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A NOVEL DIAGONAL LOADING METHOD FOR ROBUST ADAPTIVE BEAMFORMING

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Abstract—The diagonal loading method is a simple and efficient method to improve the robustness of beamformers. However, how to determine the ideal diagonal loading level has not been adequately addressed. In this paper, it is observed in the simulation that the peak of the main beam is moved with the diagonal loading level when there exists a Direction of Arrival (DOA) estimation error. Based on the observation, a novel diagonal loading method is proposed, and a tradeoff exists between the robustness and the interference suppression capability by controlling the peak location of the main beam. As long as the DOA estimation error is less than the half of the width of main beam, the proposed beamformer will not suppress the Signal of Interest (SOI) as interference. Numerical experiments prove the effectiveness of the proposed method.

1. INTRODUCTION

Beamforming is an effective way to reach some specific criteria, such as minimum variance and maximum signal-to-interference-plus-noise ratio (SINR), by utilizing antenna arrays. The simplest way of beamforming is data-independent beamforming [1]. The data-dependent beamformer is more complex with better performance than data-independent beamformer. A well-known representative example is standard Capon beamformer (SCB) [2]. If the array steering vector corresponding to the SOI is accurately known, the Capon beamformer

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has much better interference suppression capability than the data-independent beamformer.

However, in many practical applications, the steering vector of the SOI cannot be obtained precisely due to the differences between the estimated signal DOA and the true DOA or between the assumed array response and the true array response (array calibration errors). With the imprecise SOI steering vector, the performance of the SCB may be even worse than that of the data-independent beamformers [3]. In order to overcome this problem, the robust beamformer has to be adopted.

Many approaches have been proposed to make the adaptive beamformer robust against imprecise SOI steering vector. For the typical array imperfection, i.e., DOA estimation error of SOI, many solutions have been proposed, such as convex quadratic constraints [4], Bayesian approach [5] and uncertainty set based method [6]. All these methods belong to a class of popular robust beamforming methods, the diagonal loading method [7]. However, the selection of the optimal diagonal loading level is still not clear in practice.

Recently, some advanced robust methods have been proposed [8–11]. In these papers, the array magnitude responses for the array steering vectors in a polyhedron or sphere set are constrained to exceed unity in optimization. It was proven that these beamformers also belong to the family of the diagonal loading method [10]. If the knowledge of the array steering vector uncertainty can be precise, the diagonal loading level can be decided. However, it is not clear how to choose the uncertainty in which the actual steering vector is expected to lie [11]. There are also other diagonal loading methods for the specific application [12], but it is not always useful. More recently, there are very few papers concerned with the user parameter free methods [13, 14].

In this paper, a totally different diagonal loading method is proposed based on a very clear physical intuition. The main objective of the proposed diagonal loading method is to deal with the estimated DOA error, and the array calibration error is not included in this paper. Firstly, a simulation is performed to obtain the insight into the relationship between the beampattern and diagonal loading level. It is observed that the peak of the main beam is moved with increasing of the diagonal loading level. When the peak of the main beam is located at the estimated DOA of SOI, the beamformer becomes robust against the DOA estimation error. However, it is also observed that the interference suppression capability is decreased with increasing of the diagonal loading level. Therefore, there is a tradeoff between the robustness and interference suppression capability. Then, a novel diagonal loading method is proposed.

In Section 2, the problem of interest is formulated. In Section 3, the proposed diagonal loading method for robust beamforming is presented. Numerical examples illustrating the performance of the proposed method are given in Section 4. Finally, Section 5 contains the conclusions.

2. PROBLEM FORMULATION

Consider a uniform linear array with M sensors illuminated by a far field narrow band SOI and J uncorrelated interferers. With the assumption that the noise is spatially white and uncorrelated with the SOI and interferences, the received signal vector at the snapshot time index k can be written as

$$\mathbf{y}\,(k) = \mathbf{d}\,(k) + \mathbf{i}\,(k) + \mathbf{n}\,(k) \tag{1}$$

where $\mathbf{y}\,(k)$ is the vector of received signals. $\mathbf{d}\,(k)$, $\mathbf{i}\,(k)$ and $\mathbf{n}\,(k)$ are the desired signal, interference and noise components, respectively.

With the assumption that the individual vector components of this model are mutually uncorrelated, the theoretical covariance matrix $\mathbf{R}$ of the array output vector can be denoted as follows.

$$\mathbf{R} = \sigma_0^2 \mathbf{a}_0(\theta_0)\mathbf{a}_0^H(\theta_0) + \sum_{j=1}^{J} \sigma_j^2 \mathbf{a}_j \mathbf{a}_j^H + \mathbf{Q} \tag{2}$$

where $\mathbf{R}$ is the theoretical covariance matrix; σ_0^2 and σ_j^2 are the powers of the SOI and the interference; $\mathbf{a}_0(\theta_0)$ and $\mathbf{a}_j$ are the steering vectors which are the functions of the location parameters of the emitting sources (e.g., DOA); θ_0 is the real DOA of SOI; $(*)^H$ denotes the conjugate transpose; and $\mathbf{Q}$ is the noise covariance matrix.

In practical applications, $\mathbf{R}$ cannot be obtained directly. It is replaced by the sample covariance matrix $\hat{\mathbf{R}}$, where

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{y}\,(k)\mathbf{y}^H\,(k) \tag{3}$$

where $\hat{\mathbf{R}}$ is the sample covariance matrix, and K denotes the number of snapshots. When there are enough snapshots, the estimated covariance matrix $\hat{\mathbf{R}}$ is almost equal to the real covariance matrix $\mathbf{R}$.

The SCB can be derived from the following linearly constrained optimization problem

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{R} \mathbf{w}$$
$$\text{subject to} \quad \mathbf{w}^H \mathbf{a}_0(\theta_0) = 1$$

where $\mathbf{w}$ is the adaptive array weight.

The solution is easily derived

$$\mathbf{w}_0 = \frac{\mathbf{R}^{-1}\mathbf{a}_0(\theta_0)}{\mathbf{a}_0^H(\theta_0)\mathbf{R}^{-1}\mathbf{a}_0(\theta_0)} \tag{4}$$

When the adaptive array weight is obtained, the output signal $r(k)$ is therefore given by

$$r(k) = \mathbf{w}_0^H \mathbf{y}(k) \tag{5}$$

As discussed previously, the performance of SCB may be even worse than that of the data-independent beamformers if there is some DOA estimation error of SOI. Then the SCB will heavily suppress the SOI as interference to minimize the output power.

The diagonal loading method is a simple and effective way to make the SCB more robust by adding a real weighting identity matrix to the covariance matrix. The new adaptive array weight can be expressed as

$$\mathbf{w}_d = \frac{(\mathbf{R} + \lambda I)^{-1}\mathbf{a}_0(\theta_0 + \Delta\theta)}{\mathbf{a}_0^H(\theta_0 + \Delta\theta)(\mathbf{R} + \lambda I)^{-1}\mathbf{a}_0(\theta_0 + \Delta\theta)} \tag{6}$$

where $\mathbf{w}_d$ is the new adaptive array weight; the subscript d denotes diagonal loading; $\Delta\theta$ is the DOA error of SOI due to the estimator; I is the identity matrix; and λ is the real diagonal loading level. To simplify denotation, $\mathbf{a}_0(\theta + \Delta\theta)$ will be denoted as $\bar{\mathbf{a}}_0$ in the following sections.

The quality of SOI estimation is typically measured by the signal-to-interference-plus-noise ratio (SINR) as following

$$SINR = \frac{\sigma_0^2 \left|\mathrm{w}_d^H \mathrm{a}_0\right|^2}{\mathrm{w}_d^H \left(\sum\limits_{j=1}^{J} \sigma_j^2 \mathrm{a}_j \mathrm{a}_j^H + \mathrm{Q}\right) \mathrm{w}} \tag{7}$$

Although there are some works to determine the diagonal loading level [7–11], most of these approaches either are in an ad-hoc way or need user parameters that might not be available in practice. Therefore, user parameter-free method to determine the diagonal loading level is high desired.

3. THE PROPOSED DIAGONAL LOADING METHOD

For the diagonal loading method, the amplitude response for the angle θ can be expressed as

$$\left|\mathbf{w}_d^H \mathbf{a}(\theta)\right|^2 = \mathbf{a}^H(\theta)\,\mathbf{w}_d \mathbf{w}_d^H \mathbf{a}(\theta) \tag{8}$$

$$= \frac{\mathbf{a}^H(\theta)\,(\lambda I + \mathbf{R})^{-1}\,\bar{\mathbf{a}}_0 \bar{\mathbf{a}}_0^H\,(\lambda I + \mathbf{R})^{-1}\,\mathbf{a}(\theta)}{\left[\bar{\mathbf{a}}_0^H\,(\lambda I + \mathbf{R})^{-1}\,\bar{\mathbf{a}}_0\right]^2} \tag{9}$$

where $\mathbf{a}(\theta)$ is the steering vector for the DOA θ $(-90^\circ \le \theta \le 90^\circ)$.

The covariance matrix $\mathbf{R}$ is a positive definitive matrix and can be expressed as

$$\mathbf{R} = \sum_i a_i \mathbf{v_i} \mathbf{v_i}^H \tag{10}$$

where a_i is the eigenvalue, and $\mathbf{v_i}$ is its corresponding eigenvector of $\mathbf{R}$. Therefore, the upper Equation (9) can be transformed as

$$\left|\mathbf{w}_d^H \mathbf{a}(\theta)\right|^2 = \frac{\left(\sum_i \frac{\mathbf{a}^H(\theta)\mathbf{v}_i \mathbf{v}_i^H \bar{\mathbf{a}}_0}{a_i+\lambda}\right)\left(\sum_i \frac{\bar{\mathbf{a}}_0^H \mathbf{v}_i \mathbf{v}_i^H \mathbf{a}(\theta)}{a_i+\lambda}\right)}{\left(\sum_i \frac{\bar{\mathbf{a}}_0^H \mathbf{v}_i \mathbf{v}_i^H \bar{\mathbf{a}}_0}{a_i+\lambda}\right)^2} \tag{11}$$

It is clear that the beampattern is relative to the diagonal loading level. The beampatter will be changed with variable diagonal loading level. In order to obtain the insight into the relationship between diagonal loading level and array beampattern, a simulation is presented in Fig. 1. The number of elements is $M = 10$. The DOAs of the SOI

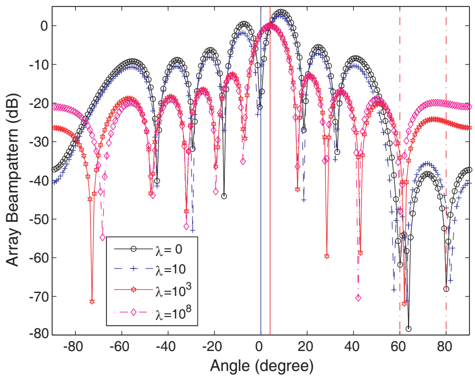

Figure 1. Comparison of the beampatterns of different diagonal loading levels ($\Delta\theta = 4^\circ$).

and two interferences are $0°$, $60°$, $80°$. The powers of the SOI and two interferences are $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = 10\,\text{dB}$. The DOA estimation error of the SOI is $\Delta\theta = 4°$.

It is noticed in Fig. 1 that, when there exists a DOA estimation error, the peak of the main beam is gradually moved toward the estimated DOA with increasing diagonal loading level. When the diagonal loading level is zero, the beamformer is transformed back to SCB, and the SOI is suppressed heavily as interference. The SOI suppression will be decreased with moving the peak toward the estimated DOA. Hence, the beamformer becomes robust against the DOA estimation error of SOI. However, it is also noticed that the interference suppression capability is also decreased with increasing the diagonal loading level.

Therefore, the following results can be obtained from Fig. 1, which motivate the proposed diagonal loading method. The peak of the main beam will be gradually moved toward the estimated DOA of SOI with increasing diagonal loading level. The greater is the diagonal loading level, the nearer to the estimated DOA is the peak of the main beam, and the weaker is the interference suppression capability.

Remember that the main problem of the diagonal loading method is how to determine the diagonal loading level which makes the beamformer robust to the DOA estimation error of SOI. It is apparent that an ideal solution is to move the maximum of the main beam to the estimated DOA precisely. Then the allowable DOA estimation error will be at least half of the width of the main beam. In this case, the SOI will not be suppressed heavily as interference. Hence, in order to find the ideal robust beamformer, the problem becomes how to move the maximum of the main beam to the estimated DOA.

Throughout this paper, it is assumed that the DOA difference between the SOI and any interference is greater than half of the width of the main beam, and the broadside of the uniform linear array is directed to the desired signal. In order to obtain the condition of the peak locating at the estimated DOA, let us look into the main beam. When the amplitude responses for the angles included in the main beam are symmetrical centering at the estimated DOA of SOI, the peak is located at the estimated DOA of SOI. Therefore, a sufficient condition to ensure that the maximum located at the estimated DOA is the exact left-right symmetry at the estimated DOA. It means that the amplitude response according to the left is equal to that according to the right. A direct method is to make the amplitude responses of $\hat{\theta}_0 + \delta\theta$ and $\hat{\theta}_0 - \delta\theta$ equal where the $\hat{\theta}_0$ is the estimated DOA of SOI, and the $\delta\theta$ is less than or equal to half of the width of the mainbeam. Without loss of generality, we make $\delta\theta \leq \frac{1}{2}\theta_{md}$. The θ_{md} is the approximate width of the main beam which can be expressed as $\theta_{md} = \frac{50.7\lambda}{Md}$ [1].

Therefore, in order to find the ideal diagonal loading level, the diagonal loading level should move the maximum to the estimated DOA. Here, the ideal diagonal loading level is meant to assure that the peak of the main beam is located at the estimated DOA of SOI. Mathematically, it can be expressed as following

$$\left|\mathbf{w}_d^H \mathbf{a}\left(\hat{\theta}_0 + \delta\theta\right)\right|^2 - \left|\mathbf{w}_d^H \mathbf{a}\left(\hat{\theta}_0 - \delta\theta\right)\right|^2 = 0 \tag{12}$$

It means that the amplitude response at the $\hat{\theta}_0 + \delta\theta$ is precisely equal to that of the $\hat{\theta}_0 - \delta\theta$.

It is also observed from Fig. 1 that the peak of the main beam locates the estimated DOA precisely when the $\lambda \to \infty$. With $\lambda \to \infty$, the Capon beamformer becomes the classical beamformer, so the peak of the main beam is always located at the estimated DOA of SOI.

Based on the observation from Fig. 1, the interference suppression capability is the worst one when the diagonal loading level goes to infinity. A possible explanation is as follows. In some cases, the diagonal loading method may be explained to add white noise to the received signal. Hence, when the diagonal loading level goes to infinity, the added white noise also goes to infinity. Then, the interferences can be omitted compared with the added white noise. Fortunately, there exists a tradeoff between the robustness and interference suppression capability by controlling the peak location of the main beam.

Then, the diagonal loading level can be obtained by making

$$\left|\mathbf{w}_d^H \mathbf{a}\left(\hat{\theta}_0 + \delta\theta\right)\right|^2 - \left|\mathbf{w}_d^H \mathbf{a}\left(\hat{\theta}_0 - \delta\theta\right)\right|^2 = \varepsilon \tag{13}$$

where ε can be regarded as the tradeoff parameter. The tradeoff parameter controls the robustness and interference suppression capability.

At the same time, it is also noted that the following function is not convex function for all circumstances.

$$f(\lambda) = \left|\mathbf{w}_d^H \mathbf{a}\left(\hat{\theta}_0 + \delta\theta\right)\right|^2 - \left|\mathbf{w}_d^H \mathbf{a}\left(\hat{\theta}_0 - \delta\theta\right)\right|^2 - \varepsilon \tag{14}$$

Therefore, there may exist multiple solutions. For all solutions, the beamformers are same robust. However, the interference suppression capability is different for different solutions. Remember that the interference suppression capability will decrease with increasing diagonal loading level. Therefore, the minimal diagonal loading level among all solutions is desired. It is not necessary to find all solutions. The problem is solved by finding the minimal solution.

As the function is not the convex, there is no guarantee to find the minimal solution with any efficient method. However, many simulations prove that there is an efficient method to find a suitable solution, e.g., a Newton's method with the initial parameter $\lambda = 0$. To simplify the process, we can make $\delta\theta = 1^\circ$ which can ensure the symmetry if the width of the main beam is greater than 2°. A lot of simulations indicate that the tradeoff parameter can be set to a constant when the number of sensors is fixed, i.e., the width of the main beam is approximately fixed. Therefore, there is no user parameter for given array which is desired for many practical applications. It is also important to point out that the diagonal loading method is useless when the DOA estimation error is greater than half of the width of the main beam as all other diagonal loading methods.

4. SIMULATION

In this section, the proposed diagonal loading method is compared with the SCB and the method in the reference [10] (RCB) by simulations. For all the simulations considered below, a uniform linear array is assumed with $M = 10$ sensors and half-wavelength sensor spacing. Enough snapshots are assumed, e.g., $K = 200$. The powers of the two interferences are $\sigma_1^2 = \sigma_2^2 = 20\,\text{dB}$, and the power of white noise is $0\,\text{dB}$. The DOAs of two interferences are $\theta_1 = 60^\circ$ and $\theta_2 = 80^\circ$. The real DOA of the SOI is $\theta_0 = 0^\circ$. For the RCB, according to [10], the parameter of the array steering vector should be greater than the corresponding uncertainty of the array steering vector, which is made as 4.5 in all simulations. As the previous discussion, the approximate width of main beam is fixed, and the tradeoff parameter is set to constant as $\varepsilon = 0.05$ for the proposed method.

In Fig. 2 and Fig. 3, the beampatterns are compared for SCB, RCB and the proposed method. The real vertical line and three dot vertical lines denote the locations of real signal DOA, estimated signal DOA, and interference DOAs, respectively. In Fig. 2, there is no DOA estimation error, and the power of SOI is $SNR = 10\,\text{dB}$. The beampattern of the proposed method is the same as that of SCB. However, the RCB beampattern has no nulls at the DOAs of all interferences. In this case, it is clear that the interference suppression capability of the proposed method is better than that of RCB. In Fig. 3, the DOA estimation error is $\Delta\theta = 6^\circ$, and the power of SOI is $SNR = 30\,\text{dB}$. Due to the DOA estimation error, the SCB and RCB would heavily suppress the SOI as interference. However, the proposed method would not suppress the SOI as interference because the DOA estimation error is less than the half of width of the main beam. The observations show that the proposed method is more robust than RCB against the DOA estimation error.

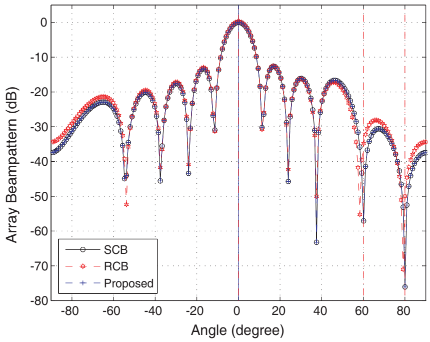

Figure 2. Comparison of the beampatterns of SCB, RCB and proposed method ($\Delta\theta = 0$, $SNR = 10\,\text{dB}$, $INR_1 = INR_2 = 20\,\text{dB}$).

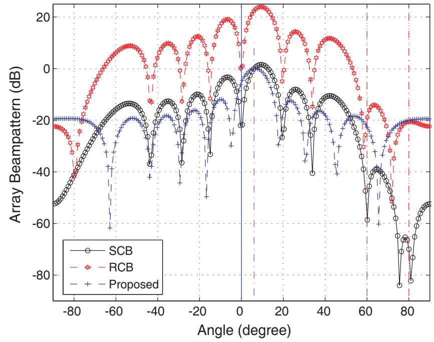

Figure 3. Comparison of the beampatterns of SCB, RCB and proposed method ($\Delta\theta = 6°$, $SNR = 30\,\text{dB}$, $INR_1 = INR_2 = 20\,\text{dB}$).

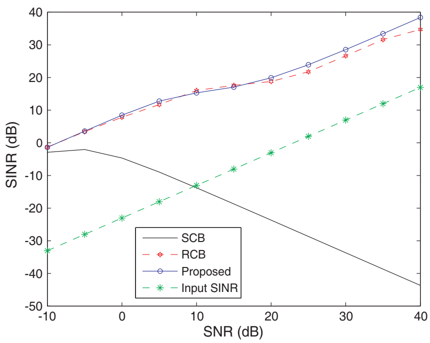

Figure 4. Comparison of the SINRs of SCB, RCB and proposed method ($\Delta\theta = 3°$, $INR_1 = INR_2 = 20\,\text{dB}$).

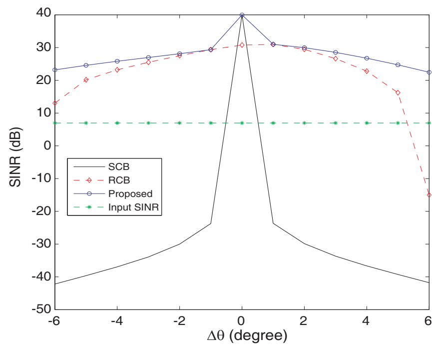

Figure 5. Comparison of the SINRs of SCB, RCB and proposed method ($SNR = 30\,\text{dB}$, $INR_1 = INR_2 = 20\,\text{dB}$).

In Fig. 4 and Fig. 5, the output SINRs are compared for the SCB, RCB and the proposed method. Here, the output SINRs is defined as Equation (7). For Fig. 4, the DOA estimation error is fixed as $\Delta\theta = 3°$. We vary the SNR by changing the power of SOI. For low SNR, the SINR of the proposed method is almost the same as that of RCB. However, for high SNR, the SINR of the proposed method is greater than that of RCB. It indicates that the proposed method is more robust than RCB against SNR. In Fig. 5, the SNR is fixed as 30 dB, and the DOA estimation error is variable. With some DOA estimation error, the SINR of the proposed method is almost same as the RCB except the zero DOA estimation error. The greater is the DOA estimation error, the higher is the SINR of the proposed method than the RCB. It indicates that the proposed method is more robust than the RCB against DOA estimation error.

5. CONCLUSION

In this paper, a novel diagonal loading method for robust beamforming is proposed. The motivation is based on the observations that the peak of the main beam will be shifted with the diagonal loading level when there exists a DOA estimation error. An ideal solution is to make the maximum of the main beam locate at the estimated DOA. It is achieved with the diagonal loading level going to infinity, and the SOI will not be suppressed as interference. However, the interference suppression capability will decrease with increasing diagonal loading level. Therefore, there exists a tradeoff between the robustness and interference suppression capability. In order to make the beamformer robust, the proposed diagonal loading method locates the maximum of the mainbeam almost at the estimated DOA. The proposed beamformer is robust when the DOA estimation error is less than half of the width of the main beam. The simulations show that the proposed method is more robust than the RCB and that there is no user parameter for any given uniform linear array.

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