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Design and Synthesis of Dual-Band Microwave Bandpass Filter Based on Hybrid Polynomials

Elden Zee* and Peng Wen Wong

Abstract—This paper presents a method of designing a bandpass filter using hybrid polynomials. Two different approaches are discussed in this paper. The first method uses class hybridization of a lowpass Chebyshev and highpass maximally flat to achieve the hybrid filtering function (HFF). The second method employs both Chebyshev polynomials of the first and second kinds to form a modified Chebyshev polynomial. Both methods achieve a narrowband dual-band lowpass prototype (DBLPP) without much deviation from classical methods of synthesis. The designs can be adapted into a modified interdigital prototype which will be shown in this paper. The results and measurements reflect a good adherence to the theoretical calculations.

1. INTRODUCTION

In recent years, the mobile telecommunication world has experienced a boom in terms of its global traffic. This has pushed the advancement of filter technology through emerging technologies. One concept of increased importance recently is the capability to support the coexistence of multiple bandwidths. Due to its increased importance, multiband filter class has been the subject of researchers worldwide. Although many methods have been proposed by past researchers, the design challenge still persists [1–6].

The concept of hybrid polynomials revolves around the modification of the characteristic polynomial found in the transfer function to form a HFF. Since the characteristic polynomial defines the response of the overall filter design, by altering the characteristic polynomials, the dual-band response can be solved at the grassroots level, i.e., in the lowpass prototype (LPP) itself, allowing classical approaches for single-band to be utilized. In comparison to many 'dual-diplex' approaches [3, 7–10], there is also no need for impedance matching circuit which is a common issue found in cascading networks. Compared to design techniques such as the cascaded formation of two prototypes [11] to form a dual-band filter, the presented methods in this paper provide more analytical flexibility over the designing process. This is possible due to formation of the dual-band response resolved at the characteristic polynomials prior to the lowpass prototype which gives a better control over the designing process.

The first method presented here will form a transfer function through the product of two classes of transfer functions, one of lowpass and the other of highpass. The second method will form a modified Chebyshev polynomials from the Chebyshev polynomials I kind and Chebyshev II kind. Both methods will result in an HFF that can be implemented in a classical synthesis and frequency transformation process. In order to realize it in distributed elements, the dual-band lowpass prototype DBLPP is creatively modified using a method similar to that in [12] before submitting to Richard's transformation for the distributed DBBPF with center frequency of 1 GHz as a proof of concept. Both methods will form a narrowband modified interdigital prototype that is similar in design. The fabricated prototypes are tested, and the results agree with the concept presented here.

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* Corresponding author: Elden Zee Chien Pheng (elden.zee@gmail.com).
The authors are with the Department of Electrical and Electronic Engineering, Universiti Teknologi PETRONAS, Malaysia.

2. THEORY OF HFF THROUGH CLASS HYBRIDIZATION

Let's assume the lowpass function to be $L(p)$ and the highpass function to be $H(p)$. The multiplication of the two functions can be depicted simply as:

$$I(p) = L(p) * H(p) \tag{1}$$

which produces a dual-band response. The introduction of a highpass transfer function into the original lowpass function creates a notch at DC which will divide the single-band into a dual-band response, thus forming a dual-band response at its LPP.

Now a lowpass Chebyshev type I with transfer function:

$$|S_{21}|^2 = \frac{1}{1 + \varepsilon^2 T(\omega)_N^2} \tag{2}$$

where ε is the ripple level, and N is the filter order. A highpass maximally-flat transfer function of

$$|S_{21}|^2 = \frac{\omega^{2M}}{\omega^{2M} + (k\omega_0)^{2M}} \tag{3}$$

where k is the scaling factor, and M is the filter order. To adjust the bandwidth of the response, a scaling factor k is introduced with $0 < k < 1$. This will also adjust the bandwidth of the response. Also, with the lowpass prototype cutoff at 1 and the introduction of a scaling factor, it is possible to produce a narrowband response.

The concept described in this method provides big room for explorations in the possibilities and potentials of the types of hybrid. It is believed that the class hybridization will inherit any features found in the base classes, and thus by performing hybridization, the distinct characteristics of each class is imported into the dual-band responses.

3. DBLPP THROUGH CLASS HYBRIDIZATION

The HFF from the transfer functions gives us

$$|S_{21}|^2 = \frac{1}{1 + \varepsilon^2 T(\omega)_N^2} \times \frac{\omega^{2M}}{\omega^{2M} + (k\omega_0)^{2M}} \tag{4}$$

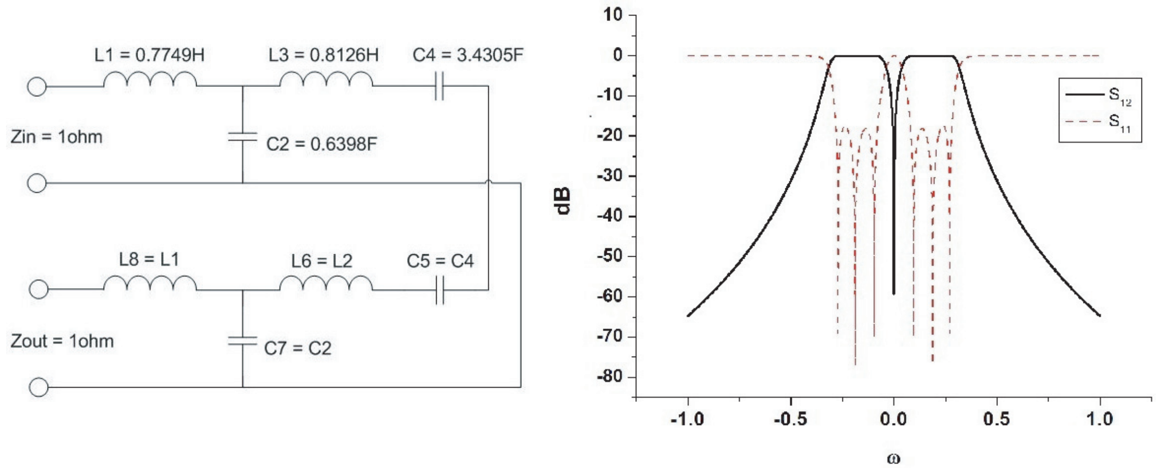
Let's set the order N for the lowpass at 3 and M for the highpass at 1. This gives a 4th order HFF. ω_0 is set at 1 rad/s with the highpass scaling factor of k at 0.1. ε from a return loss of 20 dB will be approximated to 0.01. Based on unitary condition and selecting only the left half poles, both S_{21} and S_{11} are obtained for our HFF. S_{21} is found to be

$$S_{21} = \frac{p}{\begin{matrix}(p + 1.1742)(p + 0.5871 - j1.335) \\ (p + 0.5871 + j1.3357)(p + 0.1)\end{matrix}} \tag{5}$$

where $p = j\omega$. From the unitary condition, S_{11} is found to be

$$S_{11} = \frac{\begin{matrix}(p + 0.4335 + j0.3108)(p + 0.1455 + j0.9260) \\ (p + 0.1455 - j0.9260)(p + 0.4335 - j0.3108)\end{matrix}}{\begin{matrix}(p + 1.17429)(p + 0.5871 - j1.3357) \\ (p + 0.5871 + j1.3357)(p + 0.1)\end{matrix}}. \tag{6}$$

From the transfer and reflection functions, the lumped ladder network can be synthesized through the classical continuous fraction method. In order to ease transformation into the distributed line realization, a symmetrical network is formed by adding a mirror circuit. Fig. 1 depicts the ladder network obtained and its simulated response after tuning. An inverter-coupled configuration is then implemented as shown in Fig. 2. Ideal inverters at $K = 1$ is used. By converting the design into its inverter-coupled configuration, it is now easier to adapt Richard's transformation for distributed element realization. Note the inverter and capacitor designated K33 and C33 respectively in the figure which is derived from capacitors C4 and C5. The capacitor C33 is the most sensitive element in the design because it produces the transmission zero at DC.



(a) (b)

Figure 1. (a) Ladder network DBLLP configuration; (b) Simulated DBLLP response.

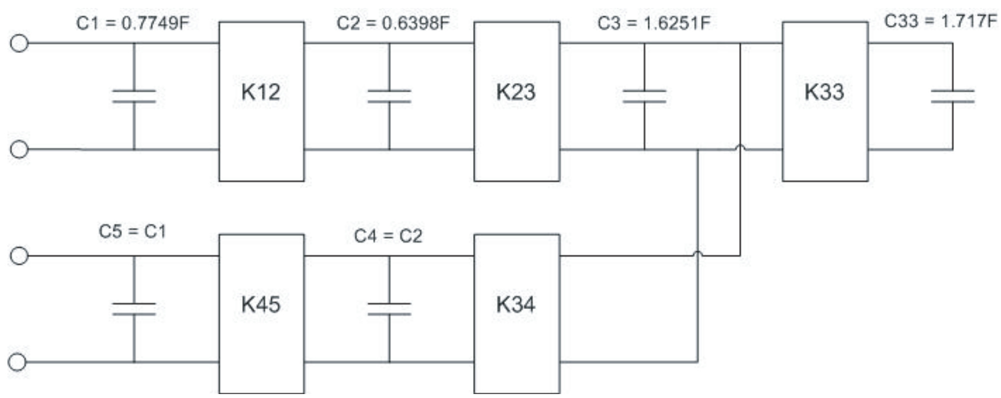


Figure 2. Inverter-coupled DBLPP configuration.

4. MODIFIED INTERDIGITAL DBBPF USING CLASS HYBRIDIZATION

From the inverter-coupled DBLPP, the prototype can be transformed into a modified interdigital DBBPF. To realize a narrowband DBBPF, the admittance scaling method as depicted in [13] is used. The scaled admittance matrix varies for the middle row and columns which results in modified design equations for coupling and line admittances.

$$[Y] = \frac{1}{t}\begin{bmatrix} 1 & -n_1\left(1-t^2\right)^{1/2} & 0 & \dots \\ -n_1\left(1-t^2\right)^{1/2} & n_1^2\left(1+C_1/\alpha\right) & -n_1n_2\left(1-t^2\right)^{1/2}K_{12} & \\ 0 & -n_1n_2\left(1-t^2\right)^{1/2}K_{12} & n_2^2\left(C_2/\alpha\right) & \\ \vdots & \vdots & & \ddots \end{bmatrix} \tag{7}$$

The existence of a central stub to the interdigital design means that slightly modified design equations are needed. Assuming all $K_{r,r+1}$ to be unity, the design equations are formed with reference from [13] and given as:

$$Y_{r,r+1} = n_r n_{r+1} \tag{8}$$
$$Y_0 = Y_{N+1} = 1 - n_1 \tag{9}$$
$$Y_{01} = Y_{N,N+1} = n_1 \tag{10}$$

$$\begin{aligned} Y_1 &= Y_N = 1 - n_1 - n_1 n_2 && (11) \\ Y_r &= 1 - n_{r-1} n_r K_{r-1,r} - n_r n_{r+1} && (12) \end{aligned}$$

where $r = 1$ to $N - 1$. Except

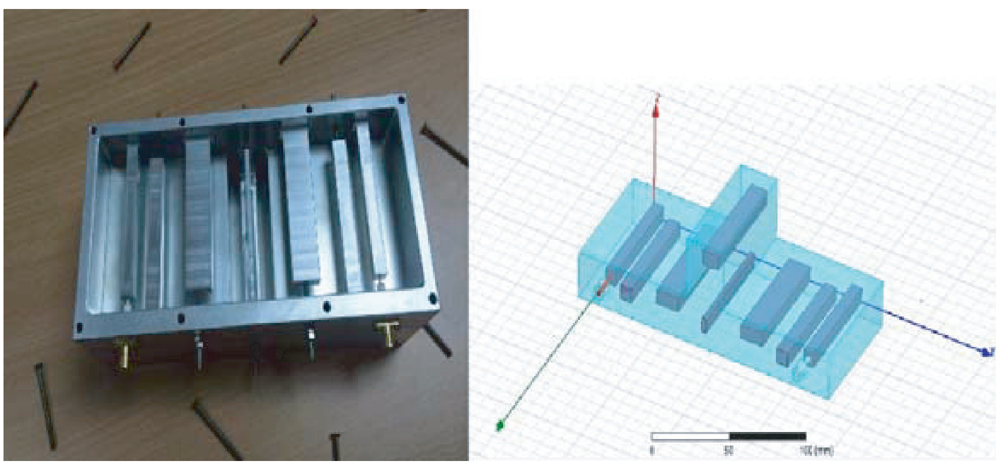
$$\begin{aligned} Y_s &= 1 - n_{s-1} n_s - n_s n_{s+1} - n_s n_{ss} && (13) \\ Y_{ss} &= 1 - n_s n_{ss} && (14) \\ Y_{s,ss} &= n_s n_{ss} && (15) \end{aligned}$$

where $s = \frac{N}{2}$. The scaling factors are given as

$$\alpha = \frac{1}{\tan(\theta_1)} \tag{16}$$

$$n_r = n_{ss} = \sqrt{\frac{\alpha}{C_r}}, \quad n_1 = n_N = \frac{1}{\sqrt{(1 + \frac{C_1}{\alpha})}} \tag{17}$$

The physical dimensions are derived from the even-mode fringing capacitance and coupling capacitance of coupled rectangular bars [13]. The modelling is done using HFSS software and shown in Fig. 3 and Fig. 4. Notice the modified interdigital design with a central stub in its symmetrical structure. The measured response from the prototype, shown in Fig. 5, gives a bandwidth of 35 MHz with an average of 13 dB return loss. The DBBPF is formed with the notch centered at 0.98 GHz.



(a) (b)

Figure 3. (a) Modified DBBPF interdigital filter prototype; (b) 3D HFSS DBBPF model.

5. THEORY OF HFF THROUGH MODIFIED CHEBYSHEV POLYNOMIALS

Let's look at the Chebyshev polynomials of the first kind. In a lowpass prototype, in order to form a dual-band response, a transmission zero needs to be introduced at $\omega = 0$. The modification is done to the Chebyshev polynomials which is given as [14]

$$T_N(\omega) = \frac{2\omega T_n(\omega) - T_{n-1}(\omega)}{\omega} \tag{18}$$

when n is odd. $N = n + 1$. It is important to note that the normalization is not applicable to $n =$ even because ω will be cancelled out due to common factor in both numerator and denominator. To resolve this and allow application when $n =$ even, ω^2 can be used as denominator instead. From Eq. (18) it is found that the amplitude increases or decreases rapidly towards infinity as ω approaches zero. It is always true that $T_N(\pm 1) = 1$ at the cutoff point.

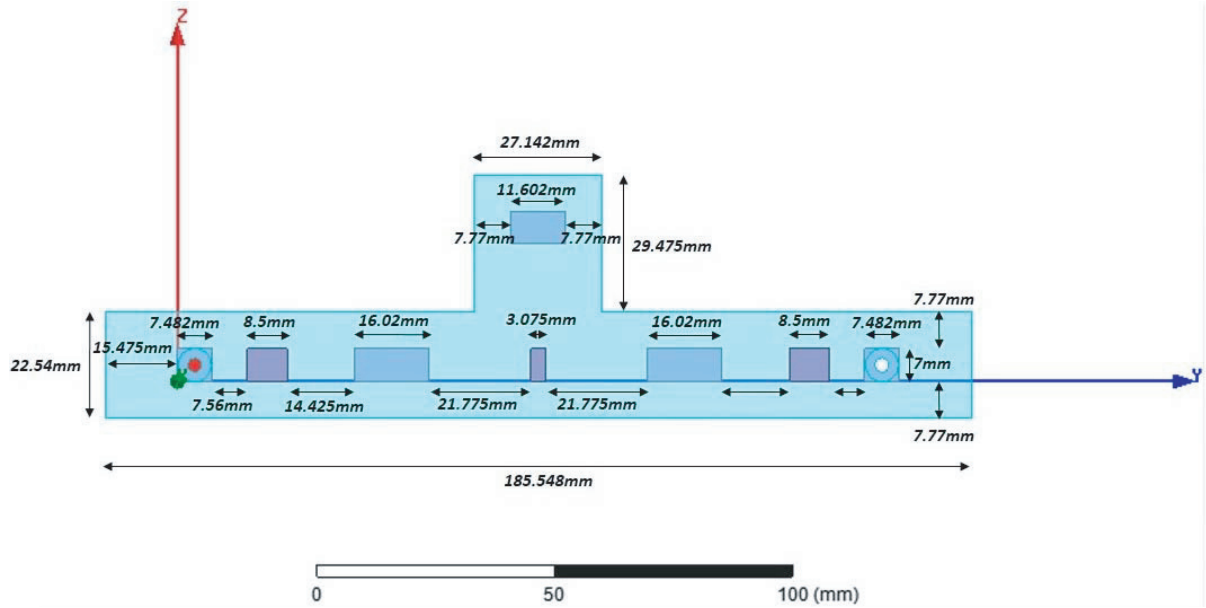


Figure 4. 2D HFSS DBBPF with measurements.

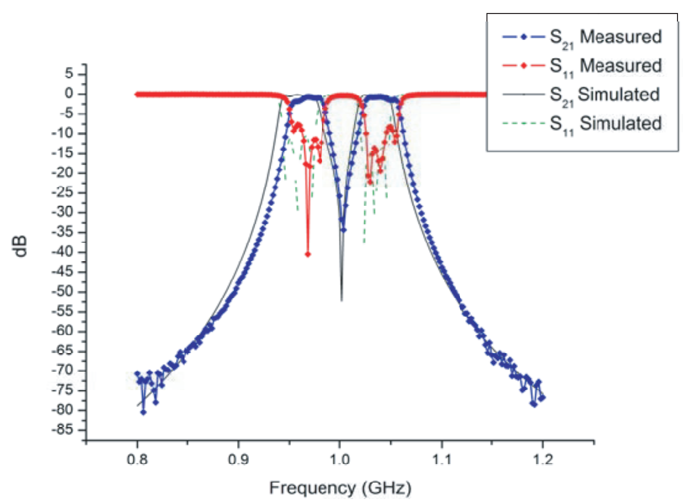


Figure 5. Comparison of simulated response and measured response.

The introduction of ω will distort the equiripple characteristic of the polynomials. Therefore, the Chebyshev polynomials of the second kind is introduced and merged with Eq. (18). This gives us the new filtering function as,

$$H_N^2(\omega) = \frac{T_N^2(\omega) + k_1 U_M^2(\omega)}{k_2} \tag{19}$$

where $T_N(\omega)$ and $U_M(\omega)$ are Chebyshev polynomials of the first and second kinds, respectively. M is the order of Chebyshev polynomial of the second kind and chosen to be same as N to maintain the overall order of the HFF. The filtering function can be simplified as

$$H_N^2(\omega) = \frac{T_N^2(\omega) + k_1 \left(2 \sum_{j\,even}^{M} T_j(\omega) - 1\right)^2}{k_2} \tag{20}$$

where k_1 is a constant. The value is chosen such that the condition of equiripple

$$H_N^2(\omega_c) = \frac{\partial}{\partial \omega} H_N^2(\omega)\bigg|_{=0} \tag{21}$$

for $-1 < \omega < +1$ is satisfied. k_2 is the normalizing constant where

$$k_2 = T_N^2(\omega_c) + k_1 \left(2 \sum_{j\,even}^{M} T_j(\omega) - 1 \right)^2. \tag{22}$$

Hence the transfer function of a dual-band lowpass prototype may be shown as

$$TF = \frac{1}{1 + \varepsilon^2 H_N^2(\omega)} \tag{23}$$

where ϵ is the prescribed ripple level at passband.

It can be seen from the theory that the degree of the filter has direct influence on the selectivity. Besides, it is observable that to achieve DBLPP using this method, only the degree of the filter and the ripple level need to be specified.

6. DBLPP THROUGH MODIFIED CHEBYSHEV POLYNOMIALS

Let's select the transmission and reflection responses of order $N = M = 5$. 5th order means that the denominator of ω^2 is used instead for the Chebyshev polynomial 1st kind. Based on unitary condition and selecting the left half poles, $S_{11}(p)$ is obtained. The transmission and reflection responses of order $N = M = 5$ are shown in Fig. 6. For given reflection function, the input impedance is formulated in which the DBLPP can be synthesized using the classical ladder synthesis. Fig. 6 shows the DBLPP ladder network where the first four elements contribute to the transmission poles at both passbands. A symmetric inverter-coupled network is then formed by introducing a mirrored network for the ease of distributed realization which is similar to the design found in Fig. 2.

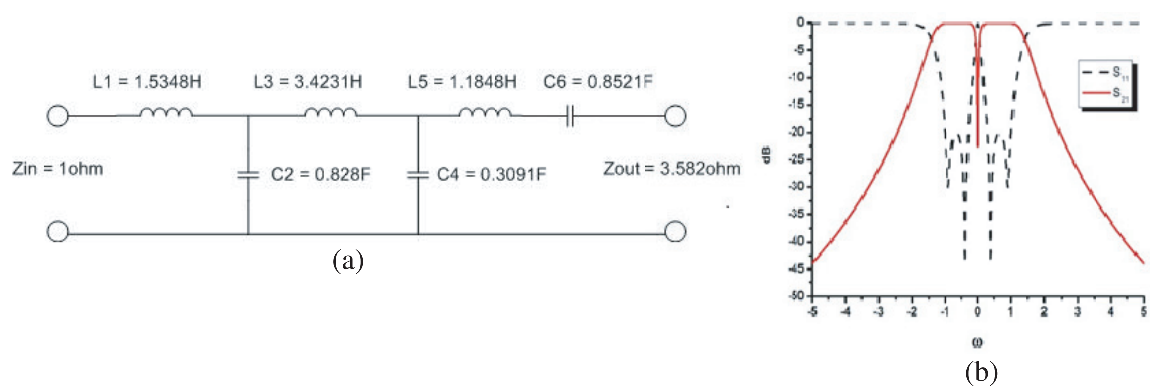
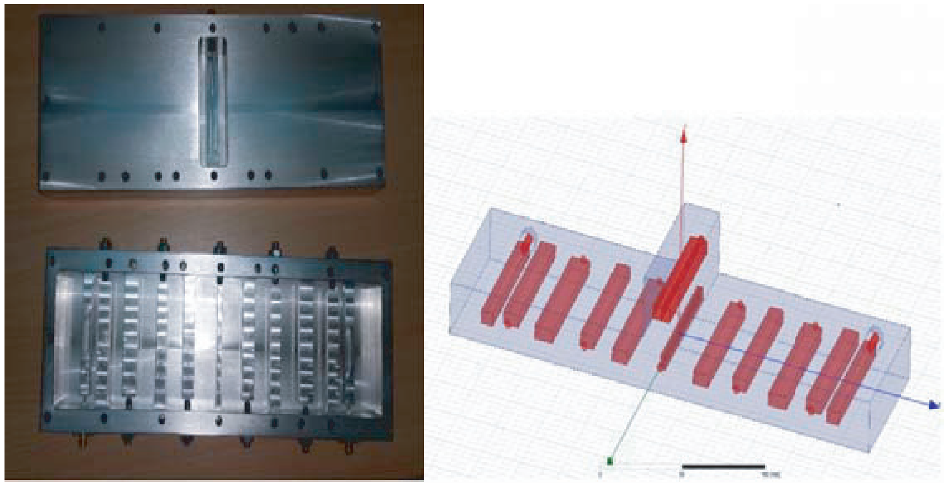


Figure 6. (a) Ladder network DBLLP configuration; (b) Simulated DBLLP response.

7. MODIFIED INTERDIGITAL DBBPF USING MODIFIED CHEBYSHEV POLYNOMIALS

From the inverter-coupled network, the Richard's transformation can be utilized from the lowpass prototype which results in a modified interdigital coupled line structure. For narrowband realization, classical admittance matrix scaling technique and UE elements at input and output are employed. The admittance matrix and design equations are similar to Equations (8)–(18) in the first method discussed.

The physical dimension is obtained based on the even-mode fringing and coupling capacitance of rectangular bars [13]. Shown in Fig. 7 is the filter structure designed based on modified interdigital



(a) (b)

Figure 7. (a) Modified DBBPF interdigital filter prototype; (b) 3D HFSS DBBPF model.

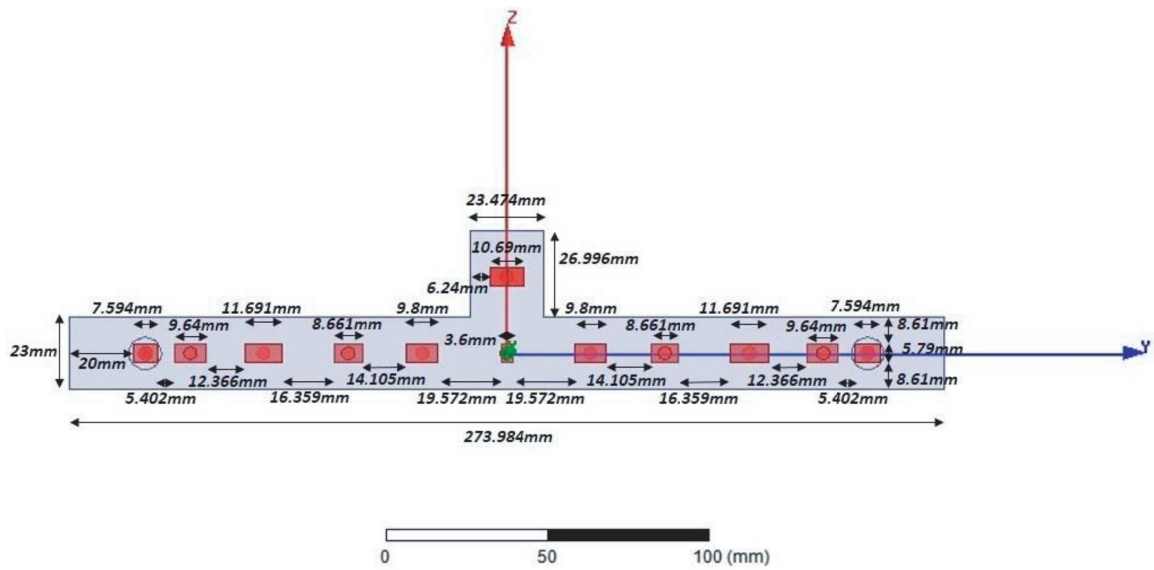


Figure 8. 2D HFSS model with measurements.

structure in HFSS. The 2D front view with measurements is shown in Fig. 8. The filter prototype produces a symmetrical dual-band response with 50 MHz bandwidth as shown in Fig. 9. The passband return loss is 15 dB, and a notch at center frequency of 0.98 GHz is obtained.

8. COMPARISON OF CLASS HYBRIDIZATION AND MODIFIED CHEBYSHEV APPROACH

In comparison, the modified Chebyshev offers better performances and robust analytical model compared to the class hybridization approach. By observing the modified polynomials, better design analysis can be done prior to the synthesis process. However, the class hybridization still holds potential for more improvements and explorations with different combinations of classes possible in theory. Besides, due to the hybrid nature, the selectivity of the inner and outer band edges are more controllable by adjusting the order of the lowpass and highpass classes respectively. Both methods allow for the formation of a lumped DBLPP.

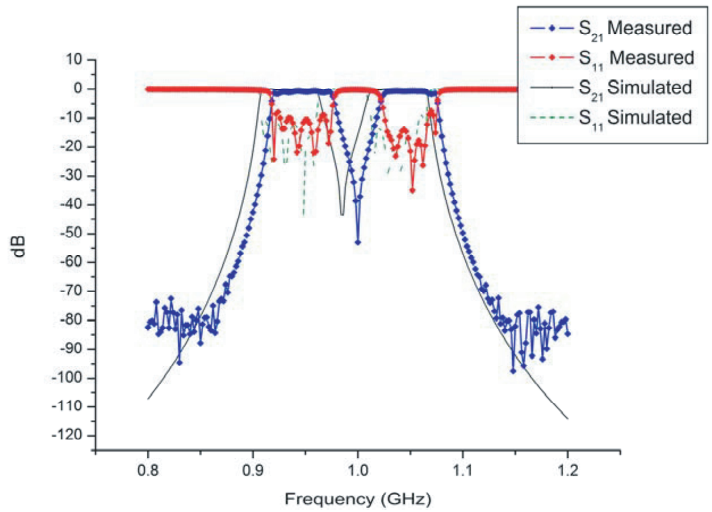


Figure 9. Comparison of simulated response and measured response.

9. CONCLUSION AND RECOMMENDATIONS

This paper discusses two new methods of synthesis of a dual-band filter based on the concept of hybrid polynomials. The first method performs hybridization of two different classes of filter of lowpass and highpass types. The second method presented employs the Chebyshev first and second kinds to produce a dual-band Chebyshev HFF. Both methods are capable of undergoing Richard's transformation to the distributed element. A modified interdigital design is used to achieve the DBBPF. Measured responses from both fabricated prototypes are shown to adhere to the design theory discussed in this paper. The method suggested in this paper can potentially be improved for asymmetrical response and extended to allow designing of tri-band filters.

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