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STUDY OF FOCUSING OF A CYLINDRICAL INTERFACE OF CHIRAL NIHILITY-CHIRAL NIHILITY MEDIA USING MASLOV'S METHOD

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Abstract—Reflection of electromagnetic plane wave from a planar chiral nihility-chiral nihility interface is calculated as a special case of two different chiral media by assuming that permittivities and permeabilities of the both media approach to zero. That is, $\epsilon_i \to 0$, $\mu_i \to 0$, and chiralities $\kappa_i \neq 0$, $i = 1, 2$. These results are used to find the geometrical optics reflected fields of a cylindrical chiral nihility-chiral nihility interface, when it is excited by a plane wave. Using the Maslov's method, field expression which yields finite values around the focus of cylindrical interface is also determined.

1. INTRODUCTION

Asymptotic Ray Theory (ART) is widely used to describe the electromagnetic waves in both homogeneous and inhomogeneous isotropic media, but it fails in the vicinity of caustics [1]. A method which remedies this defect has been proposed by Maslov [2]. Maslov's method makes use of representation of the geometric optics (GO) field in terms of mixed coordinates consisting of space coordinate $R(x, y, z)$ and wave vector (k_x, k_y, k_z). For details about Maslov's method the reader is referred to [3, 4].

In the present discussion, reflected fields associated with a cylindrical interface of two lossless, homogenous, and reciprocal chiral nihility media are studied. For more details about the concept of chiral nihility, reader is suggested to study the article [5]. Location of the

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caustic and field expressions yielding finite field around the caustic are calculated. In Section 2, reflection from a planar interface of two chiral media is determined. Assuming that the permittivities and permeabilities of both media approach to zero, reflection from a chiral nihility-chiral nihility planar interface is determined. In Section 3, location of the caustic is calculated and non singular field expression, using Maslov's method, are derived. In Sections 3.1 and 3.2, parabolic and circular cylindrical interface are considered as examples. Time dependency used throughout the communication is $\exp(-i\omega t)$.

2. PLANAR INTERFACE OF TWO CHIRAL MEDIA

Consider a planar interface, which is located at $z = 0$, of two isotropic chiral media as shown in Figure 1. Half space $z < 0$ is of chiral medium with constitutive parameters $(\epsilon_1, \mu_1, \kappa_1)$ whereas chiral half space, $z > 0$, has constitutive parameters $(\epsilon_2, \mu_2, \kappa_2)$. Assume that chiral-chiral interface is excited by an oblique incident and left circularly polarized (LCP) uniform plane wave. Total field in each half space

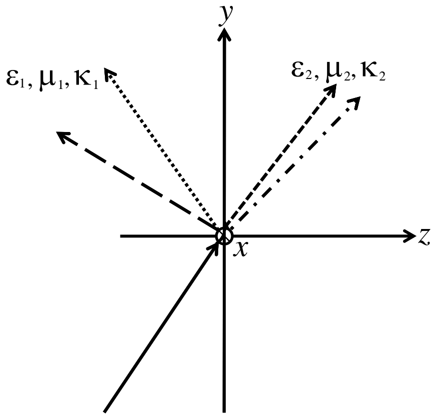

Figure 1. Reflection and transmission in chiral-chiral medium: Incident LCP (solid lines), reflected LCP (round dotted line), reflected RCP (long dashed line), refracted LCP (square dotted line), refracted RCP (long dashed dotted line).

may be written, in terms of eigen vectors, as

$$\begin{aligned}
\mathbf{E}_{\text{inc}} &= \left(\hat{x} - \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y + ik_{z1}^{+}z) \\
\mathbf{E}_{\text{ref}} &= A\left(\hat{x} + \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y - ik_{z1}^{+}z) \\
&\quad + B\left(\hat{x} - \frac{ik_{z1}^{-}}{k_1^{-}}\hat{y} - \frac{ik_y}{k_1^{-}}\hat{z}\right)\exp(ik_y y - ik_{z1}^{-}z) \qquad (1a) \\
\mathbf{E}_{\text{tra}} &= D\left(\hat{x} - \frac{ik_{z2}^{+}}{k_2^{+}}\hat{y} + \frac{ik_y}{k_2^{+}}\hat{z}\right)\exp(ik_y y + ik_{z2}^{+}z) \\
&\quad + C\left(\hat{x} + \frac{ik_{z2}^{-}}{k_2^{-}}\hat{y} - \frac{ik_y}{k_2^{-}}\hat{z}\right)\exp(ik_y y + ik_{z2}^{-}z)
\end{aligned}$$

and corresponding magnetic fields are

$$\begin{aligned}
\mathbf{H}_{\text{inc}} &= \frac{i}{\eta_1}\left(\hat{x} - \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y + ik_{z1}^{+}z) \\
\mathbf{H}_{\text{ref}} &= \frac{i}{\eta_1}\left[A\left(\hat{x} + \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y - ik_{z1}^{+}z)\right. \\
&\quad \left. - B\left(\hat{x} - \frac{ik_{z1}^{-}}{k_1^{-}}\hat{y} - \frac{ik_y}{k_1^{-}}\hat{z}\right)\exp(ik_y y - ik_{z1}^{-}z)\right] \qquad (1b) \\
\mathbf{H}_{\text{tra}} &= \frac{i}{\eta_2}\left[D\left(\hat{x} - \frac{ik_{z2}^{+}}{k_2^{+}}\hat{y} + \frac{ik_y}{k_2^{+}}\hat{z}\right)\exp(ik_y y + ik_{z2}^{+}z)\right. \\
&\quad \left. - C\left(\hat{x} + \frac{ik_{z2}^{-}}{k_2^{-}}\hat{y} - \frac{ik_y}{k_2^{-}}\hat{z}\right)\exp(ik_y y + ik_{z2}^{-}z)\right]
\end{aligned}$$

where

$$k_{(1,2)}^{\pm} = \omega\left(\sqrt{\mu_{(1,2)}^{\pm}\epsilon_{(1,2)}^{\pm}} \pm \kappa\right)$$

are wave numbers.

Superscript $\pm$ correspond to the LCP and right circularly polarized (RCP) circularly polarized plane waves, respectively. Unknown coefficients, A, B, C, and D may be determined using the boundary conditions. Application of the following boundary conditions

$$\begin{aligned}
E_1^t &= E_2^t \\
H_1^t &= H_2^t
\end{aligned}$$

at $z = 0$, yields the unknown coefficients as

$$\begin{aligned}
A &= \frac{1}{X}\left[(k_{z1}^{+}k_{1}^{-} - k_{z1}^{-}k_{1}^{+})(k_{z2}^{-}k_{2}^{+} + k_{z2}^{+}k_{2}^{-})(\eta_1^2 + \eta_2^2)\right. \\
&\quad +2\eta_1\eta_2(2k_{z1}^{+}k_{z1}^{-}k_{2}^{-}k_{2}^{+} + k_{z1}^{+}k_{z2}^{-}k_{1}^{-}k_{2}^{+} + k_{z1}^{-}k_{z2}^{-}k_{1}^{+}k_{2}^{+} \\
&\quad \left.+k_{z2}^{+}k_{z1}^{+}k_{1}^{-}k_{2}^{-} + k_{z2}^{+}k_{z1}^{+}k_{1}^{+}k_{2}^{-} + 2k_{z2}^{-}k_{z2}^{+}k_{1}^{+}k_{1}^{-})\right] \\
B &= -\frac{2k_{z1}^{+}k_{1}^{-}(\eta_1^2 - \eta_2^2)(k_{z2}^{-}k_{2}^{+} + k_{z2}^{+}k_{2}^{-})}{X} \\
C &= \frac{4\eta_2 k_{z1}^{+}k_{2}^{-}(\eta_1 - \eta_2)(k_{z1}^{-}k_{2}^{+} - k_{z2}^{+}k_{1}^{-})}{X} \\
D &= \frac{4\eta_2 k_{z1}^{+}k_{2}^{+}(\eta_1 + \eta_2)(k_{z1}^{-}k_{2}^{-} + k_{z2}^{-}k_{1}^{-})}{X} \\
X &= (k_{z1}^{+}k_{1}^{-} + k_{z1}^{-}k_{1}^{+})(k_{z2}^{-}k_{2}^{+} + k_{z2}^{+}k_{2}^{-})(\eta_1^2 + \eta_2^2) \\
&\quad +2\eta_1\eta_2\left[k_{z1}^{-}k_{z2}^{+}k_{2}^{-}k_{1}^{+} - k_{z1}^{-}k_{z2}^{-}k_{1}^{+}k_{2}^{+} + 2k_{z2}^{-}k_{z2}^{+}k_{1}^{+}k_{1}^{-}\right. \\
&\quad \left.+2k_{z1}^{-}k_{z1}^{+}k_{2}^{+}k_{2}^{-} + k_{z1}^{+}k_{z2}^{-}k_{1}^{-}k_{2}^{+} - k_{z1}^{+}k_{z2}^{+}k_{1}^{-}k_{2}^{-}\right]
\end{aligned}$$

In the next section, field expressions are derived for a chiral nihility-chiral nihility planar interface. Chiral nihility is assumed as special case of chiral media with both permittivities and permeabilities approach to zero [5].

2.1. Planar Interface of Chiral Nihility-Chiral Nihility Media

Under the assumption of chiral nihility that is $\epsilon_i \to 0$ and $\mu_i \to 0$, $(i = 1, 2)$, relations for waves number simplifies to

$$\begin{aligned}
k_{(1,2)}^{\pm} &= \pm\omega\kappa \\
k_{z(1,2)}^{+} &= -k_{z(1,2)}^{-} \\
k_{(1,2)}^{+} &= -k_{(1,2)}^{-}
\end{aligned}$$

Corresponding electric and magnetic fields in both half spaces take the following form

$$\begin{aligned}
\mathbf{E}_{\text{inc}} &= \left(\hat{x} - \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y + ik_{z1}^{+}z) \\
\mathbf{E}_{\text{ref}} &= A\left(\hat{x} + \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y - ik_{z1}^{+}z) \\
&\quad +B\left(\hat{x} - \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y + ik_{z1}^{+}z) \qquad \text{(2a)}
\end{aligned}$$

$$\begin{aligned}\mathbf{E}_{\text{tra}} = & D\left(\hat{x} - \frac{ik_{z2}^{+}}{k_2^{+}}\hat{y} + \frac{ik_y}{k_2^{+}}\hat{z}\right)\exp(ik_y y + ik_{z2}^{+} z) \\ & + C\left(\hat{x} + \frac{ik_{z2}^{+}}{k_2^{+}}\hat{y} + \frac{ik_y}{k_2^{+}}\hat{z}\right)\exp(ik_y y - ik_{z2}^{+} z)\end{aligned}$$

and corresponding magnetic fields are

$$\begin{aligned}\mathbf{H}_{\text{inc}} = & \frac{i}{\eta_1}\left(\hat{x} - \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y + ik_{z1}^{+} z) \\ \mathbf{H}_{\text{ref}} = & \frac{i}{\eta_1}\left[A\left(\hat{x} + \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y - ik_{z1}^{+} z)\right. \\ & \left. - B\left(\hat{x} - \frac{ik_{z1}^{+}}{k_1^{+}}\hat{y} + \frac{ik_y}{k_1^{+}}\hat{z}\right)\exp(ik_y y + ik_{z1}^{+} z)\right] \qquad (2b) \\ \mathbf{H}_{\text{tra}} = & \frac{i}{\eta_2}\left[D\left(\hat{x} - \frac{ik_{z2}^{+}}{k_2^{+}}\hat{y} + \frac{ik_y}{k_2^{+}}\hat{z}\right)\exp(ik_y y + ik_{z2}^{+} z)\right. \\ & \left. - C\left(\hat{x} + \frac{ik_{z2}^{+}}{k_2^{+}}\hat{y} + \frac{ik_y}{k_2^{+}}\hat{z}\right)\exp(ik_y y - ik_{z2}^{+} z)\right]\end{aligned}$$

where

$$\eta_i = \eta_0 \lim_{\epsilon_i,\mu_i \to 0} \sqrt{\frac{\mu_i}{\epsilon_i}}, \quad i = 1, 2$$

It may be noted that reflected field with coefficient B is a backward wave propagating parallel to the incident wave as shown in Figure 2.

Unknown coefficients are

$$\begin{aligned}A &= \frac{\eta_1\eta_2[k_{z1}^{+2} k_2^{+2} - k_{z2}^{+2} k_1^{+2}]}{X} \\ B &= -\frac{k_{z1}^{+} k_{z2}^{+} k_1^{+} k_2^{+}(\eta_1^2 - \eta_2^2)}{X} \\ C &= -\frac{\eta_2 k_{z1}^{+} k_2^{+}(\eta_1 - \eta_2)(k_{z2}^{+} k_1^{+} - k_{z1}^{+} k_2^{+})}{X} \\ D &= \frac{\eta_2 k_{z1}^{+} k_2^{+}(\eta_1 + \eta_2)(k_{z1}^{+} k_2^{+} + k_{z2}^{+} k_1^{+})}{X} \\ X &= (k_{z1}^{+} k_{z2}^{+} k_1^{+} k_2^{+})(\eta_1^2 + \eta_2^2) + \eta_1\eta_2(k_{z2}^{+2} k_1^{+2} + k_{z1}^{+2} k_2^{+2})\end{aligned}$$

In the next section, above calculated reflection coefficients for a planar interface are used to find field expressions for a cylindrical chiral nihility-chiral nihility interface.

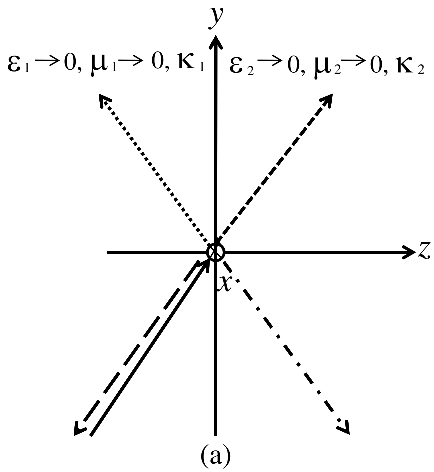

Figure 2. (a) Reflection and transmission in chiral nihility-chiral nihility medium (Poynting vector representation), and (b) reflection and transmission in chiral nihility-chiral nihility medium (Wave vector representation): Reflected backward wave (long dashed line), refracted backward wave (long dashed dotted line).

3. CYLINDRICAL INTERFACE OF TWO CHIRAL NIHILITY MEDIA

Consider a cylindrical interface of two chiral nihility media as is shown in Figure 3. The contour of the interface is described by following relation

$$\zeta = g(\xi)$$

where (ξ, ζ) are the cartesian coordinates of point on surface of cylindrical interface. Surface of the cylindrical interface is excited by a LCP plane wave. The unit vector normal to interface is

$$\hat{\mathbf{n}} = \hat{y} \sin\psi + \hat{z} \cos\psi \tag{3}$$

where ψ is the angle made by normal with z-axis and is given by

$$\sin\psi = -\frac{g'(\xi)}{\sqrt{1+g'(\xi)^2}} \tag{4a}$$

$$\cos\psi = \frac{1}{\sqrt{1+g'(\xi)^2}} \tag{4b}$$

$$\tan\psi = -g'(\xi) \tag{4c}$$

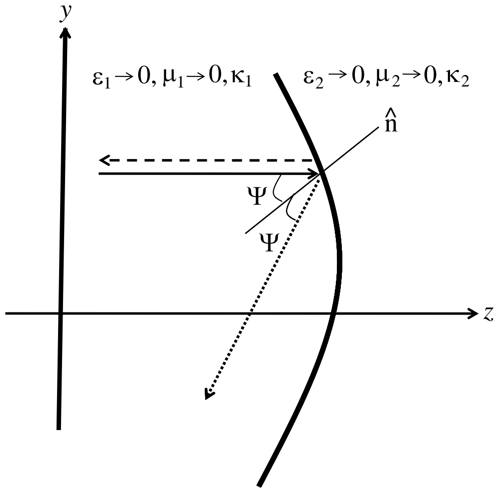

Figure 3. Cylindrical reflector in chiral nihility-chiral nihility medium (Poynting vector representation): Reflected backward wave (long dashed line), refracted forward wave towards caustic (long dashed dotted line).

Expression for LCP incident wave is given below

$$E_{\text{inc}} = \exp(ik_{z1}^{+} z)$$

Factor describing the polarization has been omitted from the expression. Both LCP and RCP are produced as reflected waves, RCP reflected wave is backward wave and travels parallel to the incident plane wave and does not contribute at the focus. Whereas LCP reflected wave is forward wave and according to Snell's law of reflection it makes an angle of ψ with unit normal and is given by

$$E_{\text{ref}}^{L} = \exp\left[-ik_1^{+}(\hat{y}\sin 2\psi + \hat{z}\cos 2\psi)\right] \tag{5}$$

Wave vector of reflected LCP wave is given by

$$\mathbf{P}_{LL} = -\hat{y}\sin 2\psi - \hat{z}\cos 2\psi$$

here subscript LL corresponds to the reflected LCP wave when incident LCP wave is considered and will follow in rest of the discussion. Coordinates of the point on the reflected ray are given by the solution of Hamiltonian's equations

$$y = \xi + p_y^r \tau, \quad z = g(\xi) + p_z^r \tau$$

where $p_y^r = -\sin 2\psi$ and $p_z^r = -\cos 2\psi$ are components of wavevector for reflected field.

Jacobian of transformation, $J_{0LL} = \frac{D(\tau)}{D(0)}$, can be found by using relation

$$D(\tau) = \frac{\partial(y,z)}{\partial(\xi,\tau)} = \frac{\partial y}{\partial \xi}\frac{\partial z}{\partial \tau} - \frac{\partial z}{\partial \xi}\frac{\partial y}{\partial \tau} \tag{6}$$

as

$$J_{0LL} = 1 - 2\tau \tfrac{\partial \psi}{\partial \xi} \tag{7}$$

The GO field for the reflected ray contributing at the focus is [4]

$$u_{LL} = A_{0LL}(\xi) \exp\left\{ik_1^+(\phi_{0LL}(\xi) + \tau)\right\} J_{0LL}^{-1/2} \tag{8}$$

Initial fields and initial phase, on the surface of the contour, for reflected LCP wave are given as

$$\begin{aligned} A_{0LL} &= \frac{\eta_1\eta_2(\cos^2\psi - \cos^2\phi)}{(\eta_1^2 + \eta_2^2)\cos\psi\cos\phi + \eta_1\eta_2(\cos^2\psi + \cos^2\phi)} \\ \phi_{0LL} &= \xi \end{aligned}$$

where relations

$$\begin{aligned} k_{z1}^+ &= k_1^+ \cos\psi \\ k_{z2}^+ &= k_2^+ \cos\phi \end{aligned}$$

have been used. ϕ is the angle of transmitted wave with the normal. The location for the caustic, where Jacobian becomes zero, is given as

$$\tau = \frac{1}{2}\frac{\partial \xi}{\partial \psi}$$

Since GO becomes infinite at caustic, so it is required to obtain finite field at caustic by using any other method. It is decided to obtain the finite field around caustic by using the Maslov's method through the following relation [4].

$$\begin{aligned} u(r) &= \sqrt{\frac{ik}{2\pi}} \int_{-\infty}^{\infty} A_{0LL}(\xi) \left[J_{0LL}\frac{\partial p_{zLL}}{\partial z}\right]^{-1/2} \\ &\quad \times \exp\left[ik_1^+(\phi_{0LL} + \tau - z_{0LL}p_z + zp_{zLL})\right] dp_{zLL} \end{aligned} \tag{9}$$

here $p_z = \frac{dz}{d\tau}$ and z_{0LL} is the value of z at stationary point. The quantity $J_{0LL}\frac{\partial P_{zLL}}{\partial z}$ can be calculated using following relation

$$\frac{\partial P_{zLL}}{\partial z} = \frac{\partial P_{zLL}}{\partial \xi}\frac{\partial \xi}{\partial z} \tag{10}$$

It is found that

$$J_{0LL}\frac{\partial p_{zLL}}{\partial z} = 2\sin^2 2\psi \frac{\partial \psi}{\partial \xi} \tag{11}$$

In next discussion, two cases have been considered to find the field. One case deals with parabolic interface and other is with circular interface.

3.1. Interface Having Parabolic Contour

Consider a parabolic interface whose contour is described by

$$\zeta = g - \frac{\xi^2}{4g} \tag{12}$$

where (ξ, ζ) are cartesian coordinates of a point on the surface of contour. Now by using (3)

$$\sin\psi = \frac{\xi}{\sqrt{\xi^2 + 4g^2}} \tag{13a}$$

$$\cos\psi = \frac{2g}{\sqrt{4g^2 + \xi^2}} \tag{13b}$$

$$\tan\psi = \frac{\xi}{2g} \tag{13c}$$

Using (13) in (7) Jacobian is determined as follow

$$J_{0LL} = 1 - \tau\frac{\cos^2\psi}{g}$$

Location of the caustic is given by

$$\tau = \frac{g}{\cos^2\psi} \tag{14}$$

and the GO field for the reflected ray can be obtained from (8) by using above result for J_{0LL}. Finite field around caustic may be obtained from expression (9) for Maslov's method using

$$J_{0LL}\frac{\partial P_{zLL}}{\partial z} = \sin^2 2\psi\frac{\cos^2\psi}{g}$$

3.2. Interface Having Circular Cylindrical Contour

Consider a cylindrical interface with counter

$$\zeta = \sqrt{a^2 - \xi 2}$$

where a is the radius of cylinder. The GO reflected field and field using Maslov's method for circular cylindrical chiral nihility-chiral nihility interface may be obtained similarly as done in the last section with following

$$J_{0LL} = 1 - 2\frac{\tau}{a\cos\psi}$$

and

$$J_{0LL}\frac{\partial P_{zLL}}{\partial z} = 2\frac{\sin^2 2\psi}{a\cos\psi}$$

4. CONCLUSIONS

Reflection of plane wave from a chiral nihility-chiral nihility planar interface is determined. Chiral nihility-chiral nihility planar interface has been achieved as a special case of two chiral media by assuming permittivities and permeabilities of both media tending to zero. Two reflected waves are produced, that is, co-polarized and cross-polarized reflected waves. Cross polarized reflected wave is a backward wave and propagates parallel to the incident plane wave whereas co-polarized plane wave reflects by making same angle with the normal as the incident wave. These results are utilized to determine the GO fields due to a cylindrical interface and it is found that only co polarized component contributes at focus. GO fields contain singularity at focus and uniform field expression, also valid at caustic, is determined by using Maslov's method.

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