=} 2\,|R|\,\gamma A \tag{6}$$

Collecting real and imaginary parts in (6) dictates that simultaneously $\gamma = (1 + \alpha\tau_{2r})$ and $\gamma = -j\alpha\tau_{2i}$ thus leading to $\alpha = -1/\tau_2$ and hence

$$\gamma = je^{-ja_{\tau 2}}\sin a_{\tau 2} \tag{7}$$

The complex AUT field is therefore given by

$$S = Ae^{j\phi_R} = \frac{e^{j\phi_R}}{2|R|\gamma}\left(I_1 - \frac{1}{\tau_2}I_2\right) \tag{8}$$

Because of the $\sin a_{\tau 2}$ term in (8), phase steps close to π cannot be used. The solution (8) can be generalized for the case of two arbitrary phase shifts, where say H_2 is combined with hologram H_3, recorded with phase shift τ_3, as follows:

$$S = \frac{R}{|R|}\left[\frac{X_i\Upsilon_i - X_r + X_r\Upsilon_r}{|\Upsilon|^2 - 1} + j\frac{X_i\Upsilon_r + X_i - X_r\Upsilon_i}{|\Upsilon|^2 - 1}\right] \tag{9}$$

where

$$X = X_r + jX_i = \frac{1}{|R|}\left(\frac{I_2}{\tau_2} + \frac{I_3}{\tau_3}\right) \tag{10}$$

and

$$\Upsilon = \Upsilon_r + j\Upsilon_i = \frac{1}{2}\left(e^{-2ja_{\tau 2}} + e^{-2ja_{\tau 3}}\right) \tag{11}$$

and similarly to (5)

$$I_3 = H_3 - \left(I_o + |R|^2 \left(|\tau_3|^2 - 1\right)\right) \tag{12}$$

As expected in holography, the reference wave magnitude $|R|$ and phase ϕ_R must be known a priori. In practice this could be done by creating holograms of the reference horn with a small calibrated horn acting as AUT. The final far-field precision will inevitably depend on how accurately the near-field reference wave can be measured or perhaps calculated. The three power measurements include hologram intensities H_1, H_2 and AUT power density $|A|^2$ with the reference field switched off. Additionally, the complex reference modifier τ must be known precisely as can be appreciated in Fig. 2, which shows the asymmetrical nature of the sensitivity of the solution to calibration errors in the phase shift. In order to improve the symmetry of the solution we suggest in Section 2.3 using phase shifts of 120° and 240° instead of 90° and 180°. As mentioned in the introduction, this choice reduces sensitivity to errors in the phase change, avoids solution problems in certain parts of the complex plane when noise is present and provides three correct solutions that can be averaged to recuperate signal to noise ratio.

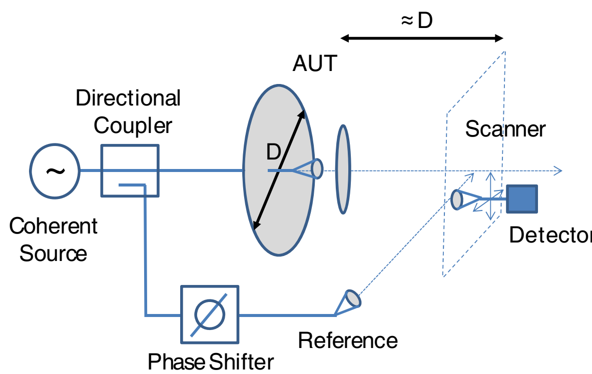

Figure 1. Proposed holographic system configuration for near-field antenna measurements. The reference horn is positioned to avoid illuminating the AUT.

2.2. Two Step PSH with Excess Reference Power (C2H)

A special case of PSH eliminates the requirement to measure $|A|^2$ and hence only two holograms are required, in addition to the reference wave. Since we suppose $|R|$ is known, the value of $|A|^2$ can be determined from H_1 and H_2 provided special condition C of Equation (18) is met as follows.

We first acquire a solution for I_o in a similar way to [7] for $\tau_1 = 1$ and $\tau_2 = \tau = \tau_r + j\tau_i$, as

$$I_o^{\pm}(H_1, H_2, R, \tau) = \frac{T_1 \pm \tau_i \sqrt{T_2 |R|^4 + T_3 |R|^2 + T_4}}{|\tau|^2 - 2\tau_r + 1} \tag{13}$$

where

$$T_1 = |R|^2 \left(\left(|\tau|^2 - 1\right)\tau_r + 1 + \tau_i^2 - \tau_r^2\right) + H_1 \left(|\tau|^2 - \tau_r\right) + H_2 (1 - \tau_r) \tag{14}$$

$$T_2 = 4\tau_r \left(|\tau|^2 + 1\right) - |\tau|^4 - 2\left(\tau_r^2 - \tau_i^2\right) - 4|\tau|^2 - 1 \tag{15}$$

$$T_3 = 2(H_1 + H_2)\left(|\tau|^2 + 1 - 2\tau_r\right) \tag{16}$$

$$T_4 = 2H_1H_2 - H_1^2 - H_2^2 \tag{17}$$

Notice that in the special case where $\tau = \exp(\pm jn\pi)$ for odd integer n then (13) reduces to $I_o = \frac{1}{2}(H_1 + H_2)$ as expected. When $\arg(\tau) \neq n\pi$, the correct solution is dictated by the sign of C in the following equation.

$$C = \frac{T_5 \left(|\tau|^2 - 2\tau_r + 1\right) - T_6}{\tau_i} \tag{18}$$

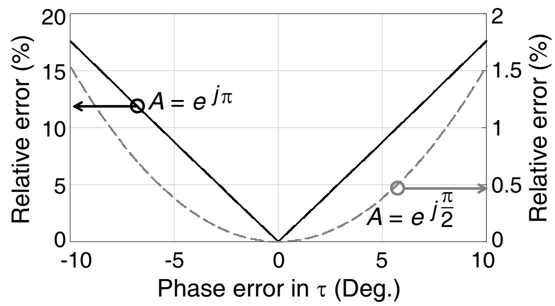

Figure 2. The calculated relative error in A, using either Equations (8) or (9), is shown as a function of the calibration error in the phase shifter τ. The two extreme cases show the asymmetric nature of this error in the complex plane.

where

$$T_5 = \frac{1}{2}\left(H_1 + H_2 - |R|^2\left(|\tau|^2 - 1\right) - 2\,|R|\,(A_r + A_r\tau_r + A_i\tau_i)\right) \tag{19}$$

$$T_6 = H_2 + |\tau|^2 H_1 + |R|^2\left(1 - \tau_r^2 + \tau_i^2\right) + \tau_r\left(|R|^2\,|\tau|^2 - H_1 - H_2 - |R|^2\right) \tag{20}$$

Not unexpectedly, the complex value of $A = A_r + jA_i$ is a prerequisite to the calculation of C. This method must therefore choose a sufficiently strong reference level so that everywhere $C > 0$ and the correct solution for I_o is always the positive term.

2.3. Three Step PSH (3H)

This is also a non-switching method that avoids measuring $|A|^2$, but replaces it with a third hologram with reference wave phase shift τ_3. The approach places no constraint on the relative levels of R and S and is attractive since it avoids the relatively slow process of switching on/off the reference wave. Phase shifting is fast, so all three measurements can be performed sequentially. To use the combination of second and third holograms having $\tau_2 = \tau_{2r} + j\tau_{2i}$ and $\tau_3 = \tau_{3r} + j\tau_{3i}$, an additional solution for $I_o^{\pm}(H_2, H_3, R, \tau_2, \tau_3)$ is required as follows:

$$I_o^{\pm} = -\frac{1}{2T_6}\left(T_5 \pm \sqrt{T_5^2 - 4T_6T_7}\right) \tag{21}$$

where

$$K_2 = |R|^2\left[|\tau_2|^2 - 1\right]; \quad K_3 = |R|^2\left[|\tau_3|^2 - 1\right] \tag{22}$$

$$T_1 = \frac{\tau_{2i}\tau_{3r} - \tau_{3i}\tau_{2r}}{|\tau_2|\,|\tau_3|}; \quad T_2 = \frac{1}{\left[2T_1\,|R|\,|\tau_2|\,|\tau_3|\right]^2} \tag{23}$$

$$T_3 = \tau_{3i}\,(H_2 - K_2) - \tau_{2i}\,(H_3 - K_3) \tag{24}$$

$$T_4 = \tau_{3r}\,(H_2 - K_2) - \tau_{2r}\,(H_3 - K_3) \tag{25}$$

$$T_5 = T_2\left[2T_3\,(\tau_{2i} - \tau_{3i}) + 2T_4\,(\tau_{2r} - \tau_{3r})\right] - 1 \tag{26}$$

$$T_6 = T_2\,|\tau_2 - \tau_3|^2; \quad T_7 = |R|^2 + T_2\left(T_3^2 + T_4^2\right) \tag{27}$$

Given a set of three holograms $\{H_1, H_2, H_3\}$ we can choose three pairs $\{(H_1, H_2), (H_1, H_3), (H_2, H_3)\}$, leading to a total of six possible values of I_o according to the twin values given by Equations (13) and (21). These are:

$$\{I_o\} = \begin{matrix} I_o^{+}(H_1, H_2, R, \tau_2) & I_o^{-}(H_1, H_2, R, \tau_2) \\ I_o^{+}(H_1, H_3, R, \tau_3) & I_o^{-}(H_1, H_3, R, \tau_3) \\ I_o^{+}(H_2, H_3, R, \tau_2, \tau_3) & I_o^{-}(H_2, H_3, R, \tau_2, \tau_3) \end{matrix} \tag{28}$$

At least three values in the set $\{I_o\}$ of (28) will be identical in noiseless conditions for arbitrary values of τ_1, τ_2. Given the presence of noise, the correct solutions are never identical so a search is made for the closest matching group of three solutions. This is achieved by first sorting and then finding the closest three values on a squared error basis. However, in the particular case of phase shifts of 90° and 180° when noise is present this approach is not valid over the entire complex plane. This special case requires a more costly search based on the solutions A^n_{12}, A^n_{13} for each of the $n = 1, \ldots, 6$ values of $I^n_o = \{I_o\}$ calculated using (8) or (9) for hologram pairs (H_1, H_2) and (H_1, H_3). The solution A^n_{23} based on the paired holograms (H_2, H_3) gives a further means of comparison. The difference between solutions is calculated as (29) and the best solution for I_o is I^m_o corresponding to $\xi^m = \min\{\xi\}$.

$$\xi^n = \frac{|A^n_{23} - A^n_{12}| + |A^n_{23} - A^n_{13}|}{|A^n_{12}| + |A^n_{13}| + |A^n_{23}|} \tag{29}$$

The final value of A is the average of $\{A^m_{12}, A^m_{13}, A^m_{23}\}$. As a bonus, the extra third hologram measurement gives a noticeable improvement in SNR compared to PSH (A2H) where only one solution is calculated.

2.4. Noise Model

The robustness of the PSH (3H) field reconstruction algorithm has been tested with simple pre-detector and post-detector noise mechanisms as shown in Equations (30) and (31), where $N_{1,2}(t)$ are taken as normal distributions with variances σ_1^2, σ_2^2 respectively.

$$|A+R+N_1(t)|^2 = I_o^2+2\mathrm{Re}(AR)+2\mathrm{Re}(AN_1(t))+2\mathrm{Re}(RN_1(t))+|N_1(t)|^2 \tag{30}$$

$$|A+R|^2 + |N_2(t)|^2 = I_o^2 + 2\mathrm{Re}(AR) + |N_2(t)|^2 \tag{31}$$

The noise power density of the *beat* noise in (30) is proportional to the product of the total coherent field strength $A + R$ with the standard deviation σ_1 of the equivalent voltage noise source $N_1(t)$. In comparison, post-detector noise power density, for example from thermal noise, is proportional to the variance σ_2^2 of the equivalent instantaneous noise voltage source.

2.5. Verification with a 372 GHz Antenna FEKO Simulation

The antenna used to test the algorithmic performance in a typical near-field measurement configuration is a conventional Cassegrain with a rotated main parabolic reflector [8]. Geometric Optics (GO) with GTD diffraction was carried out with the commercial software FEKO, at both 372 GHz and 186 GHz. Fig. 3 shows the sensitivity of the complex near-field to an independent error in just one of the phase shifts, indicating that for accurate far-field performance the phase shift should be known to within ±0.1°. If all phase shifts errors have a common factor, then the near-field error is also approximately a complex factor, and hence is not so critical.

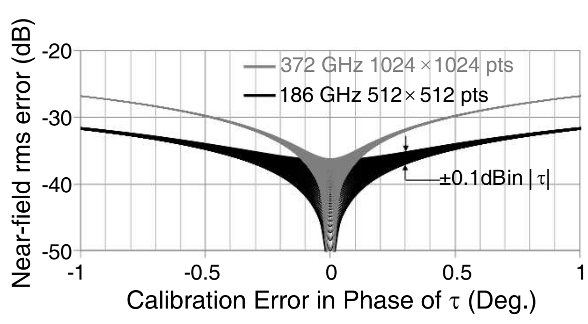

Figure 3. Sensitivity of the near-field solution to independent calibration errors in one of the two phase-shifter constants, $\tau_{1,2}$. Magnitude variations in $\tau_{1,2}$ are ±0.1 dB. Near-field data sets correspond to the AUT in [8]. $\sigma_2 = 0.001$.

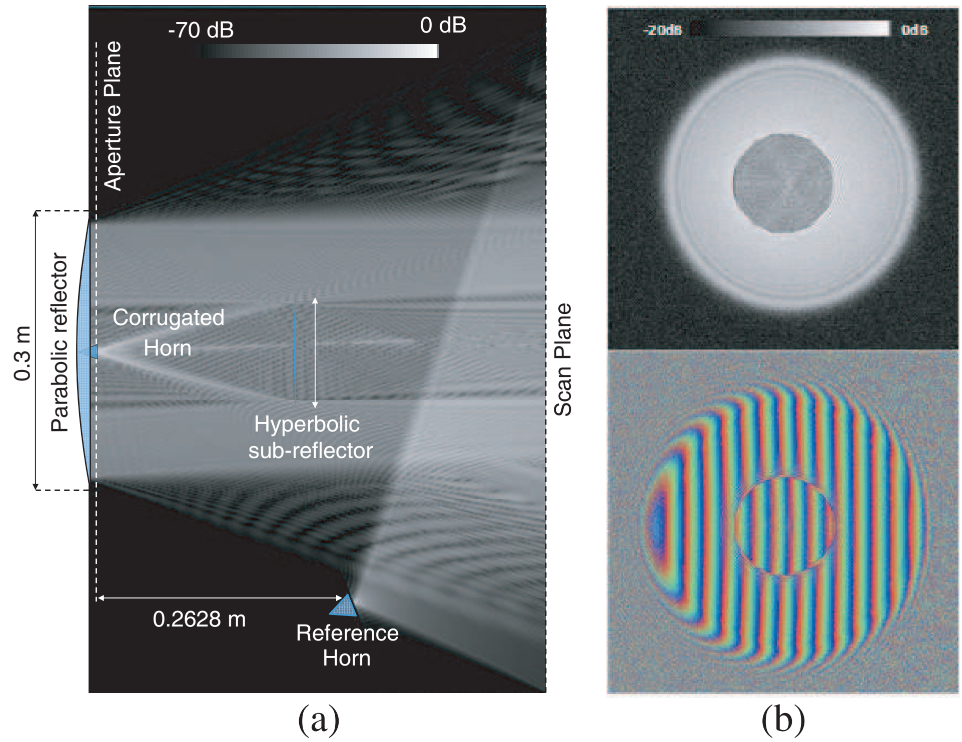

Figure 4. The back-propagated field of a 372 GHz 0.3 m diameter dual reflector antenna using FEKO to model the near-field and PSH (3H) for holographic reconstruction. (a) Horizontal axial plane reconstruction. In this case the near-field is noiseless. (b) Transverse reconstruction of the aperture plane focused at the sub-reflector plane. The near-field SNR is 20 dB with respect to near-field beam peak. The dimensions of the antenna are identical to those in [8].

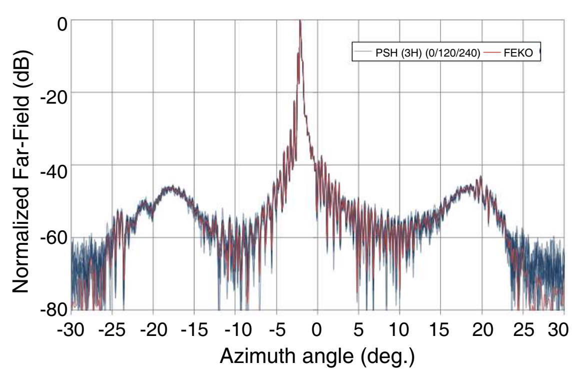

Figure 5. Predicted far-field of the 372 GHz 0.3 m diameter dual reflector antenna showing FEKO reference far-field and PSH (3H, 0°/120°/240°) holographic reconstruction. SNR corresponds to 20 dB with respect to near-field beam peak.

Figure 4(a) shows the back-propagated complex field recovered by PSH (3H) (under noiseless conditions) superimposed on the back-propagated reference wave and demonstrates how the approach here is much more compact than that reported by this author in [1]. Fig. 4(b) shows the AUT aperture image referred to the plane of the sub-reflector, showing perfect solution symmetry using PSH (3H) with only 20 dB SNR (at beam peak). The main-reflector is configured for a 2° beam-shift which appears as a horizontal displacement and the GO integration used by FEKO can be seen in the sub-reflector region of the image.

The predicted far-field shown in Fig. 5 is a superposition of 10 patterns generated with the pre-detector noise mechanism of Equation (30) and compared to the FEKO reference in red. The expected 2° beam shift and the sub-reflector spillover side-lobes are faithfully predicted.

3. CONCLUSIONS

A new three-step 120° PSH algorithm with solution symmetry and improved SNR is a good candidate for THz near-field/far-field predictions. The phase of the phase-shifter is a critical parameter that should be calibrated to better than about $\pm 0.1^\circ$ for accurate far-field prediction. The technique could have good data acquisition speed as it avoids switching on/off the reference wave, once the reference wave is known. Unlike Phase Retrieval which requires complete patterns, this method does not have any special sampling requirements and works on a point by point basis. In common with standard complex near-field measurements, these latter features are very convenient for antenna near-field alignment and spatial under-sampling using directive probes.

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