Progress In Electromagnetics Research C, Vol. 77, 195–203, 2017

Bagley Power Divider with Uniform Transmission Lines for Arbitrary Power Ratio and Terminated in Different Impedances

Young-Chul Yoon[1] and Young Kim[2, *]

Abstract—This paper presents an unequal-split Bagley power divider that consists of uniform transmission lines and is terminated in different impedances. This divider should be only adjusted by altering the length of the transmission lines. Such alteration of the transmission lines will result in an arbitrary power ratio between output ports. The Bagley divider consists of uniform transmission lines of same characteristic impedance value despite different impedances for input and output port termination. For analysis, two Bagley dividers are considered, one 3-way and one 5-way divider, both with arbitrary power ratio and arbitrary termination impedances. A good agreement can be observed between the simulated and measured results.

1. INTRODUCTION

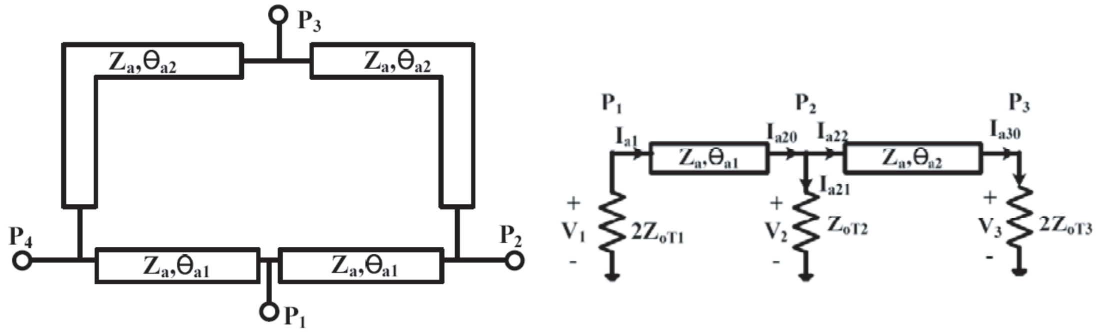
In the modern wireless industry, many types of power dividers of equal or unequal-split have been used in Doherty power amplifiers [1], phase array antennas [2] and mixers [3]. Typically unequal-split dividers use a high impedance and a low impedance lines between input and output ports to split the signal asymmetrically. To implement a divider with a high split power ratio, high impedance lines were required. Because of the narrow width of the high impedance lines, the realization of transmission lines in microstrip technology is difficult. Realization methods of high impedance lines such as defected ground structure (DGS) [4], shorted coupled lines [5], electromagnetic bandgap (EBG) [6] and grooved substrate [7] have been researched.

Recently, new methods for divider design have been introduced [8–14]. Instead of using high impedance lines to realize a divider of high split power ratio, uniform transmission lines of varying length are used. To obtain a high split power ratio from the divider, the electrical lengths between the input and output ports only need to be varied.

In a conventional Bagley divider with equal-split power division [15], the electrical lengths between the adjacent input and output ports are equal to 90°, whereas the electrical lengths between adjacent output ports are equal to 180°. Many researchers for Bagley divider have performed various studies such as arbitrary transmission line lengths between output port [16], dual-band operation using T and π networks [17], nonlinear transmission lines used [18], dual-band unequal split [19], and implementation of unequal split divider [20]. When the Bagley divider uses active element and/or passive elements, matching networks are needed to obtain the desired output performance. If the Bagley divider utilizes uniform transmission lines terminated in arbitrary impedances, no matching circuit is required. For this method, the fabrication is easy, and the total size of the circuits is reduced.

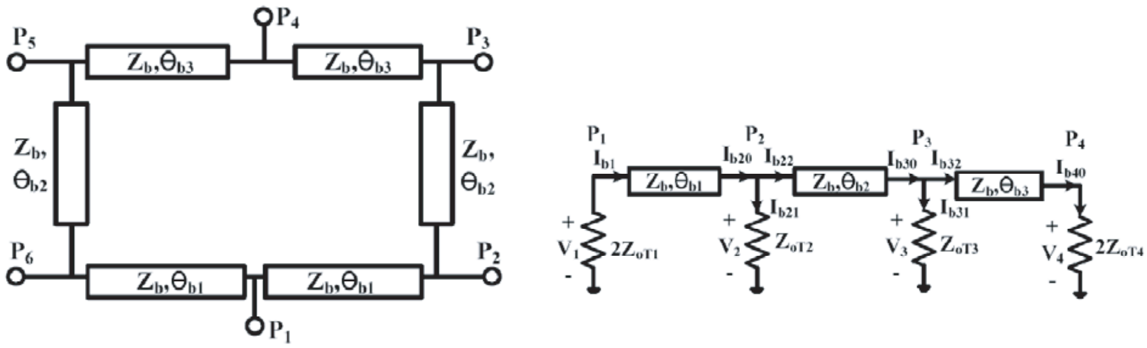
In this paper, we propose a 3-way and a 5-way Bagley power dividers with uniform transmission lines for arbitrary power ratio and terminated in different impedances. The Bagley divider consists of

Received 11 May 2017, Accepted 3 September 2017, Scheduled 11 September 2017
* Corresponding author: Young Kim (youngk63@gmail.com).
[1] Department of Electronics Engineering, Catholic Kwandong University, 522, Naegok-dong, Gangneung-shi, Gangwon-do, Korea. [2] School of Electronic Engineering, Kumoh National Institute of Technology 61 Daehak-ro, Gumi, Gyungbuk, Korea.



(a) (b)

Figure 1. 3-way Bagley power divider. (a) Configuration. (b) Equivalent circuits.



(a) (b)

Figure 2. 5-way Bagley power divider. (a) Configuration. (b) Equivalent circuits.

uniform transmission lines of different electrical lengths to realize arbitrary power ratio and different termination impedances.

Figures 1 and 2 show the configurations of the 3-way and 5-way Bagley dividers and their equivalent circuits.

2. THEORY AND DESIGN

Figure 1 shows the configuration of a 3-way Bagley divider and the equivalent circuits for this 3-way Bagley divider looking from port 1. The transmission line impedances are Z_a, and the electrical lengths are θ_{a1} and θ_{a2}. Each port's impedances are Z_{oT1}, Z_{oT2}, and Z_{oT3}.

To derive the design equations, we apply the transmission line theory and KCL at the ports.

Applying KCL at port 1 gives:

$$I_{a1} = \frac{1}{2} Y_{oT1} V_1 \tag{1}$$

Equation (1) takes into account the matching conditions ($S_{11} = 0$) at the input port.

In addition, we can derive the equations between port 1 and port 2:

$$\begin{pmatrix} V_1 \\ I_{a1} \end{pmatrix} = \begin{pmatrix} \cos\theta_{a1} & jZ_a \sin\theta_{a1} \\ jZ_a \sin\theta_{a1} & \cos\theta_{a1} \end{pmatrix} \begin{pmatrix} V_2 \\ I_{a20} \end{pmatrix} \tag{2}$$

Applying KCL at port 2 gives:

$$I_{a20} = I_{a21} + I_{a22} \tag{3}$$

where

$$I_{a21} = Y_{oT2} V_2 \tag{4}$$

In addition, we can derive the equations between port 2 and port 3:

$$\begin{pmatrix} V_2 \\ I_{a22} \end{pmatrix} = \begin{pmatrix} \cos\theta_{a2} & jZ_a \sin\theta_{a2} \\ jZ_a \sin\theta_{a2} & \cos\theta_{a2} \end{pmatrix} \begin{pmatrix} V_3 \\ I_{a30} \end{pmatrix} \tag{5}$$

Applying KCL at port 3 gives:

$$I_{a30} = \frac{1}{2} Y_{oT3} V_3 \tag{6}$$

Using Equations (1) $\sim$ (6), the insertion loss and isolation equations are derived as:

$$S_{21} = \frac{V_2}{V_1} = -j\frac{Z_a}{2Z_{oT1}} \sin\theta_{a1} + \cos\theta_{a1} \tag{7}$$

$$S_{31} = \frac{V_3}{V_1} = \frac{\left(\frac{Z_a}{Z_{oT2}} - \frac{Z_a}{2Z_{oT1}}\right)\cos\theta_{a1} + j\left(1 - \frac{Z_a^2}{2Z_{oT1}Z_{oT2}}\right)\sin\theta_{a1}}{\frac{Z_a}{2Z_{oT3}}\cos\theta_{a2} - j\sin\theta_{a2}} \tag{8}$$

$$S_{32} = \frac{V_3}{V_2} = \frac{\left(\frac{1}{Z_{oT2}} - \frac{1}{2Z_{oT1}}\right)\cos\theta_{a1} + j\left(\frac{1}{Z_a} - \frac{Z_a}{2Z_{oT1}Z_{oT2}}\right)\sin\theta_{a1}}{-\frac{\cos\theta_{1a}\cos\theta_{3a}}{2Z_{oT3}} - \frac{\sin\theta_{1a}\sin\theta_{2a}}{2Z_{oT1}} + j\left(\frac{\sin\theta_{1a}\cos\theta_{3a}}{4Z_{oT1}Z_{oT3}} - \frac{\cos\theta_{1a}\sin\theta_{2a}}{Z_a}\right)} \tag{9}$$

For a unit incident power at port 1, the output power ratios are given as:

$$P_2 : P_3 : P_4 = M_a : N_a : M_a \tag{10}$$

$$P_2 = P_4 = |S_{21}|^2 = \cos^2\theta_{a1} + K_a^2 \sin^2\theta_{a1} = M_a \tag{11}$$

$$P_3 = |S_{31}|^2 = \frac{A_a^2\cos^2\theta_{a1} + B_a^2\sin^2\theta_{a1}}{J_a^2\cos^2\theta_{a2} + \sin^2\theta_{a2}} = N_a \tag{12}$$

where

$$K_a = \frac{Z_a}{2Z_{oT1}},\ J_a = \frac{Z_a}{2Z_{oT3}},\ A_a = \frac{Z_a}{Z_{oT2}} - \frac{Z_a}{2Z_{oT1}},\ B_a = 1 - \frac{Z_a^2}{2Z_{oT1}Z_{oT2}} \tag{13}$$

If we define the dividing power ratio between ports, we can calculate the required electrical lengths of the divider branch on the basis of Equations (11) and (12). The corresponding equations are as follows:

$$\theta_{a1} = \sin^{-1}\left(\sqrt{\frac{1-M_a}{1-K_a^2}}\right) \tag{14}$$

$$\theta_{a2} = \sin^{-1}\left(\sqrt{\frac{X_a/N_a - J_a^2}{1-J_a^2}}\right) \tag{15}$$

where

$$X_a = A_a^2\cos^2\theta_{a1} + B_a^2\sin^2\theta_{a1} \tag{16}$$

Figure 2 shows the configuration of a 5-way Bagley divider and the equivalent circuit looking from port 1. The transmission line impedances are Z_b, and electrical lengths are θ_{b1}, θ_{b2}, and θ_{b3}. Each port's impedances are Z_{oT1}, Z_{oT2}, Z_{oT3}, and Z_{oT4}.

We apply the same procedure as that of the 3-way divider design method. The following design equations can be derived:

$$S_{21} = \frac{V_2}{V_1} = -j\frac{Z_b}{2Z_{oT1}} \sin\theta_{b1} + \cos\theta_{b1} \tag{17}$$

$$S_{31} = \frac{V_3}{V_1} = \cos\theta_{b1}\cos\theta_{b2} - \sin\theta_{b2}\sin\theta_{b1}\left(1 - \frac{Z_b^2}{2Z_{oT1}Z_{oT2}}\right) + j\left[\left(\frac{Z_b}{Z_{oT2}} - \frac{Z_b}{2Z_{oT1}}\right)\cos\theta_{b1}\sin\theta_{b2} - \frac{Z_b}{2Z_{oT1}}\cos\theta_{b2}\sin\theta_{b1}\right] \tag{18}$$

$$S_{41} = \frac{V_4}{V_1} = \frac{-j\frac{1}{\sin\theta_{b1}}\cdot S_{21} + \left(j\frac{\cos\theta_{b2}}{\sin\theta_{b2}} - \frac{Z_b}{Z_{oT3}}\right)\cdot S_{31}}{\frac{Z_b}{2Z_{oT4}}\cos\theta_{b3} + j\sin\theta_{b3}} \tag{19}$$

$$S_{32} = \frac{V_3}{V_2} = \frac{\left(\begin{array}{l}\cos\theta_{b1}\cos\theta_{b2} - \sin\theta_{b2}\sin\theta_{b1}\left(1 - \frac{Z_b^2}{2Z_{oT1}Z_{oT2}}\right) \\ +j\left[\left(\frac{Z_b}{Z_{oT2}} - \frac{Z_b}{2Z_{oT1}}\right)\cos\theta_{b1}\sin\theta_{b2} - \frac{Z_b}{2Z_{oT1}}\cos\theta_{b2}\sin\theta_{b1}\right]\end{array}\right)}{\cos\theta_{b1} - j\frac{Z_b}{2Z_{oT1}}\sin\theta_{b1}} \tag{20}$$

$$S_{42} = \frac{V_4}{V_2} = \frac{-j\frac{1}{\sin\theta_{b1}}\cdot S_{21} + \left(j\frac{\cos\theta_{b2}}{\sin\theta_{b2}} - \frac{Z_b}{Z_{oT3}}\right)\cdot S_{31}}{\left(\begin{array}{l}\left(\frac{Z_b}{2Z_{oT4}}\cos\theta_{b3}\cos\theta_{b1} + \frac{Z_b}{2Z_{oT1}}\sin\theta_{b3}\sin\theta_{b1}\right) \\ +j\left(\sin\theta_{b3}\cos\theta_{b1} - \frac{Z_b^2}{4Z_{oT1}Z_{oT4}}\cos\theta_{b3}\sin\theta_{b1}\right)\end{array}\right)} \tag{21}$$

$$S_{43} = \frac{V_4}{V_3} = \frac{-j\frac{1}{\sin\theta_{b1}}\cdot S_{21} + \left(j\frac{\cos\theta_{b2}}{\sin\theta_{b2}} - \frac{Z_b}{Z_{oT3}}\right)\cdot S_{31}}{\left(\frac{Z_b}{2Z_{oT4}}\cos\theta_{b3} + j\sin\theta_{b3}\right)\cdot S_{31}} \tag{22}$$

If the power dividing ratios of the proposed 5-way divider are $P_2 : P_3 : P_4 : P_5 : P_6 = M_b : N_b : L_b : N_b : M_b$, we calculate the electrical lengths of the divider branch based on the S-parameter equations (17) $\sim$ (19). The corresponding equations are as follows:

$$\theta_{b1} = \sin^{-1}\left(\sqrt{\frac{1-M_b}{1-A_1^2}}\right) \tag{23}$$

$$\theta_{b2} = \cot^{-1}\left(\frac{-X_2 - \sqrt{X_2^2 - X_1X_3}}{X_1}\right) \tag{24}$$

$$\theta_{b3} = \sin^{-1}\left(\sqrt{\frac{F_b - A_3^2}{1-A_3^2}}\right) \tag{25}$$

where

$$A_1 = \frac{Z_b}{2Z_{oT1}},\ A_3 = \frac{Z_b}{2Z_{oT4}},\ X_1 = B_1^2 + D_1^2 - N_b,\ X_2 = E_1B_1 + C_1D_1$$

$$X_3 = E_1^2 + C_1^2 - N_b,\ B_1 = \cos\theta_{b1},\ C_1 = \left(\frac{Z_b}{Z_{oT2}} - \frac{Z_b}{2Z_{oT1}}\right)\cos\theta_{b1}$$

$$D_1 = -\frac{Z_b}{2Z_{oT1}}\sin\theta_{b1},\ E_1 = \left(-1+\frac{Z_b^2}{2Z_{oT1}Z_{oT2}}\right)\sin\theta_{b1},\ F_b = \frac{\left(\frac{Z_b}{Z_{oT3}}S_{31}\right)^2 + \left(\frac{\cos\theta_{b2}}{\sin\theta_{b2}}S_{31} - \frac{1}{\sin\theta_{b1}}S_{21}\right)^2}{L_b}$$

In Equations (14), (15), and (23) $\sim$ (25), if the calculated electrical lengths are negative, the fixed results should be $+360^\circ$, and the transmission line with electrical length θ more than 180° is functionally equal to $\theta - 180^\circ$

3. SIMULATION AND EXPERIMENTAL RESULTS

For verification, we design and simulate a 3-way 1 : 2 : 1 and a 5-way 1 : 2 : 4 : 2 : 1 Bagley dividers at an operating frequency of 2 GHz, with a Teflon substrate of a dielectric constant of 2.5, thickness of 0.787 mm and conductor thickness of 0.035 mm. The simulation is carried out using the Microwave Office software developed by National Instruments.

For a 1 : 2 : 1 power ratio between ports of the 3-way Bagley divider, when the impedance of the transmission line is 40 Ω, and ports 1, 2, and 3 impedances are changed from 40 Ω to 80 Ω, the S-parameters of divider are verified. Figure 3 shows the characteristics of the divider when port 1 impedance varies from 40 Ω to 80 Ω under the condition that ports 2 and 3 impedances are fixed at 60 Ω and 70 Ω. The characteristics of the divider are confirmed under the condition that ports 2 and 3 impedances are varied in the same manner. Using this verification method, we set the port impedances at $Z_{oT1} = 50\,\Omega$, $Z_{oT2} = 60\,\Omega$, $Z_{oT3} = 70\,\Omega$ for best performance. The characteristics of the divider using the conditions previously determined, when the uniform transmission line impedance value varies from 30 Ω to 70 Ω are shown in Figure 4. We select $Z_a = 40\,\Omega$ as the condition that satisfies the best characteristics. According to the split power ratio, the calculated parameters are $M_a = \frac{1}{4}$, $N_a = \frac{1}{2}$. The calculated electrical lengths using Equations (14) and (15) are $\theta_{a1} = 114°$ and $\theta_{a2} = 52.1°$.

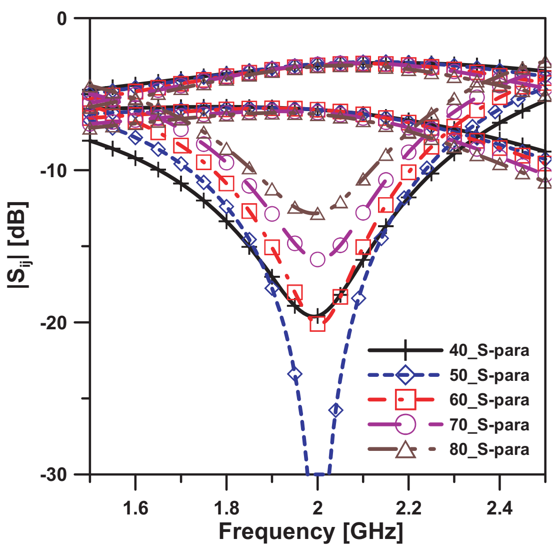


Figure 3. S-parameters when port 1 impedance changes from 40 Ω to 80 Ω. (1 : 2 : 1 3-way, $Z_o = 40\,\Omega$, port 2/3 impedances are 60 Ω, 70 Ω).

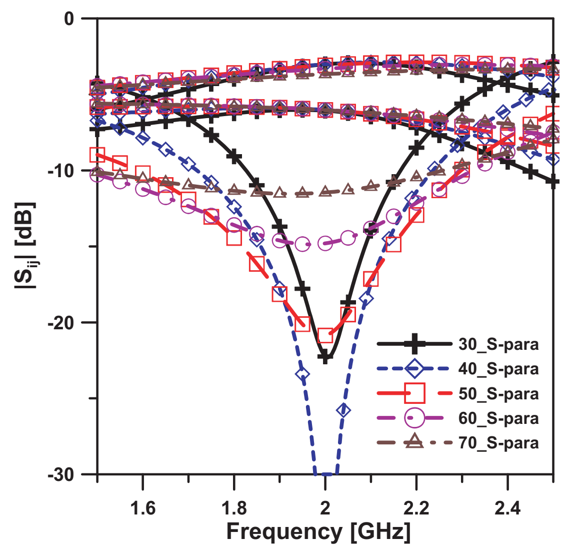


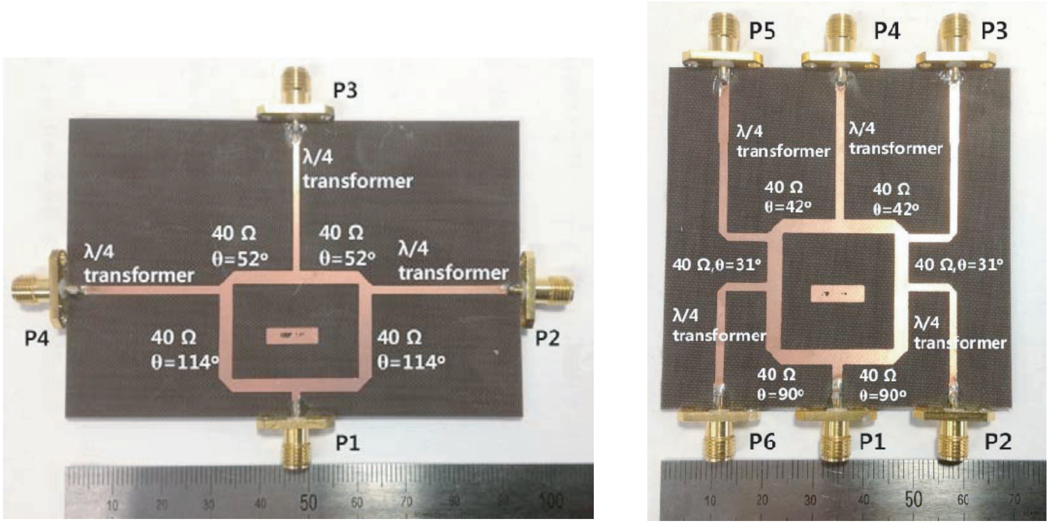
Figure 4. S-parameters when characteristics impedance changes from 30 Ω to 70 Ω. (1 : 2 : 1 3-way, port 1/2/3 impedances are 50 Ω, 60 Ω, 70 Ω).

To design a 5-way 1 : 2 : 4 : 2 : 1 divider, through the same process as the design method of a 3-way Bagley divider, we set the port impedances at $Z_{oT1} = 50\,\Omega$, $Z_{oT2} = 75\,\Omega$, $Z_{oT3} = 75\,\Omega$ and $Z_{oT4} = 60\,\Omega$ and uniform transmission line impedance values of $Z_b = 40\,\Omega$. According to the power ratio, we calculate $M_b = \frac{1}{10}$, $N_b = \frac{1}{5}$, $L_b = \frac{2}{5}$. The calculated electrical lengths using Equations (23) $\sim$ (25) are $\theta_{b1} = 90°$, $\theta_{b2} = 31.11°$ and $\theta_{b3} = 42.13°$.

To verify the characteristics of the impedance transformer for 50 Ω matching, the impedance transformers are connected back-to-back, and the losses of 0.04 dB are measured.

Figure 5(a) depicts a photograph of the 3-way 1 : 2 : 1 Bagley divider terminated in different impedances. Compared with the conventional Bagley divider, the proposed Bagley divider has a compact size of 61.5% electrical length reduction between $\theta_{\text{proposed}} = 332°$ and $\theta_{\text{conventional}} = 540°$.

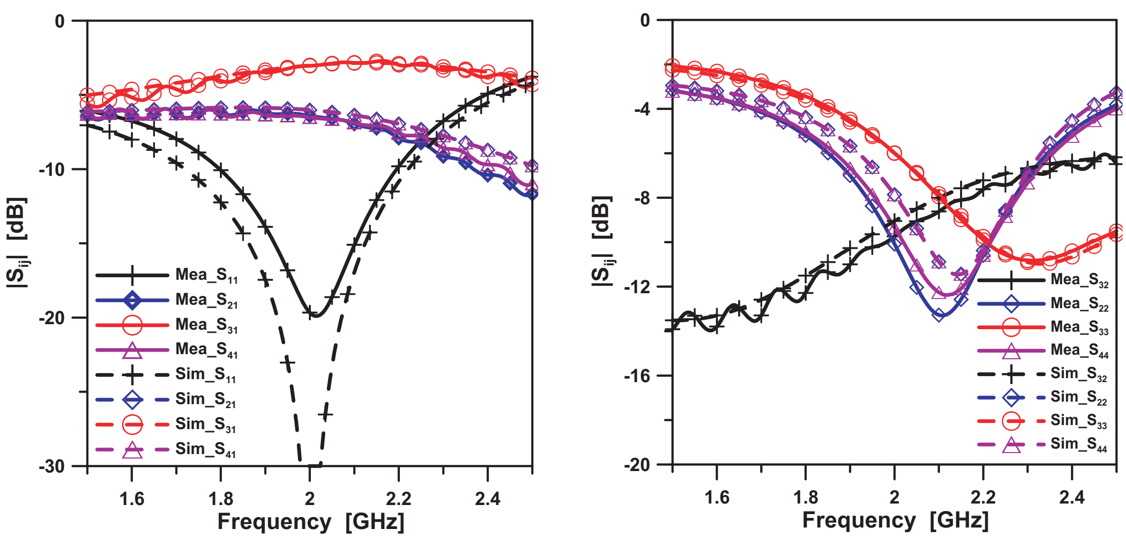
Figure 5(b) shows a photograph of the 5-way 1 : 2 : 4 : 2 : 1 Bagley divider with 50 Ω, 75 Ω, 75 Ω, 60 Ω terminations. The size of the proposed Bagley divider is reduced by 36.3% compared with a conventional divider.



(a) (b)

Figure 5. Photograph of the implemented Bagley divider. (a) 1 : 2 : 1 3-way. (b) 1 : 2 : 4 : 2 : 1 5-way.

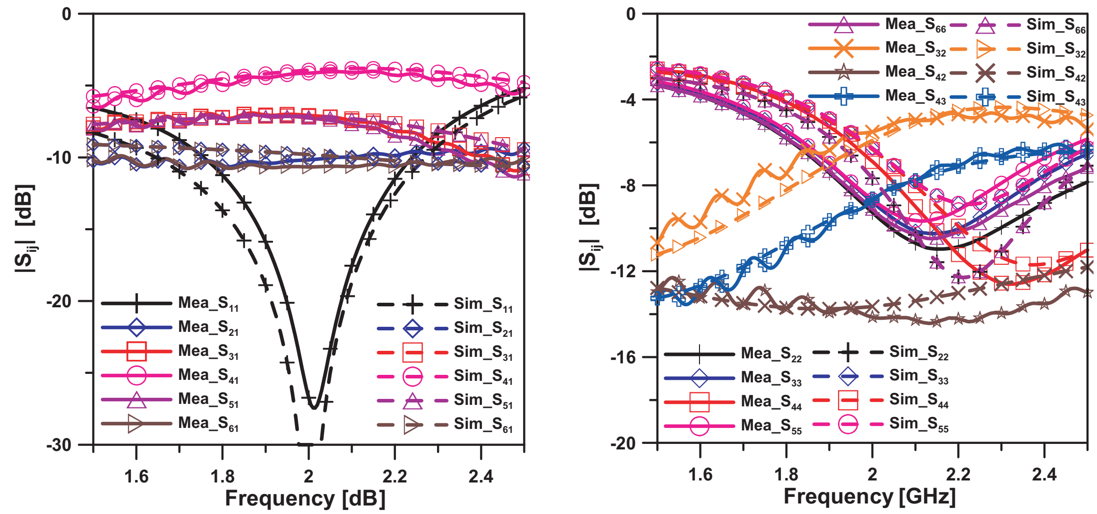
Figure 6 shows the measured and simulated S-parameters of the proposed 3-way Bagley divider, which have insertion losses of 6.4 dB and 3.1 dB, and input return losses of 19.7 dB at a center frequency of 2 GHz. In addition, the S-parameters of P_2, P_3 and P_4 have output return losses of 10 dB, 6 dB, and 9.4 dB, and the isolation loss between the output ports is 9.7 dB.



(a) (b)

Figure 6. Measured and simulated S-parameters of the 1 : 2 : 1 3-way Bagley divider. (a) $|S_{11}|$, $|S_{21}|$, $|S_{31}|$, $|S_{41}|$, (b) $|S_{22}|$, $|S_{33}|$, $|S_{44}|$, $|S_{32}|$.

Figure 7 represents the measured and simulated S-parameters of the proposed 5-way Bagley divider, which have insertion losses of 10.2 dB, 7.1 dB, 4.1 dB, 7.2 dB, and 10.6 dB at ports P_2, P_3, P_4, P_5 and P_6, respectively, as well as input return losses of 26.7 dB at a center frequency of 2 GHz. In addition, the S-parameters of P_2, P_3, P_4, P_5 and P_6 have output return losses of 9.3 dB, 8.8 dB, 6.9 dB, 8.7 dB, 9.1 dB, and the isolation loss between the output ports is 14 dB. The waviness of the measured transmission coefficients shown in Figures 6 and 7 appears because of the phase difference of the reach path of the output port for the signal flow, and the reflection coefficients of output port are large.



(a) (b)

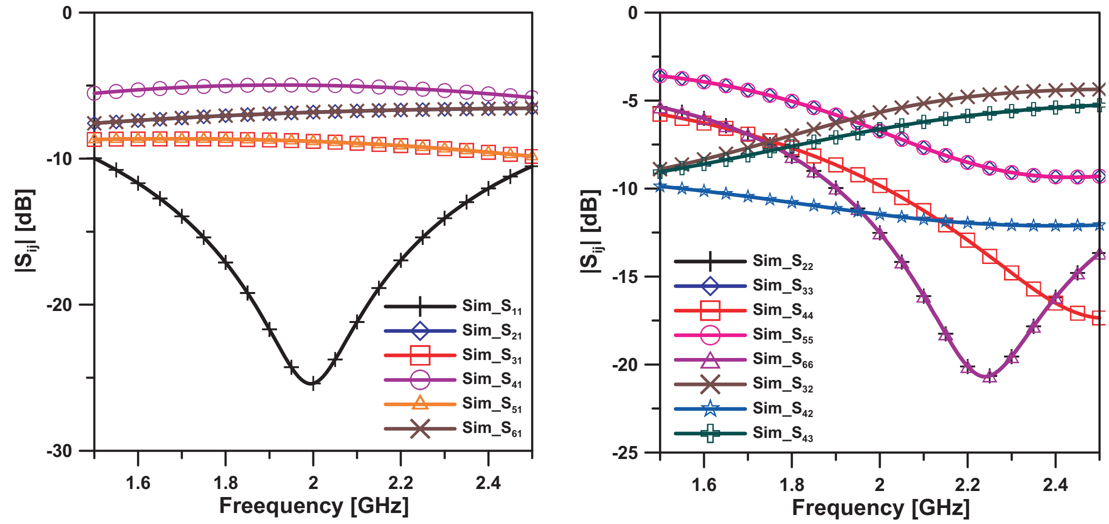
Figure 7. Measured and simulated S-parameters of the $1:2:4:2:1$ 5-way Bagley divider. (a) $|S_{11}|$, $|S_{21}|$, $|S_{31}|$, $|S_{41}|$, $|S_{51}|$, $|S_{61}|$, (b) $|S_{22}|$, $|S_{33}|$, $|S_{44}|$, $|S_{55}|$, $|S_{66}|$, $|S_{32}|$, $|S_{42}|$, $|S_{43}|$.

Figure 8 depicts the simulated S-parameters of the $2:3:5:3:2$ 5-way Bagley divider with the port impedances $Z_{oT1} = 50\,\Omega$, $Z_{oT2} = 60\,\Omega$, $Z_{oT3} = 70\,\Omega$, and $Z_{oT4} = 40\,\Omega$ and uniform transmission line impedance values of $Z_b = 50\,\Omega$. This shows that the characteristics of the power divider can be satisfied even when the power ratio, port impedance, and characteristic impedance of the transmission line are different from each other.

Table 1 shows the comparison with other Bagley power dividers.

Table 1. Comparison with other Bagley power dividers.

Items	Power division	Frequency	Electrical length (without output matching transformer)	Transmission line impedances (Ω)	Port impedances (Ω)
[16]	equal 3/5-way	Single 1 GHz	270°	100/33	50
[17]	equal 3-way	Dual-band 0.5/1 GHz	120°/360°	69/48 33/50	50
[18]	equal 3-way	Single 1 GHz	201°	57.7 (Non-TL)	50
[19]	unequal 3/5-way	Dual-band 0.5/1 GHz	240°/360°	62.48/40 63/79.3	50
[20]	unequal 3/5-way	Single 1 GHz	360°/540°	50/70.7 37.8/28.3/141.4	50
This work	unequal 3/5-way	Single 2 GHz	332°/326°	40	50/60/70/75



(a) (b)

Figure 8. Simulated S-parameters of the 2 : 3 : 5 : 3 : 2 : 5-way Bagley divider. (a) $|S_{11}|$, $|S_{21}|$, $|S_{31}|$, $|S_{41}|$, $|S_{51}|$, $|S_{61}|$, (b) $|S_{22}|$, $|S_{33}|$, $|S_{44}|$, $|S_{55}|$, $|S_{66}|$, $|S_{32}|$, $|S_{42}|$, $|S_{43}|$.

4. CONCLUSION

This paper presents an unequal-split Bagley power divider with uniform transmission lines for arbitrary power ratio and terminated in different impedances. In order to obtain the Bagley divider with an arbitrary power ratio and different impedance termination conditions, we use a uniform impedance transmission line and adjust only the electrical lengths between the ports. This design method allows the removal of the impedance transformer for port matching and reduces overall circuit size. The proposed Bagley divider exhibits good impedance matching and insertion losses at an operating frequency of 2 GHz.

ACKNOWLEDGMENT

This paper was supported by Research Fund, Kumoh National Institute of Technology.

REFERENCES

1. Kim, J. H., B. Fehri, S. Boumaiza, and H. Wood, "Power efficiency and linearity enhancement using optimized asymmetrical Doherty power amplifiers," *IEEE Trans. Microwave Theory & Tech.*, Vol. 59, No. 2, 425–434, Feb. 2011.
2. Hansen, R. C., *Phase Array Antennas*, John Wiley & Sons, Hoboken, New Jersey, 2009.
3. Au-Yeung, C.-F. and K.-K. M. Cheng, "IMS reduction in CMOS double-balanced mixer using multibias dual-gate transistor," *IEEE Trans. Microwave Theory & Tech.*, Vol. 54, No. 1, 4–9, Jan. 2006.
4. Lim, J. S., S. W. Lee, C. S. Kim, J. S. Park, D. Ahn, and S. W. Nam, "A 4 : 1 unequal Wilkinson power divider," *IEEE Microw. Wireless Compon. Lett.*, Vol. 11, No. 3, 124–126, Mar. 2001.
5. Chen, Y.-T., P.-H. Deng, B.-L. Chen, W. Lo, and T.-H. Hsieh, "Design of a microstrip out-of-phase power divider using shorted-coupled-line transformer," *2015 Asia-pacific Microwave Conference*, Nanjing, China, Dec. 2015.
6. Ko, Y. J., J. Y. Park, and J. U. Bu, "Fully integrated unequal Wilkinson power divider with EBG CPW," *IEEE Microw. Wireless Compon. Lett.*, Vol. 13, No. 7, 276–278, Jul. 2003.

7. Moradian, M. and H. Oaraizi, "Application of grooved substrates for design of unequal Wilkinson power dividers," *IET Electron. Lett.*, Vol. 44, No. 1, 32–33, Jan. 2008.
8. Horst, S., R. Bairavasubramanian, M. M. Tentzeris, and J. Papapolymerou, "Modified Wilkinson power dividers for millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory* & *Tech.*, Vol. 55, No. 11, 2439–2446, Nov. 2007.
9. Zhu, Y.-Z., X.-J. Zhang, X.-F. Wu, C. Li, and G.-Y. Fang, "Novel Wilkinson power divider with uniform impedance line," *2008 Asia-pacific Microwave Conference*, Macau, China, Dec. 2008.
10. Cheng, K.-K. M. and P.-W. Li, "A novel power divider design with unequal power diving ratio and simple layout," *IEEE Trans. Microwave Theory* & *Tech.*, Vol. 57, No. 6, 1589–1594, Jun. 2009.
11. Qi, T., S. He, Z. Dai, and W. Shi, "Novel unequal power divider with $50\,\Omega$ characteristic impedance lines," *IEEE Microw. Wireless Compon. Lett.*, Vol. 26, No. 3, 180–182, Mar. 2016.
12. Park, M.-J. and B.-J. Lee, "Design of ring couplers for arbitrary power division with $50\,\Omega$ lines," *IEEE Microw. Wireless Compon. Lett.*, Vol. 21, No. 4, 185–187, Apr. 2011.
13. Feng, L., Q.-X. Chu, Z. Gong, and Z. Lin, "Compact broadband Gysel power divider with arbitrary power dividing ratio using microstrip/slotline phase inverter," *IEEE Trans. Microwave Theory* & *Tech.*, Vol. 60, No. 5, 1226–1234, May 2012.
14. Yoon, Y.-C. and Y. Kim, "Unequal Bagley power divider using uniform transmission lines," *Progress In Electromagnetics Research Symposium*, Spring (*PIERS*), 1184–1187, St Petersburg, Russia, May 22–25, 2017.
15. Burberry, R. A., *VHF and UHF Antennas*, Peter Peregrinus Ltd., London, 1992.
16. Sakagami, I., T. Wuren, M. Fujii, and M. Tahara, "Compact multi-way power dividers similar to the Bagley polygon," *IEEE Int. Microwave Symposium*, 419–422, Honolulu, Jun. 2007.
17. Qaroot, A., K. Shamaileh, and N. Dib, "Design and analysis of dual-frequency modified 3-way Bagley power dividers," *Progress In Electromagnetics Research C*, Vol. 20, 67–81, 2011.
18. Shamaileh, K., A. Qaroot, and N. Dib, "Design of miniaturized 3-way Bagley polygon power divider using non-uniform transmission lines," *Antennas and Propagation Society International Symposium (APSURSI)*, Spokane, Jul. 2011.
19. Omar, A.-A. and N. Dib, "Design and analysis of dual-frequency unequal split Bagley power dividers," *Antennas and Propagation Society International Symposium (APSURSI)*, Chicago, Jul. 2012.
20. Omar, A.-A., N. Dib, K. A. Shamaileh, and A. Sheta, "Design and analysis of unequal split Bagley power dividers," *International Journal of Electronics*, Vol. 102, No. 3, 500–513, 2015.