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OPTIMIZATION OF NON-UNIFORM CIRCULAR ARRAYS WITH COVARIANCE MATRIX ADAPTATION EVOLUTIONARY STRATEGY

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Abstract—In this paper, a covariance matrix adaptation evolutionary strategy (CMA-ES) is employed for optimization design of non-uniform circular antenna arrays. To achieve minimum sidelobe levels with the constraint of a specific first null beamwidth, the CMA-ES is utilized to find out the optimal weights and geometry of the circular array. The three various circular ring arrays are solved via CMA-ES, and the results are presented for arrays of varying configurations. The design results obtained with CMA-ES are compared to the existing array designs in the literature and to those found by the other evolutionary algorithms. Comparison with the results of other algorithms reveals the superiority of the CMA-ES.

1. INTRODUCTION

Antenna arrays have been widely used in applications including radar, sonar, air and space navigation, geophysical exploration, astronomy, and many other systems [1]. In particular, circular antenna arrays are very popular due to their advantages, such as an all-azimuth scan capability (i.e., the array can perform a 360° scan around its center), and the ability to keep the beam pattern invariant. In addition, circular arrays are less sensitive to mutual coupling as compared to linear and rectangular arrays since these do not have edge elements Moreover in direction-of-arrival (DOA) applications circular arrays provide almost invariant azimuth-angle coverage.

Hence many approaches of analysis and synthesis for circular antenna arrays have been investigated [2–17]. In [2, 3], the method

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of moments (MoM) and finite-difference time-domain (FDTD) are respectively utilized to analyze circular antenna arrays. In [4–6], the weights were optimized to reduce the peak sidelobe level (PSLL) in uniform circular arrays. Barkat and Benghalia [7] applied particle swarm optimization and the full-wave method for synthesis of superconducting circular array. Recently, work on non-uniform circular arrays has been concerned with optimization of their patterns [8–10]. Naturalistic optimization methods such as GA [8], PSO [9], and modified invasive weed optimization (modified IWO) [10] have been employed for optimizing the amplitudes and the positions that provide a radiation pattern with a minimum PSLL reduction for a constraint of beamwidth. However, many of the current difficulties with using GA, PSO, DE, IWO, and related techniques lies in the evolutionary settings of the algorithm. These settings play a vital role in the productivity of the optimization. For antenna engineers, choosing proper control parameters for the evolutionary algorithms is a very tedious and time-consuming task. The covariance matrix adaptation evolutionary strategy (CMA-ES) with very few settings required by the users is one of the most powerful evolutionary algorithms for real-valued optimization [18, 19] with many successful applications [18–20]. The main advantages of the CMA-ES lie in its efficient self-adaptation of the mutation distribution that offers avoiding any human choices that impact the performance of the algorithm. Generally, in the CMA-ES a small population size is usually sufficient and only one set of strategy parameters is maintained [18]. In [20], Gregory et al.,has applied CMA-ES for optimization design of the stacked-patch antennas and ultra-wideband linear antenna array layouts, to the same problem and achieved better results as compared to those by PSO.

In this paper, CMA-ES is utilized for designing non-uniform circular arrays with minimum sidelobe levels for the constraint of a specific first null beamwidth. The design problem can be formulated as a constrained minimization optimization problem. The argument of this minimization is the amplitudes and the positions of the circular array. The objective function presented in this work directly incorporates design constraints, which is entirely different from those reported in [8–10]. The main objective is PSLL minimization, subject to a desired beamwidth for the specific first null. The three various circular ring arrays are examined by using the CMA-ES. The design results obtained with CMA-ES are better than the previously published results and those found by the other evolutionary algorithms.

2. PROBLEM FORMULATION

2.1. Circular Antenna Array

The antenna array to be considered here is a non-uniform and planar, i.e., the elements are non-uniformly spaced on a circle of radius R in the x-y plane. The geometry of the N-element circular antenna array of point sources is shown in Figure 1, where φ_n denotes the angular position of the nth element. The array factor of the circular antenna array with the non-uniform spaced is given as follows:

$$AF(\varphi) = \sum_{n=1}^{N} I_n \exp\left(j(kR\cos(\varphi - \varphi_n) + \psi_n\right) \quad (1)$$

$$kR = \frac{2\pi R}{\lambda_f} = \sum_{i=1}^{N} d_i \quad (2)$$

where I_n and ψ_n denote the amplitude and the phase excitation of the nth element; d_n is the arc distance between elements n and $n-1$; $k = 2\pi/\lambda_f$ is the free-space propagation constant, where λ_f is the operating wavelength at the frequency of interest.

To direct the peak of the main beam in the φ_0 direction, the excitation phase of the nth element is chosen to be

$$\psi_n = -kR\cos(\varphi_0 - \varphi_n) \quad (3)$$

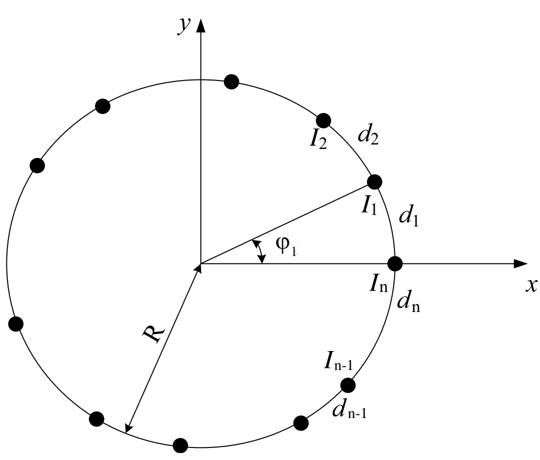

Figure 1. Geometry of N-element circular ring array.

where

$$\varphi_n = \frac{2\pi}{kR}\sum_{i=1}^{N} d_i \tag{4}$$

Then, the array factor can be rewritten as

$$AF(\varphi) = \sum_{n=1}^{N} I_n \exp\left(jkR\left[\cos(\varphi - \varphi_n) - \cos(\varphi_0 - \varphi_n)\right]\right) \tag{5}$$

2.2. Cost Function

The design objective of the circular array problem is to achieve the lowest PSLL with the constraint of a specific first null beamwidth (FNBW). Therefore the design problem can be formulated as a constrained minimization problem. To solve a constrained optimization problem, the constraints have to be incorporated into the algorithm's objective function. In [21], a comprehensive overview of the most frequently used constraint-handling techniques for evolutionary algorithms is provided, together with their advantages and drawbacks. The most often used approach is the penalty function method, where the objective function is augmented by a weighted sum of all constraint violations. The weights associated with every constraint are called penalty factors.

To circumvent this circular array design problem, the objective function is modeled based on the penalty function method, which is entirely different from those in [8–10]. The constrained optimization problem consists of minimizing the PSLL as objective function, subject to the FNBW. In this work, both the current excitation amplitude $(I_1, I_2, \ldots, I_n)^{\mathrm{T}}$ and the circular arc distance between any two adjacent elements $(d_1, d_2, \ldots, d_n)^{\mathrm{T}}$, i.e., $\mathbf{X} = (I_1, I_2, \ldots, I_n, d_1, d_2, \ldots, d_n)^{\mathrm{T}}$, are chosen as the optimal variables for optimization design of the circular antenna array. This design problem is therefore defined by the minimization of the objective function:

$$f(\mathbf{X}) = \max_{\varphi\in S} |AF(\varphi)/AF_{\max}| + \zeta\cdot\max\{0, |\mathrm{FNBW}_c - \mathrm{FNBW}_d| - 0.5\} \tag{6}$$

where S denotes the sidelobe region; $AF_{\max}$ is the peak value of the main beam; FNBW_c is the calculated fist null beamwidth, FNBW_d is the desired fist null beamwidth, and ζ is penalty factors. The second term on the right side of (6) represents the constraint on setting the FNBW_c to FNBW_d within ±0.5 deg error. The aim of this design is to effectively solve the proposed optimization problem. In this paper, the penalty factors ζ is set as 1000.

3. CMA-ES

The covariance matrix adaptation evolution strategy introduced by Hansen and Ostermeier [18] is one of the most powerful evolutionary algorithms for real-valued optimization. The main advantages of the CMA-ES lie in its invariance properties [18, 19], which are achieved by carefully designed variation and selection operators, and in its efficient self-adaptation of the mutation distribution. The standard CMA-ES relies on non-elitist (μ, λ)-selection, that is, the best μ of λ offspring form the next parent population and all former parents are discarded. For each set of strategy parameters to be adapted, several offspring have to be generated in each generation. More detail about the CMA-ES can be found in [18]. For completeness, we outline the CMA-ES algorithm as follows:

Step0: Initialize the mean point $\mathbf{x}$, the global step size σ and the covariance matrix $\mathbf{C}$. Assign $\mathbf{0}$ to the evolution paths $\mathbf{p}_\sigma$ and $\mathbf{p}_\mathbf{C}$ for the global step size and the covariance matrix respectively. The parameters are set in according to the default values in Table 1.

Step1: Compute the eigen-decomposition of the mutation covariance matrix $\mathbf{C} = \mathbf{BD}(\mathbf{BD})^{\mathrm{T}}$ such that the columns of $n \times n$ matrix $\mathbf{B}$ are the normalized eigenvectors of $\mathbf{C}$. The diagonal elements of the $n \times n$ diagonal are the square roots of the eigenvalues of $\mathbf{C}$.

Step2: Generate λ offspring candidate solutions

$$\mathbf{x}_i = \mathbf{x} + \sigma\mathbf{BD}\mathbf{z}_i, \ i = 1, \ldots, \lambda$$

where the $\mathbf{z}_i$ are the n-dimensional random vectors with independent components drawn from a normal distribution $N(0,1)$.

Step3: Evaluate the objective function values $f(\mathbf{x}_i)$ and order these λ offspring such that $\mathbf{x}_{k:\lambda}$ represents the k-best individual in the offspring population.

Step4: Recombine the best μ individuals in the offspring

Table 1. Default parameters for the CMA-ES.

Step size control
$c_\sigma = \frac{\mu_{eff}+2}{n+\mu_{eff}+3}, \quad d_\sigma = 1 + 2\cdot\max\left(0, \sqrt{(\mu_{eff}-1)/(n+1)} - 1\right) + c_\sigma$
Covariance matrix adaptation
$c_\mathbf{C} = \frac{4}{n+4}, \quad \mu_{eff} = \left(\sum_{i=1}^{u} w_i^2\right)^{-1'}$
$c_{cov} = \frac{1}{\mu_{eff}} \cdot \frac{2}{(n+\sqrt{2})^2} + \left(1 - \frac{1}{\mu_{eff}}\right) \cdot \min\left(1, \frac{2\mu_{eff}-1}{(n+2)^2+\mu_{eff}}\right)$

population by weighted recombination

$$\mathbf{x} \leftarrow \sum_{i=1}^{\mu} w_i \mathbf{x}_{i:\lambda}$$

where $\mu = \lfloor \lambda/2 \rfloor$ and $w_{i=1,\ldots\mu} = \dfrac{\ln(\mu+1) - \ln(i)}{\sum_{j=1}^{\mu}(\ln(\mu+1) - \ln(j))}$.

Step5: Update the evolution path according to

$$\mathbf{p}_\sigma \leftarrow (1 - c_\sigma)\mathbf{p}_\sigma + \sqrt{c_\sigma(2 - c_\sigma)} \cdot \sqrt{\mu_{eff}} \cdot \sum_{i=1}^{\mu} w_i \mathbf{z}_{i:\lambda}$$

and

$$\mathbf{p}_\mathbf{C} \leftarrow (1 - c_\mathbf{C})\mathbf{p}_\mathbf{C} + \sqrt{c_\mathbf{C}(2 - c_\mathbf{C})} \cdot \sqrt{\mu_{eff}} \cdot \mathbf{BD} \sum_{i=1}^{\mu} w_i \mathbf{z}_{i:\lambda}$$

Step6: Update the covariance matrix $\mathbf{C}$ according to

$$\mathbf{C} \leftarrow (1 - c_{cov})\mathbf{C} + c_{cov}\left(\frac{1}{\mu_{cov}}\mathbf{P}_\mathbf{C}\mathbf{P}_\mathbf{C}^{\mathrm{T}} + \left(1 - \frac{1}{\mu_{cov}}\right)\mathbf{C}_\mu\right)$$

where $\mathbf{C}_\mu = \mathbf{BD}\left(\sum_{i=1}^{\mu} w_i \mathbf{z}_{i:\lambda}(\mathbf{z}_{i:\lambda})^{\mathrm{T}}\right)(\mathbf{BD})^{\mathrm{T}}$

Step7: Update the global step size σ according to

$$\sigma \leftarrow \sigma \cdot \exp\left(\frac{c_\sigma}{d_\sigma}\left(\frac{\|\mathbf{p}_\sigma\|}{\mathrm{E}\,\|N(\mathbf{0},\mathbf{I})\|} - 1\right)\right)$$

where $\mathrm{E}\,\|\cdot\|$ denotes the average operator and the $\mathrm{E}\,\|N(\mathbf{0},\mathbf{I})\|$ is the average length of the n-dimensional normal distributed random vector.

Step8: Go to Step2 until the termination criterion is met.

In practice, the only parameter the user needs to choose is the population size (λ), everything else is predetermined from the problem properties. This makes a strong case for use of self-adaptive evolutionary strategies such as CMA-ES, as less user time is spent with configuring the algorithm or trial and error of evolutionary parameters (such as crossover and mutation rates of a GA). A suggested minimum population size is given by [18]

$$\lambda = 4 + \lfloor 3 \cdot \ln(n) \rfloor \tag{7}$$

a value chosen to be large enough to make a good estimation of the search space for most cost functions.

4. NUMERICAL SIMULATIONS AND RESULTS

The CMA-ES in this section is utilized for solving the three instantiations of the design problems designed by using GA [8] and PSO [9], and modified IWO [10]. The three instantiations are circular ring arrays with 8, 10, and 12 elements. In this case, we consider the peak of the radiation pattern to be directed along the x-axis, i.e., $\varphi_0 = 0$. We use experiments to evaluate the performance of CMA-ES by comparing it with GA, PSO, DE, and modified IWO. Some existing results of GA [8], PSO [9] and IWO [10] obtained from literature are also used for direct comparisons. All the experiments are run 30 times independently. Since the results reported in [8–10] were only the best ones, the best results found by the CMA-ES are used for comparisons.

The desired FNBW in this paper is assumed to be a constant, corresponding to a circular array with a uniform spacing ($d = \lambda/2$) between the elements. To meet the requirements of practical considerations, normalization is done for the current amplitudes with maximum value of the amplitude being set equal to 1. The DE-variant used here is called DE/rand/1/bin and is the most popular one in DE literature [22]. The parameters used for DE are set as follows: population size $NP = 60$, scaling factor $F = 0.5$, crossover rate $CR = 0.9$. The only setting parameter of the CMA-ES is population size that set according to (7). The maximum number of the function evaluations (MaxNFE) consumed the algorithms is listed in Table 2. The maximum number of the function evaluations used by CMA-ES is 27500, which is the same as that ($50 \times (1 + 0.1) \times 500 = 27500$) consumed by GA [8], and less than that consumed by DE, PSO [9], and modified IWO [10].

The first test case to demonstrate the effectiveness of the CMA-ES considers an 8-element circular ring array. The desired FNBW is set to 70 deg, which is the FNBW of the corresponding equally spaced

Table 2. The maximum number of the function evaluations used by CMA-ES, DE, GA, PSO, and modified IWO.

Algorithm	MaxNFE
CMA-ES	27,500
DE	300,000
GA	27,500 [8]
PSO	400000–900000 [9]
Modified IWO	100000 [10]

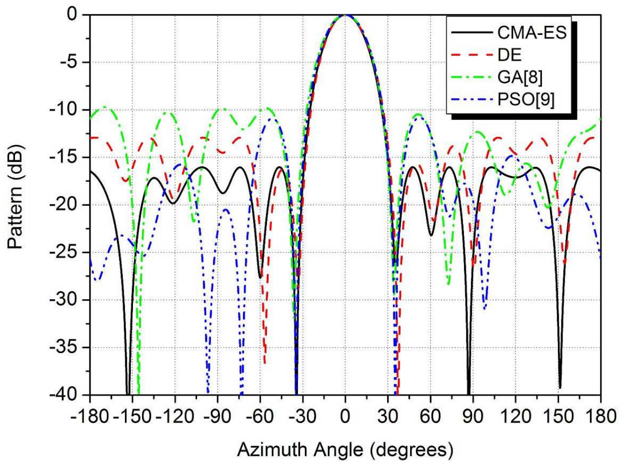

Figure 2. The optimized radiation patterns of the 8-element circular ring arrays using CMA-ES, GA, PSO, and DE, respectively.

($d = \lambda/2$) circular array. The optimization objective of this problem is to minimize PSLL while possibly maintaining the calculated FNBW the same as the desired FNBW. The formulation (6) acted as the fitness function. According to (7), the population size of the CMA-ES is set as 12. The synthesis was performed by the CMA-ES. The optimal performance parameters and the corresponding optimized variables are respectively listed in Table 3 and Table 4. The corresponding radiation pattern of the optimal array is depicted in Figure 2. As shown in Table 3, the PSLL obtained with the CMA-ES is lower by 6.21 dB, 5.22 dB, 3.1 dB, and 5.27 dB than those with the GA [8], PSO [9], DE, and modified IWO [10], respectively. The FNBW of the optimal array is 69.70 deg, which is less than that in [8–10] and obtained by DE. The 3 dB beamwidth of the optimal array is also listed in Table 3. As shown in Table 3, the 3 dB beamwidth obtained with the CMA-Es is narrower by 2.5 deg, 1.9 deg, and 0.3 deg than those with the GA, PSO and DE, respectively. It can be seen from Figure 2 and Table 3 that CMA-ES outperforms GA, PSO, DE, and modified IWO in terms of PSLL and FNBW.

The second example is that of the circular array with 10 elements. The FNBW of the 10-element equally spaced circular array is 56 deg. For CMA-ES, a value of 12 for population size was chosen since it is the minimum suggested population size according to (7). The best radiation pattern found by CMA-ES in Figure 3 has a PSLL of −15.03 dB and FNBW of 55.50 deg. The optimal performance parameters and the corresponding optimized variables are respectively listed in Table 3 and Table 4. The optimal design has a resulting PSLL of −15.03 dB, which is lower than −9.81 dB by GA [8], −12.31 dB by PSO [9], −13.03 dB by DE, and −13.78 dB by modified IWO [10]. Moreover, the 3 dB-beamwidth of the optimal array with CMA-ES is respectively smaller by 2.3 deg, 1.1 deg, and 1.0 deg than those with the GA [8], PSO [9] and DE.

In last example, CMA-ES is utilized for optimization design of 12-

Table 3. The optimal results found by five different algorithms.

No. of elements	Algorithm	PSLL (dB)	FNBW (degree)	3 dB-beamwidth (degree)
8	GA [8]	-9.81	70.50	32.20
	PSO [9]	-10.80	70.27	31.70
	DE	-12.92	70.20	30.10
	Modified IWO [10]	-10.75	70.27	NA
	CMA-ES[1]	-13.45	70.10	30.5
	CMA-ES	**-16.02**	**69.70**	**29.80**
10	GA [8]	-9.81	56.40	25.60
	PSO [9]	-12.31	55.85	24.40
	DE	-13.03	56.00	24.30
	Modified IWO [10]	-13.78	55.85	NA
	CMA-ES[1]	-14.59	55.90	**22.80**
	CMA-ES	**-15.03**	**55.50**	23.30
12	GA [8]	-11.83	46.00	20.70
	PSO [9]	-13.67	46.26	20.90
	DE	-13.98	46.60	17.20
	Modified IWO [10]	-14.23	46.23	NA
	CMA-ES[1]	-14.96	46.20	19.98
	CMA-ES	**-16.58**	**46.00**	**15.60**

[1] The results were obtained by CMA-ES with the cost function in [8].

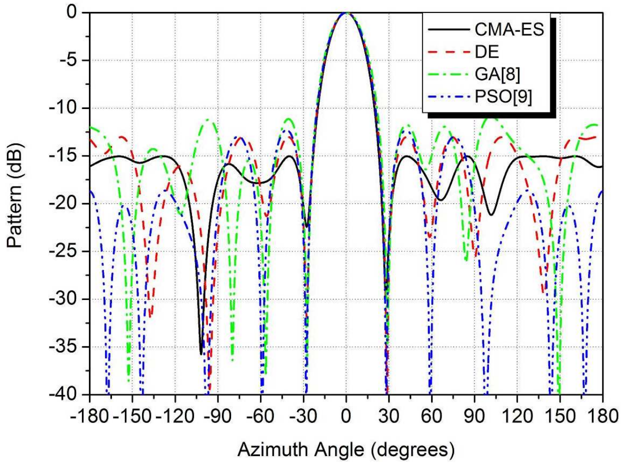

Figure 3. The optimized radiation patterns of the 10-element circular ring arrays using CMA-ES, GA, PSO, and DE, respectively.

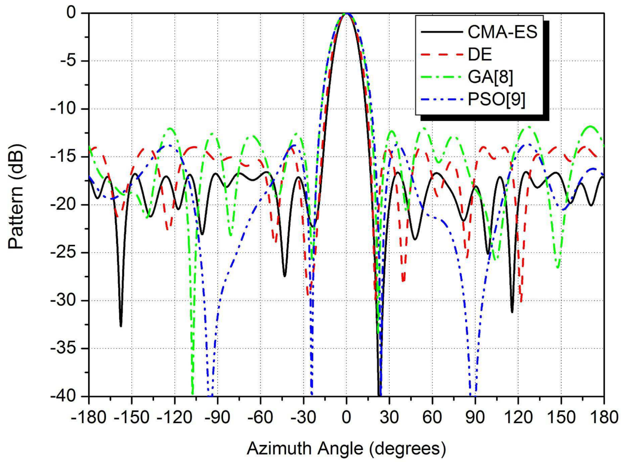

Figure 4. The optimized radiation patterns of the 12-element circular ring arrays using CMA-ES, GA, PSO, and DE, respectively.

element circular array. The FNBW of the 12-element equally spaced circular array is 46 deg. According to (7), the population size of the CMA-ES is set as 13. The radiation pattern of the optimal array with CMA-ES is shown in Figure 4. The optimal performance parameters and the corresponding optimized variables are respectively listed in

Table 3 and Table 4. The corresponding array pattern in Figure 4 has FNBW of 46 deg with a maximum PSLL 16.58 dB below the main beam. As shown in Table 3, the optimal PSLL by CMA-ES is 4.75 dB, 2.91 dB, 2.35 dB, and 2.6 dB lower than that of the best array in [8–10], and obtained by DE, respectively. Moreover, the FNBW of the three optimal arrays are almost equal. The optimal array by CMA-ES has 3 dB-beamwidth of 15.6 deg. However, compared with the optimal arrays by GA, PSO, and DE, the 3 dB-beamwidth of the array with CMA-ES is reduced by 5.1 deg, 5.3 deg and 1.6 deg, respectively.

In the above three examples, the CMA-ES with the proposed cost function is employed to optimize the circular ring arrays. However, without using the cost function proposed in this work, those results can not provide the effective of the CMA-ES. To verify the effectiveness of the CMA-ES, an experiment was added in this section. In this experiment, the cost function presented in [8] is used. The results obtained by CMA-ES with the cost function in [8] were listed in Table 3. As can be seen from Table 3, CMA-ES with the cost function in [8] outperforms GA, PSO, and modified IWO in terms of PSLL and FNBW (except the FNBW of 10-element array). The experiment results show the CMA-ES without the proposed cost function is also effective optimization technique for circular ring array designs.

Table 4. The optimal variables obtained with CMA-ES.

	8-element array		10-element array		12-element array	
	$d_n(\lambda_f)$	I_n	$d_n(\lambda_f)$	I_n	$d_n(\lambda_f)$	I_n
1	0.3658	0.7459	0.2987	0.9060	0.4519	0.7565
2	1.0431	0.3067	0.8274	0.4387	1.2291	0.5037
3	0.3490	0.3824	1.2924	0.7123	0.8276	0.3890
4	0.9651	0.8935	0.9502	1.0000	1.1110	0.6304
5	0.5299	1.0000	0.4556	0.9160	1.0408	0.6848
6	0.9463	0.4576	0.9036	0.5359	0.5106	0.8254
7	1.4389	0.8217	0.6752	0.5371	0.3254	0.6978
8	0.3161	0.4000	0.7076	0.2915	0.4881	0.6349
9			0.7576	0.9334	1.6115	0.7767
10			0.3172	0.60181	0.8415	0.3921
11					1.7681	0.7524
12					0.4644	1.0000

5. CONCLUSION

In this paper, non-uniform circular antenna array designs by using the CMA-ES have been presented. The design objective is to achieve minimum sidelobe levels with the constraint of a specific first null beamwidth in the circular array. We formulated the design problem as a constrained optimization problem. The objective function presented in this work is based on an exact penalty method in order to include design constraints. The CMA-ES is utilized to find out the optimal weights and geometry of the circular array. The design results obtained with CMA-ES are compared to the existing array designs in the literature and to those found by the other evolutionary algorithms. Comparison with the results of other algorithms reveals the superiority of the CMA-ES and confirms its potential application in electromagnetics.

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