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Robust Adaptive Wideband Beamforming Using Probability-Constrained Optimization

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Abstract—The existing robust narrowband beamformers based on probability-constrained optimization have an excellent performance as compared to several state-of-the-art robust beamforming algorithms. However, they always assume that the steering vector errors are small enough. Without this assumption, we extend the probability-constrained approach to a wideband beamformer. In addition, a novel robust wideband beamformer with frequency invariance constraints is proposed by introducing the response variation (RV) element. Our problems can be reformulated in a convex form as the iterative second order cone programming (SOCP) problem and solved effectively using well-established interior point method. Compared with existing robust wideband beamformers, a more efficient control over the beamformer's response against the steering vector errors is achieved with an improved output signal-to-interference-plus-noise ratio (SINR).

1. INTRODUCTION

In the past, beamforming was studied extensively for desired signal enhancement and interference signals suppression [1–4]. Given the exact look direction, many traditional wideband beamformers can work effectively and achieve a satisfactory output signal-to-interference-plus-noise ratio (SINR) [5, 6]. One of the most well-known beamformers is the linearly constrained minimum variance (LCMV) beamformer [7, 8], which minimizes its output power while preserving a unity gain at the look direction or subject to some more complicated constraints. However, in practice, the look direction error exists inevitably. As a result, the output performance may be disappointing since the wideband beamformers will tend to null out the desired signal as an interference signal.

Many robust beamformers have been proposed to deal with look direction errors [9, 10]. One of the most well-known methods is diagonal loading method (DL) [11]. The crucial problem of DL is how to obtain the reasonable DL factor. Another choice is the derivative constraint method which imposes additional derivative constraints on the beamformer to obtain a wider main beam [12]. In [13, 14], a class of popular robust beamformers is proposed based on worst-case performance optimization (RB-WC). There are two problems with this approach. One is its relatively high computational complexity due to its constraints imposed on a large number of sampled frequency points, the other one is that there is no mechanism to control the response consistency to the mismatched desired signal. Therefore, a potentially intolerable distortion to the desired signal may happen. To address those problems, a robust wideband beamformer (RB-FI-WC) was proposed in [15], where a good frequency response consistency in the range of interesting angle is achieved by a response variation RV constraint [16]. The approaches based on worst-case performance optimization aim at optimizing the SINR assuming that the array operates under the worst conditions irrespective of the probability of such worst-case scenario. A potential problem with this approach is that it may be overly conservative in practical applications, especially

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taking into account that the worst-case mismatch may actually seldom occur. To improve robustness of the beamformer, a less conservative robust approach based on the probability-constrained optimization is proposed which guarantees the robustness against the signal steering vector mismatch with a selected probability [17–19]. The robust narrowband beamformers based on probability-constrained optimization have an excellent performance as compared to several state-of-the-art robust beamforming algorithms. However, they all need assume that the steering vector errors are small enough.

In this paper, we extend the probability-constrained approach to a wideband beamformer without the small steering vector errors assumption, called RB-PC. In order to alleviate the high computational complexity of the RB-PC and address its frequency response inconsistency problem, we apply the RV element to the array response to ensure a good response consistency in the robust angle region, and then impose the probability-constrained on the reference frequency point in the look direction, which is named as RB-FI-PC. The simulations illustrate the effectiveness of the proposed algorithm. The rest of this paper is organized as follows. In Section 2 the conventional adaptive wideband beamforming algorithm is reviewed. The RB-PC and RB-PC method are proposed in Section 3. Simulation results and performance comparisons are given in Section 4. Conclusions are drawn in Section 5.

2. REVIEW OF ADAPTIVE WIDEBAND BEAMFORMING

A wideband array processor based on uniform linear array is shown in Fig. 1, where J is the number of taps associated with each of the M sensor channels. Let the array sensors be uniformly spaced with the inter-element spacing d less than or equal to $c/(2f_h)$, where c is the wave propagation speed and f_h is the maximum frequency of the desired signal. Its response with respect to frequency f_i and look direction θ can be written as

$$H(f_i,\theta) = \mathbf{w}^H\mathbf{S}(f_i,\theta) \tag{1}$$

where $f_i \in B = [f_l, f_h]$ is the chosen discrete frequency in the frequency range of interest B, f_l the minimum frequency, and $\mathbf{w}$ a $MJ \times 1$ coefficient vector defined as

$$\mathbf{w} = [w_{0,0},\ldots,w_{M-1,0},\ldots,w_{M-1,J-1}]^T \tag{2}$$

the $MJ \times 1$ steering vector $\mathbf{S}(f_i,\theta)$ is given by

$$\mathbf{S}(f_i,\theta) = [1,\ldots,e^{-j2\pi f_i(M-1)d\sin\theta/c},\ldots,e^{-j2\pi f_i(J-1)T_s},\ldots,e^{-j2\pi f_i((M-1)d\sin\theta/c+(J-1)T_s)}]^T \tag{3}$$

where T_s is the sampling period.

The wideband LCMV problem [7] can be formulated as

$$\min_{w} \mathbf{w}^H\mathbf{R}\mathbf{w} \quad \text{subject to } \mathbf{C}^H\mathbf{w} = \mathbf{f} \tag{4}$$

where $\mathbf{R}$ is the autocorrelation matrix of the observed array data $\mathbf{X}$, $\mathbf{C}$ a constraint matrix, and $\mathbf{f}$ the response vector.

Let us assume that the desired signal steering vector is known exactly. Then the solution of the wideband beamformer is

$$\mathbf{w}_{opt} = \hat{\mathbf{R}}_{xx}^{-1}\mathbf{C}\left(\mathbf{C}^H\hat{\mathbf{R}}_{xx}^{-1}\mathbf{C}\right)^{-1}\mathbf{f} \tag{5}$$

where $\hat{\mathbf{R}}_{xx}$ is the sample correlation matrix given by

$$\hat{\mathbf{R}}_{xx} = \frac{1}{L}\sum_{n=0}^{L-1}\mathbf{X}(n)\mathbf{X}^H(n) \tag{6}$$

with L being the number of samples available and

$$\mathbf{X}(n) = [x_0(n),\ldots,x_{M-1}(n),\ldots,x_0(n-J+1),\ldots,x_{M-1}(n-J+1)]^T \tag{7}$$

3. PROPOSED ROBUST WIDEBAND BEAMFORMER

In practical situations, the signal steering vector may be known imprecisely, that is, the actual steering vector may differ from the presumed one. An essential shortcoming of the beamformer (7) is that it is not robust against such a steering vector mismatch and can severely suppress the desired signal. The actual steering vector of the desired signal from direction θ_0 has been explicitly modeled as

$$\hat{\mathbf{S}}(f_i, \theta_0) = \mathbf{e}(f_i) + \mathbf{S}(f_i, \theta_0), \quad \|\mathbf{e}(f_i)\| \le \varepsilon \tag{8}$$

where $\mathbf{S}(f_i, \theta_s)$ is the assumed steering vector of the look direction θ_s, $\mathbf{e}(f_i)$ an error vector, $\|\bullet\|$ the Euclidian norm, and ε a small positive value.

Using the Cauchy-Schwarts inequality, it follows that

$$\left|\mathbf{w}^H \hat{\mathbf{S}}(f_i, \theta_0)\right| = \left|\mathbf{w}^H (\mathbf{S}(f_i, \theta_s) + \mathbf{e}(\mathbf{f}))\right| \ge \left|\left|\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right| - \left|\mathbf{w}^H \mathbf{e}(f_i)\right|\right| \tag{9}$$

3.1. The Conventional Probability-Constrained Optimization Method

The existing probability-constrained optimization methods all assume that the steering vector errors are small enough to satisfy

$$\left|\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right| > \left|\mathbf{w}^H \mathbf{e}(f_i)\right| \tag{10}$$

From the triangle inequality (9) it follows that

$$\left|\mathbf{w}^H (\mathbf{S}(f_i, \theta_s) + \mathbf{e}(f))\right| \ge \left|\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right| - \left|\mathbf{w}^H \mathbf{e}(f_i)\right| \tag{11}$$

Following [17–19], the probability-constrained robust wideband beamformer can be written as

$$\begin{cases} \min\limits_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}}_{xx} \mathbf{w} \\ s.t. \Pr\left\{\left|\mathbf{w}^H \hat{\mathbf{S}}(f_i, \theta_0)\right| \ge 1\right\} \ge p \quad f_i \in [f_l, f_h] \end{cases} \tag{12}$$

where p is a certain preselected probability value, and $\Pr\{\bullet\}$ stands for the probability operator. Then the constraint in (12) can be approximated as

$$\Pr\left\{\left|\mathbf{w}^H \mathbf{e}(f_i)\right| \le \left|\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right| - 1\right\} \ge p \quad f_i \in [f_l, f_h] \tag{13}$$

Let $\mathbf{e}(f_i)$ be drawn from a complex circularly symmetric Gaussian distribution with zero mean and covariance matrix $\mathbf{C_e}$. Thus $\mathbf{w}^H(\mathbf{S}(f_i, \theta_s) + \mathbf{e}(f))$ has the complex Gaussian distribution with $\mathbf{w}^H \mathbf{S}(f_i, \theta_s)$ mean and covariance matrix $\|\mathbf{C}_e^{1/2} \mathbf{w}\|^2$. Since $\mathbf{w}^H(\mathbf{e}(f))$ is circular zero mean complex Gaussian, its real and imaginary parts are real independent identically distributed Gaussian.

Using the Rayleigh-distributed, the Eq. (13) can be written as

$$\Pr\left\{\left|\mathbf{w}^H \mathbf{e}(f_i)\right| \le \left|\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right| - 1\right\} = 1 - \exp\left(\frac{\left(\left|\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right| - 1\right)^2}{\left\|\mathbf{C}_e^{1/2} \mathbf{w}\right\|^2}\right) \ge p \tag{14}$$

Based on the distortion-less response constraint $\left|\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right| \ge 1$, we can obtain

$$\sqrt{\mathbf{w}^H \mathbf{C_e} \mathbf{w}} \le \frac{1}{\sqrt{-\ln(1-p)}} \left(\left|\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right| - 1\right) \tag{15}$$

Observing that the cost function in (12) is unchanged when $\mathbf{w}$ undergoes an arbitrary phase rotation, we can select $\mathbf{w}$ such that

$$\mathrm{Re}\left\{\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right\} \ge 0, \quad \mathrm{Im}\left\{\mathbf{w}^H \mathbf{S}(f_i, \theta_s)\right\} = 0 \tag{16}$$

By Eqs. (15) and (16), the corresponding problem of the probability-constrained is formulated as follows

$$\begin{cases} \min\limits_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}}_{xx} \mathbf{w} \\ s.t. \sqrt{\mathbf{w}^H \mathbf{C_e} \mathbf{w}} \le \frac{1}{\sqrt{-\ln(1-p)}} \left(\mathbf{w}^H \mathbf{S}(f_i, \theta_s) - 1\right) \quad f_i \in [f_l, f_h] \end{cases} \tag{17}$$

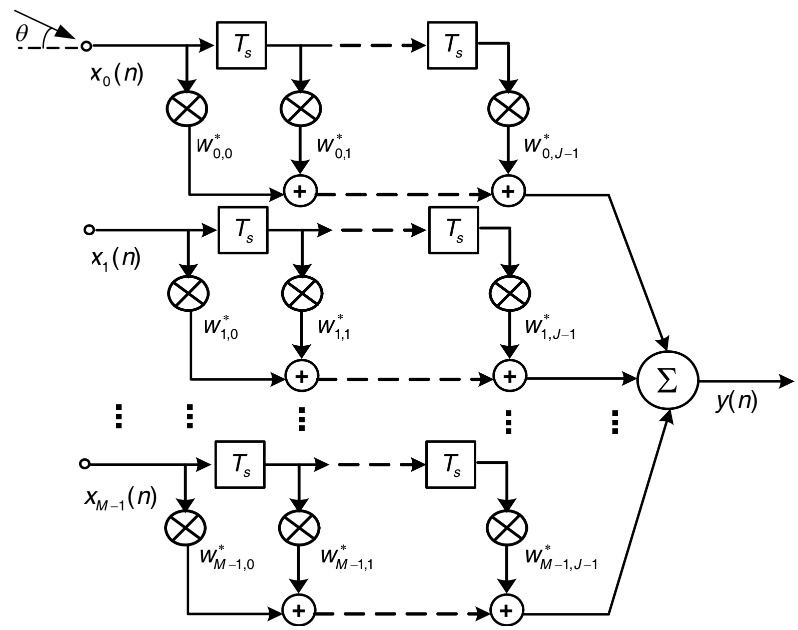


Figure 1. A general wideband beamforming structure.

3.2. Proposed RB-PC

Without the small steering vector errors assumption in (10), we propose a novel robust wideband beamformer based on probabilistic-constraint optimization which is called RB-PC.

Let us rewrite the constraint $\Pr\left\{\left|\mathbf{w}^H\hat{\mathbf{S}}(f_i,\theta_0)\right|\geq 1\right\}\geq p$ in (12) and (13) as

$$\Pr\left\{\left|\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right|-\left|\mathbf{w}^H\mathbf{e}(f_i)\right|\geq 1\right\}+\Pr\left\{\left|\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right|-\left|\mathbf{w}^H\mathbf{e}(f_i)\right|\leq -1\right\}\geq p \tag{18}$$

Using Eq. (14), we can obtain

$$\begin{aligned}&\Pr\left\{\left|\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right|-\left|\mathbf{w}^H\mathbf{e}(f_i)\right|\geq 1\right\}=\Pr\left\{\left|\mathbf{w}^H\mathbf{e}(f_i)\right|\leq\left|\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right|-1\right\}\\ &=1-\exp\left(-\frac{\left(\left|\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right|-1\right)^2}{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\right)\end{aligned}\tag{19}$$

$$\begin{aligned}&\Pr\left\{\left|\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right|-\left|\mathbf{w}^H\mathbf{e}(f_i)\right|\leq -1\right\}=1-\Pr\left\{\left|\mathbf{w}^H\mathbf{e}(f_i)\right|<\left|\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right|+1\right\}\\ &=\exp\left(-\frac{\left(\left|\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right|+1\right)^2}{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\right)\end{aligned}\tag{20}$$

$$\mathrm{Re}\left\{\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right\}\geq 0,\quad \mathrm{Im}\left\{\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right\}=0 \tag{21}$$

Using (19)–(21), the problem of the proposed RB-PC can be approximated as

$$\begin{aligned}&\min_{\mathbf{w}}\mathbf{w}^H\hat{\mathbf{R}}_{xx}\mathbf{w}\\ &s.t.\quad \mathrm{Im}\left\{\mathbf{w}^H\mathbf{S}(f_i,\theta_0)\right\}=0\\ &1-\exp\left(-\frac{\left(\mathbf{w}^H\mathbf{S}(f_i,\theta_0)-1\right)^2}{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\right)+\exp\left(-\frac{\left(\mathbf{w}^H\mathbf{S}(f_i,\theta_0)+1\right)^2}{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\right)\geq p\quad f_i\in[f_l,f_h]\end{aligned}\tag{22}$$

Let us define a variable t as

$$t=p-\exp\left(-\frac{\left(\mathbf{w}^H\mathbf{S}(f_i,\theta_s)+1\right)^2}{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\right)\tag{23}$$

Then the cost function in (22) can be written as

$$\exp\left(-\frac{\left(\mathbf{w}^H\mathbf{S}(f_i,\theta_s)-1\right)^2}{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\right)\le 1-t \tag{24}$$

Therefore, the problem of RB-PC can be written as

$$\begin{array}{l}\min\limits_{\mathbf{w}} \mathbf{w}^H\hat{\mathbf{R}}_{xx}\mathbf{w}\\ s.t.\quad \mathrm{Im}\left\{\mathbf{w}^H\mathbf{S}(f_i,\theta_s)\right\}=0\\ \sqrt{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\le\frac{1}{\sqrt{-\ln(1-t)}}\left(\mathbf{w}^H\mathbf{S}(f_i,\theta_s)-1\right)\quad f_i\in[f_l,f_h]\end{array} \tag{25}$$

The RB-PC method imposes a group of constraints on the chosen discrete frequency to prevent the mismatched desired signal from being suppressed by the beamformer. However, the inconsistency of the frequency response of the wideband beamformer may cause severe distortion to the mismatched desired signal. Moreover, too many constraints can decrease the number of degrees of freedom. As a result, the performance of interference cancellation may drop significantly.

3.3. Proposed RB-FI-PC

To improve the performance of RB-PC method, an efficient method called response variation (RV) can be employed to control consistency of the frequency response over the frequency-angle range of interest [16]. The parameter RV is given by

$$\mathrm{RV}=\frac{1}{N_fN_\Theta}\sum_{i=1}^{N_f}\sum_{j=1}^{N_\Theta}[\mathbf{S}(f_i,\theta_j)-\mathbf{S}(f_r,\theta_j)][\mathbf{S}(f_i,\theta_j)-\mathbf{S}(f_r,\theta_j)]^H \tag{26}$$

where $N_\bullet$ denotes the number of the sampling points, Θ the angle range over which RV is considered, and f_r the reference frequency. If RV is small enough, the beamformer has a better consistent frequency response over f and Θ.

The parameter RV can be transformed to

$$\mathrm{RV}=\mathbf{w}^H\mathbf{C}\mathbf{w} \tag{27}$$

where

$$\mathbf{C}=\frac{1}{N_\mathrm{B}N_\Theta}\sum_{j=1}^{N_\Theta}\sum_{i=1}^{N_f}\left|\mathbf{S}(f_i,\theta_j)-\mathbf{S}(f_r,\theta_j)\right|^2 \tag{28}$$

We define a threshold γ to constraint the parameter RV

$$\mathrm{RV}=\left\|\mathbf{L}_1^H\mathbf{w}\right\|^2\le\gamma \tag{29}$$

where $\mathbf{L}_1=\mathbf{U}_1\mathbf{\Lambda}_1^{1/2}$, with $\mathbf{\Lambda}_1$ being the eigenvalue matrix of $\mathbf{C}$, and $\mathbf{U}_1$ being the corresponding eigenvector matrix. Similar to (29), we can get

$$\mathbf{w}^H\hat{\mathbf{R}}_{xx}\mathbf{w}=\left\|\mathbf{L}_2^H\mathbf{w}\right\|^2 \tag{30}$$

Then the robust wideband beamformer with the RV constraint (RB-FI-PC) can be approximated as

$$\begin{array}{ll}\min\limits_{w} & \left\|\mathbf{L}_2^H\mathbf{w}\right\|^2\\ s.t. & \mathrm{Im}\left\{\mathbf{w}^H\mathbf{S}(f_r,\theta_s)\right\}=0\\ & \sqrt{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\le\frac{1}{\sqrt{-\ln(1-t)}}\left(\mathbf{w}^H\mathbf{S}(f_r,\theta_0)-1\right)\theta_s\\ & \|\mathbf{w}\|^2<\gamma_w\\ & \left\|\mathbf{L}_1^H\mathbf{w}\right\|^2\le\gamma\end{array} \tag{31}$$

where γ_w is a positive real-valued constant to avoid a large noise gain.

3.4. Optimization Problem Solution Using Iterative SOCP

Since the solution processing of the RB-PC method is the same as the RB-FI-PC method, next we will give the solution of RB-FI-PC by using iterative SOCP method.

Note that

$$t = p - \exp\left(-\frac{\left(\mathbf{w}^H\mathbf{S}(f_r,\theta_s)+1\right)^2}{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\right) = 1 - p - \left\{1 - \exp\left(-\frac{\left(\mathbf{w}^H\mathbf{S}(f_r,\theta_s)+1\right)^2}{\mathbf{w}^H\mathbf{C_e}\mathbf{w}}\right)\right\} < p \tag{32}$$

For initialization of the iterative SOCP, we can choose $t_0 = p$ and obtain the initial weight vector $\mathbf{w}_0$ by the SOCP method.

Using the estimated weight vector $\mathbf{w}_{k-1}$ of (31) to update the variable t_k in (23) as

$$t_k = p - \exp\left(-\frac{\left(\mathbf{w}_{k-1}^H\mathbf{S}(f_r,\theta_s)+1\right)^2}{\mathbf{w}_{k-1}^H\mathbf{C_e}\mathbf{w}_{k-1}}\right) \tag{33}$$

To guarantee the estimated $\mathbf{w}_{k-1}$ always be in feasible regions, we should check the value of the function $\Phi(\mathbf{w}_{k-1})$ before the new iteration

$$\Phi(\mathbf{w}_{k-1}) = \exp\left(-\frac{\left(\mathbf{w}_{k-1}^H\mathbf{S}(f_r,\theta_s)-1\right)^2}{\mathbf{w}_{k-1}^H\mathbf{C_e}\mathbf{w}_{k-1}}\right) - \exp\left(-\frac{\left(\mathbf{w}_{k-1}^H\mathbf{S}(f_r,\theta_s)+1\right)^2}{\mathbf{w}_{k-1}^H\mathbf{C_e}\mathbf{w}_{k-1}}\right) \leq 1 - p \tag{34}$$

If $\mathbf{w}_{k-1}$ cannot satisfy the constraint in (34), we will employ the following measures.

For any constant $\beta \geq 1$, we can get

$$\frac{\left[(\beta\mathbf{w}_{k-1})^H\mathbf{S}(f_r,\theta_s)-1\right]^2}{(\beta\mathbf{w}_{k-1})^H\mathbf{C_e}(\beta\mathbf{w}_{k-1})} \geq \frac{\left[(\beta\mathbf{w}_{k-1})^H\mathbf{S}(f_r,\theta_s)-\beta\right]^2}{(\beta\mathbf{w}_{k-1})^H\mathbf{C_e}(\beta\mathbf{w}_{k-1})} = \frac{\left(\mathbf{w}_{k-1}^H\mathbf{S}(f_r,\theta_s)-1\right)^2}{\mathbf{w}_{k-1}^H\mathbf{C_e}\mathbf{w}_{k-1}} \tag{35}$$

$$\frac{\left[(\beta\mathbf{w}_{k-1})^H\mathbf{S}(f_r,\theta_s)+1\right]^2}{(\beta\mathbf{w}_{k-1})^H\mathbf{C_e}(\beta\mathbf{w}_{k-1})} \leq \frac{\left[(\beta\mathbf{w}_{k-1})^H\mathbf{S}(f_r,\theta_s)+\beta\right]^2}{(\beta\mathbf{w}_{k-1})^H\mathbf{C_e}(\beta\mathbf{w}_{k-1})} = \frac{\left(\mathbf{w}_{k-1}^H\mathbf{S}(f_r,\theta_s)+1\right)^2}{\mathbf{w}_{k-1}^H\mathbf{C_e}\mathbf{w}_{k-1}} \tag{36}$$

It is easy to observe that the function $\Phi(\beta\mathbf{w}_{k-1})$ is a decreasing function on constant β. As a result, there always exists a suitable β to make $\beta\mathbf{w}_{k-1}$ satisfy the constraint in (34). The constant β can be directly obtained by some conventional one-dimension search schemes.

When the absolute value $|t_k - t_{k-1}|$ drops below a certain pre-specified threshold, it is assumed that the variable t has adapted optimally and the iterative procedure can stop. Otherwise the iterative process should continue until it reaches convergence. Fortunately, it only needs 2 to 3 times of iterative to achieve convergence according to numbers of experiments.

For an iterative algorithm, we need present the convergence proof of the optimization algorithm based on iterative SOCP theory. First of all, let us prove that the sequence $\{t_k\}$ is a monotone non-increasing sequence in the iteration process. For a known t_k, the Eq. (31) is a convex optimization problem. As a result, the optimum weight $\mathbf{w}_k$ can be obtained.

Using the KKT condition in (31), we can obtain t_k as

$$t_k = 1 - \exp\left(-\frac{\left(\mathbf{w}_k^H\mathbf{S}(f_r,\theta_s)-1\right)^2}{\mathbf{w}_k^H\mathbf{C_e}\mathbf{w}_k}\right) \tag{37}$$

On the other hand, we know that

$$t_{k+1} = p - \exp\left(-\frac{\left(\mathbf{w}_k^H\mathbf{S}(f_r,\theta_s)+1\right)^2}{\mathbf{w}_k^H\mathbf{C_e}\mathbf{w}_k}\right) \tag{38}$$

Therefore, we can get

$$t_{k+1} - t_k = \exp\left(-\frac{\left(\mathbf{w}_k^H\mathbf{S}(f_r,\theta_s)-1\right)^2}{\mathbf{w}_k^H\mathbf{C_e}\mathbf{w}_k}\right) - \exp\left(-\frac{\left(\mathbf{w}_k^H\mathbf{S}(f_r,\theta_s)+1\right)^2}{\mathbf{w}_k^H\mathbf{C_e}\mathbf{w}_k}\right) - (1-p) \tag{39}$$

Using Eq. (34), it is easy to find that $t_{k+1}-t_k \leq 0$, so the sequence $\{t_k\}$ is a monotone non-increasing sequence in the iteration process. Therefore, we can verify that when $k \to \infty$, there always exits a lower bound $t_{opt} > 0$ to make the sequence $\{t_k\}$ convergent. Furthermore, when $k \to \infty$, the optimum weight $\{\mathbf{w}_k\}$ can also converge to the optimum value $\mathbf{w}_{opt}$.

So we can summarize the proposed algorithm as follows

Step 1. Selecting the initial value $t_0 = p$, we can obtain the initial weight vector $\mathbf{w}_0$ by applying the SOCP method in (31).

Step 2. For $k > 1$, if the $k-1$th weight vector $\mathbf{w}_{k-1}$ is content with (34), the t_k can be obtained by (33). If not, let us choose the appropriate constant $\beta \geq 1$ through one-dimension search algorithm, and replace $\mathbf{w}_{k-1}$ by $\beta\mathbf{w}_{k-1}$ to obtain the updated t_k. Then, the weight vector $\mathbf{w}_k$ can be obtained by (31).

Step 3. If the absolute value $|t_k - t_{k-1}|$ drops below a certain pre-specified threshold, the variable t and the weight vector w have adapted optimally and the iterative procedure can stop. Otherwise, the iterative process should return to step 2 to continue until it reaches convergence.

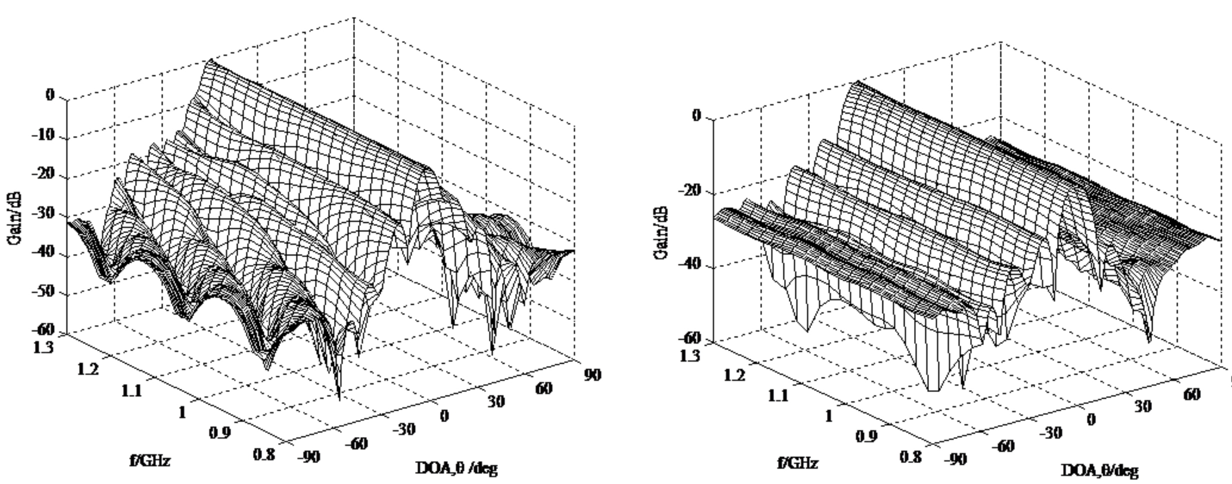
4. SIMULATIONS

In our simulation, we compare the proposed RB-FI-PC beamformer in (31) with the RB-PC beamformer in (25), the RB-WC beamformer and the RB-FI-WC beamformer in [15]. A total of 200 independent Monte-Carlo runs are used to obtain each point.

They are performed based on a ULA with $M = 15$ and $J = 20$, the frequency range of interest is between 800 MHz and 1300 MHz, the reference frequency is 1050 MHz, the number of the chosen discrete frequencies is 18. The covariance matrix $\mathbf{C_e}$ can be approximated as $\sigma^2\mathbf{I}_{MK}$ [20], where σ^2 is 0.4. Sample number is set to 512 in each simulation.

The actual desired signal comes from $\theta_0 = 0^\circ$ with a signal-to-noise ratio (SNR) of 10 dB. Two wideband interferences arrive from $-30°$ and $45°$, respectively, with a signal-to-interference (SIR) of -30 dB. In the following, the presumed look direction is $\theta_s = 4°$. For the RB -PC beamformer, p is set to be 0.95; for the RB-FI-PC beamformer, the value of $\gamma = 0.0007$ and $p = 0.95$ is chosen; for the RB-WC beamformer, ε is set to be 4.5; for the RB-FI-WC beamformer, $\varepsilon = 3.1$ and $\varsigma = 0.013$ are chosen.

The resultant beam pattern of the RB-PC and the RB-FI-PC is shown in Fig. 2(a) and Fig. 2(b), respectively, which shows an effective robustness of the wideband beamformer against look direction



(a) The RB-PC (b) The RB-FI-PC

Figure 2. The resultant beam pattern of the wideband beamformer.

errors and forms two nulls at -30° and 45° to suppress two interfering signals. Furthermore, the RB-FI-PC beamformer has a better performance in terms of frequency invariant property over the RV angle range, response variation control and interference suppression.

Figure 3 shows the output SINR versus the probability p for the wideband beamformers. Generally, the probability p cannot smaller than 0.9. To observe the change trend of output SINR with the varied p, we expand the range of p to 0.1$\sim$ 1. With the increasing value of p, we can see that the output SINRs for both proposed beamformers improve gradually. When the probability $p \geq 0.9$, the output SINRs of proposed RB-FI-PC method will stay higher than 20 dB with little fluctuation.

Let us set constraint parameters as $p = 0.95$, $\sigma^2 \in [0.1, 1]$ and $\gamma \in [0.0001, 0.001]$. Fig. 4(a) shows the output SINR versus the parameter σ^2 of the RB-PC method. Since the larger σ^2 denotes the larger steering vector mismatch, the output SINR performance decreases drastically with the increasing σ^2 in $[0.4 \sim 1]$. However, when $\sigma^2 \in [0.1 \sim 0.4]$, the larger σ^2 brings the better output SINR. Because too

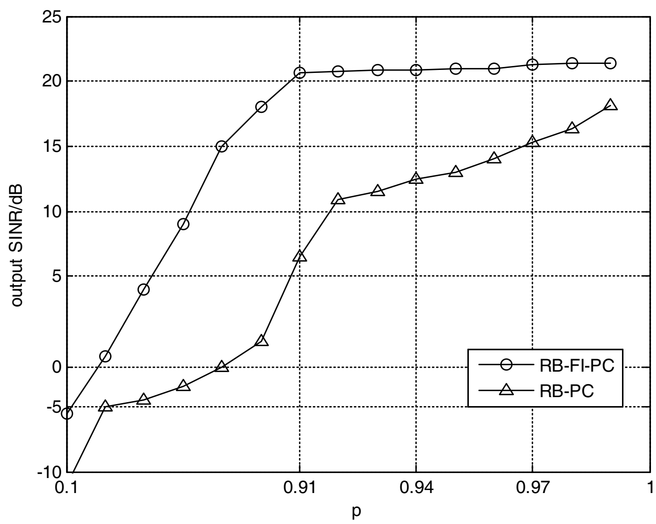
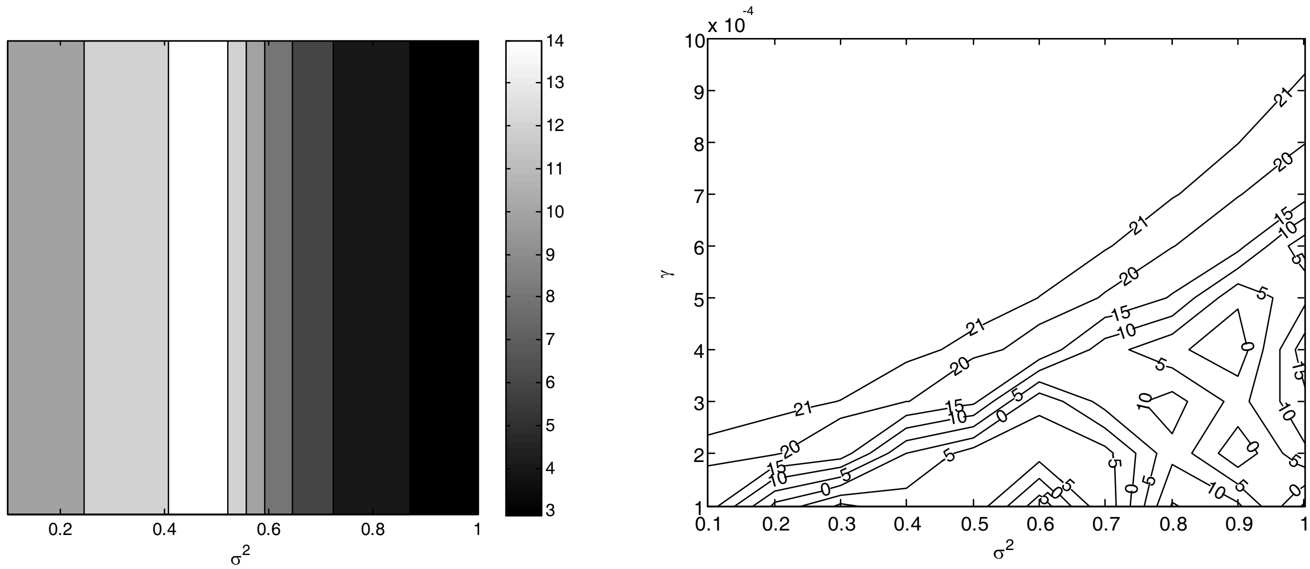


Figure 3. Output SINR versus the probability p.



(a) The RB-PC (b) The RB-FI-PC

Figure 4. Output SINR versus the parameters σ^2 and γ.

many constraints narrowed the scope of optimal solution set, the appropriate increase of σ^2 is equivalent to enlarge the set of optimal solutions, thus the output performance of SINR increased. Fig. 4(b) shows the output SINR versus the parameter σ^2 of the RB-FI-PC method. Following the increase of σ^2, in order to obtain the better SINR performance, the selection range of parameter γ is smaller. When the other conditions remain unchanged, the larger σ^2 can decrease the output SINR performance. Therefore, we should select those parameters reasonably to achieve the best output performance.

Figure 5 shows the output SINR versus the input SNR for the RB-FI-PC, the RB-FI-WC, the RB-PC, and the RB-WC beamformer. Obviously, the RB-FI-PC has obtained the highest output SINR than the other beamformers for the input SNRs in $[-2 \sim 20]$ dB. The worst-case mismatch of the RB-FI-WC and RB-WC beamformer may actually seldom occur in practical applications, so the over conservative constraint can degrade the output performance. However, when the input SNRs is lower than -2 dB, the output SINR of RB-FI-WC is slightly higher than the proposed RB-FI-PC due to the worse initial t of RB-FI-PC in the low SNR condition.

In the last, we study their performance in terms of output SINR versus look direction error, and the result is shown in Fig. 6. It can be seen that the RB-FI-PC beamformer has the best robustness than others against the look direction error. For large look direction errors, the performance of the RB-PC beamformer decreases significantly due to the extra consumption of degrees of freedom caused by the frequency response inconsistency.

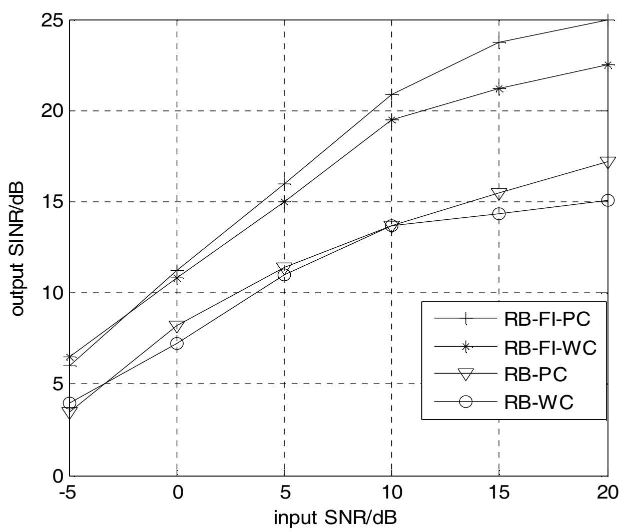


Figure 5. Output SINR versus input SNR with an angle estimation error of 4°.

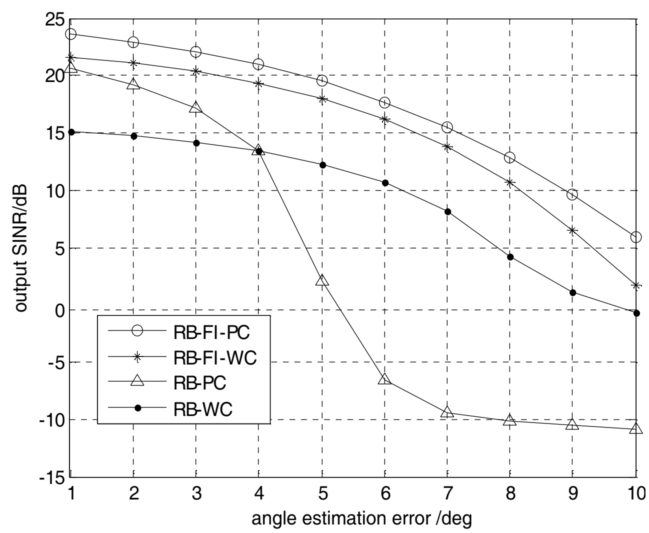


Figure 6. Output SINR versus the look direction error.

5. CONCLUSION

A novel robust wideband beamformer with frequency invariance constraints is proposed based on the probability-constrained optimization. By employing the RV element, we can control the frequency invariant property of the adaptive wideband beamformer in the look direction region over the frequency range of interest. The optimum coefficient vector is obtained by a proposed iterative SOCP method without the small steering vector errors assumption. Simulation results have validated a superior performance of the proposed wideband beamformer as compared to robust wideband beamformer based on worst-case performance optimization.

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