

## PRIMARY USER SIGNAL DETECTION BASED ON VIRTUAL MULTIPLE ANTENNAS FOR COGNITIVE RADIO NETWORKS

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**Abstract**—Primary User (PU) signal detection is critical for cognitive radio networks as it allows a secondary user to find spectrum holes for opportunistic reuse. Eigenvalue based detection has many advantages, such as it does not require knowledge on primary user signal or noise power level. However, most of the work on eigenvalue based detection methods presented in the literature require multiple sensing nodes or receiving antennas so that they cannot be directly applied to single antenna systems. In this paper, an effective eigenvalue based PU signal detection method is proposed for a cognitive user equipped with a single receiving antenna. The proposed method utilizes the temporal smoothing technique to form a virtual multi-antenna structure. The maximum and minimum eigenvalues of the covariance matrix obtained by the virtual multi-antenna structure are used to detect PU signal. Compared with the previous work, the presented method offers a number of advantages over other recently proposed algorithms. Firstly, the presented approach makes use of power method to calculate the maximum and minimum eigenvalues, it has lower computational complexity since the eigenvalue decomposition processing is avoided. Secondly, it can reduce system overhead since single antenna is used instead of multiple antennas or sensing nodes. Finally, simulation results show that performance of the proposed method is close to that of maximum-minimum eigenvalue detection using multiple antennas.

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## 1. INTRODUCTION

The expansion of wireless applications and mobile devices during recent years leads to a big radio spectrum shortage. The research studied by the Federal Communications Commission (FCC) shows that some licensed spectrum bands allocated through the current fixed spectrum allocation policy experience low utilization [1]. To address the need for intelligent spectrum allocation and improve the efficiency of spectrum utilization, the notion of cognitive radio (CR) was proposed by Mitola and Maguire in 1999 [2]. In CR networks, a secondary (unlicensed) user is allowed to utilize the spectrum resources when it does not cause intolerable interference to the primary (licensed) users. The key point for opportunistic spectrum access is to find the presence of primary signals in a given frequency band to avoid the collision. Therefore, primary user signal detection (spectrum sensing) becomes the most important component for the establishment of cognitive radio networks.

A lot of work has been done on spectrum sensing recently. In [3], a survey of spectrum sensing algorithms for cognitive radio applications is given. Various aspects of spectrum sensing problem are studied from a cognitive radio perspective and challenges associated with spectrum sensing are given. The spectrum sensing functionality can be implemented in either a non-cooperative or a cooperative fashion. Matched filter detection, energy detection and cyclostationary feature detection [4] are three classic non-cooperative spectrum sensing methods. Pros and cons of these different detection methods are discussed in [3]. Cooperative spectrum sensing can effectively improve the sensing performance in fading environment. In cooperative spectrum sensing, local sensors individually sense the channels and then send information to the network center, and the network center will make the final decision according to a certain fusion rule. OR rule, AND rule and MAJORITY rule are three common fusion rules [5]. Taricco proposes a linear cooperative spectrum sensing algorithm for cognitive radio networks in [6]. The optimal linear combining vector is acquired by solving a nonconvex optimization problem. A cooperative wideband spectrum sensing scheme based on compressed sensing is presented in [7]. It can effectively detect the wideband channel occupancy state by searching peaks in the reconstructed signal amplitude vector.

Eigenvalue based spectrum sensing methods have been widely studied recently. These methods use the eigenvalues of the sample covariance matrix to detect the primary transmitter without requiring information of primary user signals or noise power. Eigenvalue

based detection techniques studied in the literature include maximum-minimum eigenvalue (MME) detection [8], energy with minimum eigenvalue (EME) detection [9], maximum eigenvalue detection (MED) [10] and maximum eigenvalue to trace detection (MET) [11]. In [12], the simulation and performance results for MME detection and EME detection are presented for the Nakagami-m fading channel. In the eigenvalue based methods, the expression for the decision threshold has been derived based on the asymptotic or limiting distributions of extreme eigenvalues. The exact decision threshold is calculated for MME detector in [13]. By using the exact decision thresholds, the detection performance of MME detector achieves significant performance gains. An eigenvalue based spectrum sensing technique with finite number of samples and sensors is proposed in [14]. The authors express the distribution of the largest eigenvalue of finite sample covariance matrix in the form of sum of two gamma random variables.

However, all of above eigenvalue based spectrum sensing methods require multiple sensing nodes or receiving antennas. In some networks such as cognitive radio ad hoc networks [15] and cognitive radio cellular networks [16], CR users are mobile and they communicate with each other using small and flexible devices. It is impractical to equip a small mobile device with multiple antennas due to the required size of these antennas. More specifically, the space between two antennas must be at least of the order of  $\lambda/2$ ,  $\lambda$  being the wavelength used for transmissions. For the commonly used 2.4 GHz frequency band, the required distance is 6.125 cm. Even four antennas can be too big to be mounted on a laptop and the situation will get worse for a small mobile device [15]. Therefore, this paper addresses the problem of spectrum sensing in single receiver system. An effective maximum-minimum eigenvalue detection method using a single antenna (referred to as S-MME detection) is proposed for cognitive radio networks. The temporal smoothing technique is utilized to form a virtual multi-antenna structure for the implementation of proposed detection method based on single antenna. The proposed approach makes use of the power method to obtain the maximum and minimum eigenvalues to avoid the eigenvalue decomposition processing. The decision threshold is derived based on random matrix theory. The rest of the paper is organized as follows. The data model is described in Section 2. In Section 3, we introduce the framework of S-MME detection. Section 4 shows some simulation results. Finally, the conclusion is given in Section 5.

## 2. DATA MODEL

Assume that we are interested in the frequency band with central frequency  $f_c$  and bandwidth  $W$ . During a particular time interval, the frequency band may be occupied by only one primary user. Several secondary users are randomly distributed in the cognitive radio network. Each secondary user is equipped with a single antenna. In this paper, we consider the non-cooperative spectrum sensing scheme, i.e., the sensing work is completed by one secondary user.

For signal detection, there are two hypotheses: (1) hypothesis  $H_0$ : there exists only noise (no signal); (2) hypothesis  $H_1$ : there exist both noise and signal. At hypothesis  $H_0$ , the received signal of a secondary user is

$$y(t) = w(t)$$

while at hypothesis  $H_1$ , suppose that the primary signal has a carrier frequency of  $f_c + f_s$ . After demodulation to IF, the received signal of a secondary user is

$$y(t) = e^{j2\pi f_s t} b_s(t) + w(t)$$

where  $s(t)$  is the baseband representation and  $b \in \mathcal{R}^+$  the amplitude of the primary signal. The item  $w(t)$  is the complex additive white Gaussian noise.

Assume  $P$  is the sampling rate, which is much higher than the data rate of the primary signal. The data samples collected at the CR receiver are

$$y\left(\frac{n}{P}\right) = \exp\left(j\frac{2\pi}{P}f_s n\right) b_s\left(\frac{n}{P}\right) + w\left(\frac{n}{P}\right), \quad (n = 1, \dots, N) \quad (1)$$

where  $N$  is the number of samples. In the remainder of the paper, unless it is necessary to write it explicitly, the amplitude  $b$  in the data model is absorbed by  $s(t)$ , in which case, the amplitude of the primary signal is equal to  $b$  instead of 1. Then we can express (1) into vector form as

$$\mathbf{y} = \left[ \Phi_s \left(\frac{1}{P}\right) \Phi_s^2 \left(\frac{2}{P}\right) \dots \Phi_s^N \left(\frac{N}{P}\right) \right] + \mathbf{w} \quad (2)$$

where  $\Phi = e^{j(2\pi/P)f_s}$ ,  $\mathbf{w} \in \mathcal{C}^{N,1}$  is the noise vector collecting the samples of the noise term at the output of the receiving antenna.

Based on the received signal with little or no information on the primary signal and noise power, a sensing algorithm should make a decision on the existence of signal. Let  $P_d$  be the probability of detection, that is, at hypothesis  $H_1$ , the probability of the algorithm having detected the signal. Let  $P_{fa}$  be the probability of false alarm,

that is, at  $H_0$ , the probability of the algorithm having detected the signal. Obviously, for a good detection algorithm,  $P_d$  should be high and  $P_{fa}$  should be low. The requirements of the  $P_d$  and  $P_{fa}$  depend on the applications.

### 3. S-MME DETECTION METHOD

In this section, S-MME detection method is described in detail. The data information of the primary signal is collected by a secondary user equipped with a single antenna. The temporal smoothing technique is implemented to form a virtual multi-antenna structure to get the covariance matrix. The maximum and minimum eigenvalues are obtained by exploiting the power method. The decision threshold is derived based on random matrix theory.

#### 3.1. Virtual Multiple Antennas

In the data model,  $N$  samples of the primary signal are collected by a single-antenna receiver. In this subsection, we adopt a data stacking technique named temporal smoothing to form a virtual multi-antenna structure for the received data model. An  $M$ -factor temporally smoothed data matrix  $\mathbf{Y}$  is constructed by stacking  $M$  temporally shifted versions of the original data samples. As a result,  $\mathbf{Y}$  will have a virtual multi-antenna structure. On the basis of (2),  $\mathbf{Y}$  can be given as [17]

$$\mathbf{Y} = \begin{bmatrix} \Phi \left[ s \left( \frac{1}{P} \right) \quad \Phi s \left( \frac{2}{P} \right) \quad \dots \quad \Phi^{N-M} s \left( \frac{N-M+1}{P} \right) \right] \\ \Phi^2 \left[ s \left( \frac{2}{P} \right) \quad \Phi s \left( \frac{3}{P} \right) \quad \dots \right] \\ \vdots \\ \Phi^M \left[ s \left( \frac{M}{P} \right) \quad \Phi s \left( \frac{M+1}{P} \right) \quad \dots \right] \end{bmatrix} + \mathbf{W} \quad (3)$$

where  $\mathbf{W} \in \mathcal{C}^{M, N-M+1}$  represents the noise term constructed from  $\mathbf{w}$  in a similar way as  $\mathbf{Y}$  is obtained from  $\mathbf{y}$ . Assume that the primary signal is narrow band, i.e.,

$$s(t) \approx s \left( t + \frac{1}{P} \right) \approx \dots \approx s \left( t + \frac{M}{P} \right).$$

In this case, all the block rows in the right-hand term of (3) are approximately equal, which means that  $\mathbf{Y}$  has the following factorization

$$\mathbf{Y} \approx \begin{bmatrix} \Phi \\ \Phi^2 \\ \vdots \\ \Phi^M \end{bmatrix} \left[ s \left( \frac{1}{P} \right) \quad \Phi s \left( \frac{2}{P} \right) \quad \dots \right] + \mathbf{W} \triangleq \mathbf{A} \mathbf{F}_s + \mathbf{W} \in \mathcal{C}^{M, N-M+1} \quad (4)$$

where  $\mathbf{A}$ , throughout the sequel, is given by

$$\mathbf{A} = \begin{bmatrix} \Phi \\ \Phi^2 \\ \vdots \\ \Phi^M \end{bmatrix} \in \mathcal{C}^{M,1}$$

and

$$\mathbf{F}_s = \left[ s \left( \frac{1}{P} \right) \quad \Phi_s \left( \frac{2}{P} \right) \quad \dots \quad \Phi^{N-M} s \left( \frac{N-M+1}{P} \right) \right] \in \mathcal{C}^{1, N-M+1}$$

is a vector collecting  $N - M + 1$  samples of the primary signal.

Let  $\mathbf{R}$  be the covariance matrix of the received signal data model which has a virtual multi-antenna structure, that is,

$$\mathbf{R} = \frac{1}{N - M + 1} \mathbf{Y} \mathbf{Y}^H$$

where  $(\cdot)^H$  denotes Hermitian transposition. Suppose the noise and transmitted signal are uncorrelated. Substituting  $\mathbf{Y}$  by (4), we can verify that

$$\mathbf{R} \approx \mathbf{A} \sigma_s^2 \mathbf{A}^H + \sigma_n^2 \mathbf{I}_M$$

where  $\sigma_s^2 = \frac{1}{N-M+1} (\mathbf{F}_s \mathbf{F}_s^H)$ ,  $\sigma_n^2$  is the variance of the noise, and  $\mathbf{I}_M$  denotes an  $M \times M$  identity matrix.

Let  $\hat{\lambda}_{\max}$  and  $\hat{\lambda}_{\min}$  be the estimated maximum and minimum eigenvalue of  $\mathbf{R}$ . It is easy to know that the only non-zero eigenvalue of  $\mathbf{A} \sigma_s^2 \mathbf{A}^H$  is  $M \sigma_s^2$ . Obviously, when the primary signal is present,  $\hat{\lambda}_{\max} = M \sigma_s^2 + \sigma_n^2$ ,  $\hat{\lambda}_{\min} = \sigma_n^2$ . When the primary signal is absent,  $\hat{\lambda}_{\max} = \hat{\lambda}_{\min} = \sigma_n^2$ . Hence, if there is no signal,  $\hat{\lambda}_{\max} / \hat{\lambda}_{\min} = 1$ ; otherwise,  $\hat{\lambda}_{\max} / \hat{\lambda}_{\min} > 1$ . The ratio of  $\hat{\lambda}_{\max} / \hat{\lambda}_{\min}$  can be used to detect the presence of the primary signal. However,  $\hat{\lambda}_{\max}$  and  $\hat{\lambda}_{\min}$  are the estimated eigenvalues. In the next subsection, the real maximum eigenvalue  $\lambda_{\max}$  and minimum eigenvalue  $\lambda_{\min}$  of  $\mathbf{R}$  will be obtained.

### 3.2. Power Method

In this subsection, we exploit the power method to calculate  $\lambda_{\max}$  and  $\lambda_{\min}$  for the detection of primary signal. In this way, the eigenvalues can be obtained by simple algebraic operations. This method can reduce computational complexity since the eigenvalue decomposition processing is avoided.

It is well known that power method is an effective method to compute the maximum eigenvalue and the corresponding eigenvector

(named as the maximum eigenvector) for a real-valued matrix  $\mathbf{U}$ . It is easy to know that this method is still effective even if  $\mathbf{U}$  is a complex-valued matrix. For a complex-valued matrix, we have the following Theorem.

**Theorem 1:** For a Hermitian matrix  $\mathbf{U} \in \mathcal{C}^{n \times n}$ , if it has  $n$  linearly independent eigenvectors  $\mathbf{u}_1, \dots, \mathbf{u}_n$  ( $\|\mathbf{u}_i\|_2 = 1$ , for  $\forall i \in [1, \dots, n]$ ) and its eigenvalues satisfy the following relation  $|\lambda_1| > |\lambda_2| \geq \dots \geq |\lambda_n|$ . Let  $\mathbf{v}_0 = \sum_{i=1}^n \alpha_i \mathbf{u}_i$  ( $\alpha_1 \neq 0$ ). Take the vector  $\mathbf{v}_0$  as the initial vector, and form a vector sequence according to the power of matrix  $\mathbf{U}$  as follows:

$$\begin{cases} \mathbf{v}_k = \mathbf{U}\mathbf{v}_{k-1} \\ m_k = \max(\mathbf{v}_k) = v_{ki} \quad (|v_{ki}| = \max_{1 \leq j \leq n} |v_{kj}|) \\ \mathbf{v}_k = \mathbf{v}_k / m_k \quad (k = 1, 2, \dots) \end{cases} \quad (5)$$

where  $\mathbf{v}_k = [v_{k1}, \dots, v_{kn}]^T$ . Then, any one of the following statements is true:

(a)  $\lim_{k \rightarrow \infty} \mathbf{v}_k = \frac{\mathbf{u}_1}{\max(\mathbf{u}_1)}$ ,      (b)  $\lim_{k \rightarrow \infty} \frac{\max(\mathbf{v}_k)}{\max(\mathbf{v}_{k-1})} = \lambda_1$ .

*Proof:* Under the above assumptions, the iteration vector  $\mathbf{v}_k$  can be written as follows

$$\begin{aligned} \mathbf{v}_k &= \mathbf{U}\mathbf{v}_{k-1} = \mathbf{U}^k \mathbf{v}_0 = \alpha_1 \lambda_1^k \mathbf{u}_1 + \alpha_2 \lambda_2^k \mathbf{u}_2 + \dots + \alpha_n \lambda_n^k \mathbf{u}_n \\ &= \alpha_1 \lambda_1^k \left( \mathbf{u}_1 + \sum_{i=2}^n \frac{\alpha_i}{\alpha_1} \left( \frac{\lambda_i}{\lambda_1} \right)^k \mathbf{u}_i \right). \end{aligned}$$

Since  $\mathbf{U}$  is a Hermitian matrix, it is easy to know that  $\lambda_i \geq 0 \forall i \in [1, \dots, n]$ , i.e.,  $\lambda_1 > \lambda_2 \geq \dots \geq \lambda_n \geq 0$ . Using the above analysis, we have  $\lim_{k \rightarrow \infty} \left( \frac{\lambda_i}{\lambda_1} \right)^k = 0$ . Therefore, for all sufficiently large  $k$ , it is clear that  $\mathbf{v}_k = \mathbf{U}^k \mathbf{v}_0 \approx \alpha_1 \lambda_1^k \mathbf{u}_1$ . From (5),  $\mathbf{v}_k$  can be expressed as  $\mathbf{v}_k = \frac{\mathbf{U}^k \mathbf{v}_0}{\max(\mathbf{U}^k \mathbf{v}_0)}$ . Notice that  $\alpha_1 \neq 0$  and  $v_{ki} = 1$  (it is the maximal component defined in (5) for the vector  $\mathbf{v}_k$ ), it can be easily verified that

$$\begin{aligned} \mathbf{v}_k &\rightarrow \frac{\mathbf{u}_1}{\max(\mathbf{u}_1)} \\ \frac{\max(\mathbf{v}_k)}{\max(\mathbf{v}_{k-1})} &= \frac{\lambda_1^k \max \left( \alpha_1 \mathbf{u}_1 + \sum_{i=2}^n \alpha_i \left( \frac{\lambda_i}{\lambda_1} \right)^k \mathbf{u}_i \right)}{\lambda_1^{k-1} \max \left( \alpha_1 \mathbf{u}_1 + \sum_{i=2}^n \alpha_i \left( \frac{\lambda_i}{\lambda_1} \right)^{k-1} \mathbf{u}_i \right)} \rightarrow \lambda_1 \end{aligned}$$

This concludes the proof.

According to Theorem 1, the maximum eigenvalue  $\lambda_{\max}$  of the covariance matrix  $\mathbf{R}$  can be solved by the power method. Since only

one primary signal is concerned,  $\mathbf{R}$  has only one maximum eigenvalue, the other  $M - 1$  eigenvalues are all small eigenvalues. To get a more precise result, we compute the minimum eigenvalue  $\lambda_{\min}$  of  $\mathbf{R}$  as follows

$$\lambda_{\min} = \frac{\text{tr}(\mathbf{R}) - \lambda_{\max}}{M - 1}$$

where  $\text{tr}(\mathbf{R})$  represents the trace of  $\mathbf{R}$ . Finally, we get test statistic of S-MME detection:  $T = \lambda_{\max}/\lambda_{\min}$ .

We briefly investigate the computational complexity of the power method and eigenvalue decomposition when computing eigenvalues. We use  $\mathcal{O}(n^3)$  to represent the order of  $n^3$  multiplications. The eigenvalue decomposition processing solves for the complete set of eigenvalues and eigenvectors of the matrix even if the problem requires only a small subset of them to be computed. For the  $n \times n$  matrix  $\mathbf{U}$ , eigenvalue decomposition calls for  $2n^3(s + 1)$  real multiplications, where  $s$  is the maximum number of iterations required to reduce a superdiagonal element as to be considered zero by the convergence criterion [18]. Thus the computational complexity of eigenvalue decomposition is  $\mathcal{O}(n^3)$ . The idea of the power method is only to compute the principal eigenvalues and eigenvectors. The power method mainly consists of two computational steps: obtaining the iteration vector  $\mathbf{v}_k$  by computing  $\mathbf{v}_k = \mathbf{U}\mathbf{v}_{k-1}$  and  $\mathbf{v}_k = \mathbf{v}_k/m_k$  in (5). Since  $\mathbf{v}_k$  is an  $n \times 1$  vector, the computation of these two steps calls for  $4n^2$  and  $4n$  real multiplications, respectively. Suppose the number of iterations is  $L$ , then the total number of real multiplications is  $4L(n^2 + n)$ , i.e., the computational complexity of the power method is  $\mathcal{O}(n^2)$ . Therefore, the power method has lower computational complexity than eigenvalue decomposition processing when computing eigenvalues.

### 3.3. Threshold Determination

In the general model of the spectrum sensing, a threshold must be determined to compare with a test statistic of the sensing metric in order to determine the presence of a primary user. Since the eigenvalue distribution of  $\mathbf{R}$  is very complicated [19], the choice of the thresholds are difficult. In this subsection, using random matrix theory, we will find an approximation for the distribution of this random variable and derive the decision threshold based on the pre-defined  $P_{fa}$ .

When the primary signal is absent,  $\mathbf{R}$  turns to  $\mathbf{R}_n$ , the covariance matrix of the noise defined as,

$$\mathbf{R}_n = \frac{1}{N - M + 1} (\mathbf{W}\mathbf{W}^H)$$

$\mathbf{R}_n$  is nearly a Wishart random matrix [19]. The study of the eigenvalue distributions of a random matrix is a very hot topic in recent years

in mathematics as well as communication and physics. The joint probability density function (PDF) of ordered eigenvalues of a Wishart random matrix has been known for many years [19]. However, since the expression of the PDF is very complicated, no closed form expression has been found for the marginal PDF of ordered eigenvalues. Recently, researchers have found the distribution of the largest eigenvalue [20] and smallest eigenvalue [21] as described in the following theorems. For convenience, we let  $K = N - M + 1$ .

**Theorem 2** [20]: Assume that the noise is complex. Let  $\mathbf{V} = \frac{K}{\sigma_n^2} \mathbf{R}_n$ ,  $\mu = (\sqrt{K} + \sqrt{M})^2$  and  $\nu = (\sqrt{K} + \sqrt{M})(\frac{1}{\sqrt{K}} + \frac{1}{\sqrt{M}})^{1/3}$ . Assume that  $\lim_{K \rightarrow \infty} \frac{M}{K} = a$  ( $0 < a < 1$ ). Then  $\frac{\lambda_{\max}(\mathbf{V}) - \mu}{\nu}$  converges (with probability one) to the Tracy-Widom distribution of order 2 [22].

**Theorem 3** [21]: Assume that  $\lim_{K \rightarrow \infty} \frac{M}{K} = a$  ( $0 < a < 1$ ). Then  $\lim_{K \rightarrow \infty} \lambda_{\min} = \sigma_n^2(1 - \sqrt{a})^2$ .

Based on the theorems, we have the following results:

$$\lambda_{\max} \approx \frac{\sigma_n^2}{K} (\sqrt{K} + \sqrt{M})^2$$

$$\lambda_{\min} \approx \frac{\sigma_n^2}{K} (\sqrt{K} - \sqrt{M})^2.$$

The Tracy-Widom distributions were found by Tracy and Widom as the limiting law of the largest eigenvalue of certain random matrices [22]. Let  $F_2$  be the cumulative distribution function (CDF) of the Tracy-Widom distribution of order 2. There is no closed form expression for the distribution functions. It is generally difficult to evaluate them. Fortunately, based on numerical computation, there have been tables for the functions [20].

Using the theories, we are ready to derive the decision threshold for S-MME detection method. Let  $\gamma$  represent the decision threshold, then the probability of false alarm of the S-MME detection is

$$\begin{aligned} P_{fa} &= P(\lambda_{\max} > \gamma \lambda_{\min}) = P\left(\frac{\sigma_n^2}{K} \lambda_{\max}(\mathbf{V}) > \gamma \lambda_{\min}\right) \\ &\approx P\left(\lambda_{\max}(\mathbf{V}) > \gamma(\sqrt{K} - \sqrt{M})^2\right) \\ &= P\left(\frac{\lambda_{\max}(\mathbf{V}) - \mu}{\nu} > \frac{\gamma(\sqrt{K} - \sqrt{M})^2 - \mu}{\nu}\right) \\ &= 1 - F_2\left(\frac{\gamma(\sqrt{K} - \sqrt{M})^2 - \mu}{\nu}\right) \end{aligned}$$

Hence, we should choose the threshold such that

$$1 - F_2 \left( \frac{\gamma(\sqrt{K} - \sqrt{M})^2 - \mu}{\nu} \right) = P_{fa}.$$

This leads to

$$F_2 \left( \frac{\gamma(\sqrt{K} - \sqrt{M})^2 - \mu}{\nu} \right) = 1 - P_{fa},$$

or, equivalently,

$$\frac{\gamma(\sqrt{K} - \sqrt{M})^2 - \mu}{\nu} = F_2^{-1}(1 - P_{fa}).$$

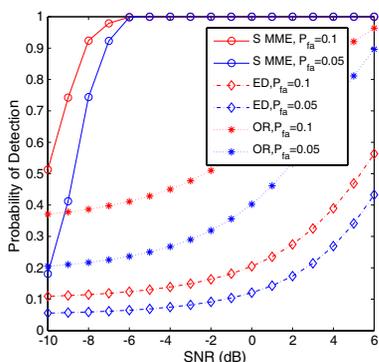
From the definitions of  $\mu$  and  $\nu$ , we finally obtain the threshold

$$\gamma = \frac{(\sqrt{K} + \sqrt{M})^2}{(\sqrt{K} - \sqrt{M})^2} \left( 1 + \frac{(\sqrt{K} + \sqrt{M})^{-2/3}}{(KM)^{1/6}} F_2^{-1}(1 - P_{fa}) \right).$$

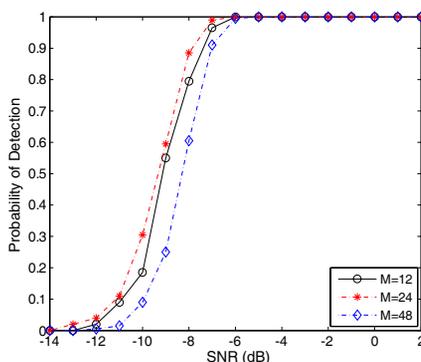
#### 4. SIMULATION RESULTS

In this section, simulation results are provided to illustrate the performance of the proposed S-MME detection method. Consider a licensed frequency band in the cognitive radio network with only one active primary user. The primary signal employs Binary Phase Shift Keying (BPSK) modulation and the center frequency is 8 MHz. The sampling rate is set to 32 MHz.  $N$  is the number of samples and  $M$  is temporal smoothing factor. The results are averaged over 1000 tests using Monte-Carlo simulations written in Matlab. The SNR of a CR receiver is defined as the ratio of the average received signal power to noise power over the licensed frequency band.

Figure 1 shows the probability of detection curves for S-MME detection, energy detection (ED) and cooperative energy detection based on OR rule. The results are taken for  $N = 256$  and SNRs varying from  $-10$  dB to  $6$  dB. In S-MME detection, the temporal smoothing factor is 8. In cooperative energy detection, the decision is made by fusing the sensing information of 4 secondary users. As shown in the figure, the proposed S-MME sensing method can achieve satisfactory detection performance even in low SNR conditions. For example, S-MME detection can detect PU signal with 100% probability at SNR of  $-6$  dB. However, the detection probabilities of ED and cooperative ED are less than 50%. From the figure, we can also see that for the same SNR, probability of detection improves as probability of false alarm increases. This reflects the tradeoff between false alarm and detection probability.



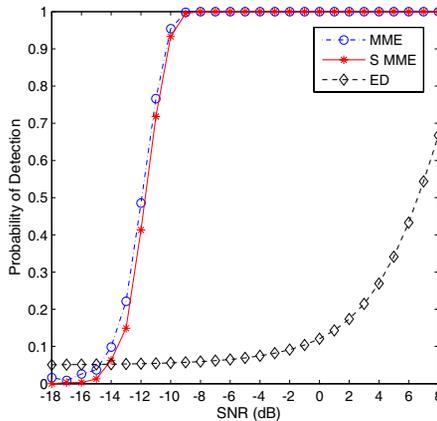
**Figure 1.** Probability of detection versus SNR for different probability of false alarm with  $N = 256$  and  $M = 8$ .



**Figure 2.** Probability of detection versus SNR for different temporal smoothing factor with  $N = 256$  and  $P_{fa} = 0.1$ .

Figure 2 shows the probability of detection versus SNR for different temporal smoothing factors. The results are taken for  $N = 256$ ,  $P_{fa} = 0.1$ , and SNRs varying from  $-14$  dB to  $2$  dB. It is shown that the detection performance becomes better when  $M$  increases from  $12$  to  $24$ . But when  $M$  turns to  $48$ , the detection performance declines. This is because  $M$  should be relatively small to  $N$  when the temporal smoothing technique is utilized. So choosing a proper temporal smoothing factor for a given number of samples is important.

Figure 3 shows the performance comparison of MME detection, S-MME detection and energy detection. In MME detection, 4 receiving antennas are used to sensing the radio environment. In S-MME detection, the temporal smoothing factor is 16. For all three methods, 512 samples are collected,  $P_{fa} = 0.05$ , and SNRs varying from  $-18$  dB to  $8$  dB. As shown in the figure, the propose S-MME detection method performs better than energy detection method. Meanwhile, we can see that both S-MME detection and MME detection can detect the PU signal with 100% probability when the SNR is more than  $-9$  dB. The performance of S-MME detection is very close to that of MME detection when the SNR is less than  $-9$  dB. For example, the detection probabilities of MME detection and S-MME detection are 0.954 and 0.933 at SNR of  $-10$  dB, respectively. The biggest performance gap between these two methods is only 0.072 with change in SNR. In others words, the proposed S-MME detection method can achieve roughly the same performance as MME detection by using a single antenna. The main reason for this result is that the processed data of



**Figure 3.** Performance comparison of different sensing methods with  $N = 512$  and  $P_{fa} = 0.05$ .

these two methods have similar structures. We exploit the temporal smoothing technique that adds structure to the data model for the implementation of S-MME detection method. The information of PU signal are perfectly contained in the data model of both methods, thus they can achieve roughly the same performance.

## 5. CONCLUSIONS

In this paper, we present a maximum-minimum eigenvalue based spectrum sensing method using a single antenna for cognitive radio networks. The temporal smoothing technique is utilized to form a virtual multi-antenna structure. The proposed approach makes use of the power method to calculate the maximum and minimum eigenvalues of the covariance matrix obtained by the virtual multi-antenna structure. To ensure a good detection performance, the decision threshold is derived based on random matrix theory. Simulations using BPSK signals are presented in order to illustrate the performance of S-MME detection method. It has been shown that, performance of S-MME detection is very close to that of MME detection with multiple antennas. Besides, S-MME detection can reduce system overhead and avoid the eigenvalue decomposition processing by utilizing power method. These advantages make S-MME detection an attractive spectrum sensing technique.

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