

## DESIGN OF A DUAL-BAND METAMATERIAL BAND-PASS FILTER USING ZEROth ORDER RESONANCE

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**Abstract**—This paper suggests the design of a novel miniaturized dual-band bandpass filter based on the composite right/left-handed (CRLH) metamaterial structure. In detail, subwavelength resonators are realized through the zeroth order resonance (ZOR), and inverter structures are proposed to control the coupling between neighboring ZOR resonators. The proposed technique is validated by the EM predictions, the proof of metamaterial properties with the ZOR field distributions and extracted constitutive parameters, and measurements. It is found that the suggested method enables the remarkable size reduction from the conventional filters such as the parallel coupled type which is designed on the basis of the half-wavelength resonance.

### 1. INTRODUCTION

As RF systems play an important role in mobile communication, the traveling users tend to pursue more convenience such as not only multi-functions but also higher mobility. The former results in the multi-band service-oriented device, and the latter drives the development of the equipment into the size reduction.

For a long time, a number of techniques have been reported to provide multiple bands for components, especially, the dual band cases which are briefly mentioned now. The work in [1] simply connects two independently designed bandpass filters. Another method is using stepped impedance resonators (SIRs) to move the harmonic up or down in the frequency [2, 3].

In [4], the traditional resonators are mixed with the DGS (Defected Ground Structure) which separates the successive elements

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on the top and bottom of the substrate. Though all these previous structures formed the dual bands, they ended up with the limitation in the effective size reduction, since their resonators basically rely on the half-wavelength resonance.

With regard to overcoming the current design limitations and going beyond the boundary, new alternative approaches have been sought, and the concept of metamaterial structures started to draw lots of attention. Without resorting to the half-wavelength resonator, the SRR next to the line and the CSRR under the line play the lumped elements for the LH and RH characteristics and make compact bandstop filters as well as BPFs, where the finite number of unit cells are placed along the uniform line [5]. Instead of the SRR or CSRR, the grounded stub and interdigital line are combined for the composite Right and Left handedness (CRLH) phenomenon [6]. The zeroth order resonance (ZOR) of no phase variation between 2 ports is introduced, and a CRLH transmission line with 18 cells is designed to support the ZOR energy-flow. The unit cells of the components in [5–9] are smaller by the factor of 10 from the conventional resonators and result in enormous size minimization. But these works could not present dual pass-band performances.

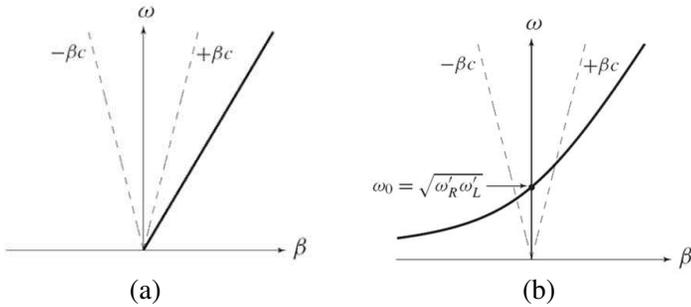
In this paper, we propose the design of a novel dual-band bandpass filter based on the metamaterial CRLH ZOR to reduce the size (without lumped elements loaded) for the passband. Also, we adopt the idea to control the coupling between metamaterial resonators to make the bandpass filter have the dual-passband for the GSM and ISM as wanted.

In the sections to come, the design process including the circuit simulation is presented. The filter is implemented, and the field distributions of the ZORs obtained by the 3D full-wave EM simulation and effective constitutive parameters are shown to prove the metamaterial characteristic. Finally, the proposed technique is validated by the fabrication and measurement which is followed by technical discussions and conclusions.

## 2. THEORY AND DESIGN

Prior to the main part of the design, the basic differences between the traditional (Right-handed) transmission line and composite Right- and Left-handed geometry are addressed with their dispersion diagrams in Fig. 1.

Figure 1(a) shows the linear relationship of the frequency and the wavenumber which takes only the positive value, and the phase velocity is kept constant. The dispersion curve of the CRLH line with



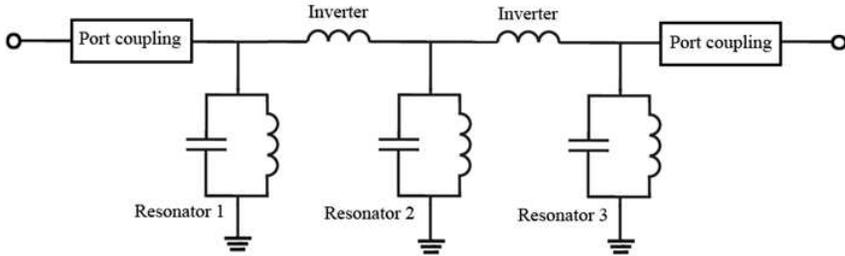
**Figure 1.** Typical dispersive characteristics: (a) Traditional transmission line. (b) Composite Right- and Left-handedness line [5].

Fig. 1(b) is nonlinear. On top of it, for a certain frequency band, the wavenumber becomes negative. This means, for the left-handed propagation, that the phase velocity has the opposite sign to the group velocity. When the right-handedness (RH) and left-handedness (LH) are combined, the total propagation constants become negative, 0 or positive. Especially, the total propagation constant goes to zero at not DC. The resonance with no phase variation (ZOR) is generated regardless of the position along the geometry, and it can be interpreted that we are free from the resonance condition of half-wavelength and can make the size-reduction possible beyond the limitation.

There are a number of metamaterial filters available in literature [6]. As shown in their designs, each structure consists of multiple cells up to 17 ~ 18. The ZOR occurs from such long structures (many half-wavelengths long), because they use the interdigital lines as pure capacitance and the stub as pure inductance for the LH property, and the simple connection along the entire periodic cells for the RH property. However, in our case, a ZOR can be generated from only one cell (much smaller than a quarter-wavelength), since we intentionally use the parasitic series inductance as the RH inductor from the interdigital lines and the parasitic shunt capacitance as the RH capacitance from the grounded stub.

Before we figure out the elements of the ZORs for the required performance, it is necessary to know the prototype of the bandpass filter for the same purpose. In our suggestion, the bandpass filter prototype is achieved by the conventional filter synthesis technique.

In Fig. 2, a filter prototype is made of the resonators and inverters. Each resonator comprises a shunt L and shunt C. And the inductive coupling is adopted for the inverter. To have passbands at 900 MHz and 2.4 GHz, the following element values can be calculated through the initial circuit design.



**Figure 2.** Brief sketch of the bandpass filter prototype to be used.

**Table 1.** Circuit elements of the bandpass filter prototype.

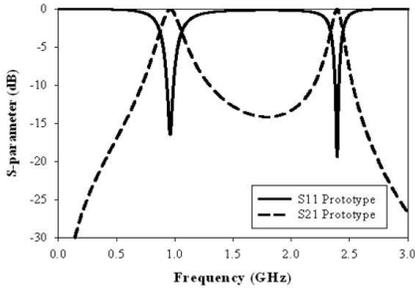
Element	Value	
	Resonator 1	Resonator 2
L for the $i$ -th shunt resonator	2.4 nH	10 nH
C1 for the $i$ -th shunt resonator	11.2 pF	10 pF
L for Inverter	0.5 nH	
Port coupling	5 nH	

Entering the circuit elements from Table 1, the frequency response as the performance of the circuit in Fig. 2 is evaluated and investigated. We plot the scattering parameters of the prototype circuit.

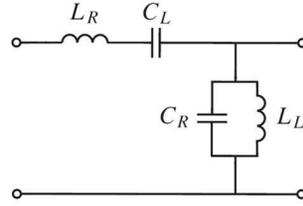
Seeing the  $s$ -parameters as the result of the prototype circuit simulation where it is assumed that there are no lossy elements in the circuit, we obtain the perfect transmission at the wanted frequency points at 900 MHz and 2.4 GHz. If lossy elements are included, the insertion loss will occur, and the quality factor of the resonator will be lower.

The bandpass filter prototype is successfully synthesized, but it is still a conventional filter. So we propose that the resonators in Fig. 2 should be replaced by the ZORs for the same frequency response and remarkable miniaturization. So the CRLH ZOR is shown in Fig. 4 where the LH line ( $L_L$  and  $C_L$ ) combined with the RH line ( $L_R$  and  $C_R$ ) for one resonator. Conspicuously, since the size of one ZOR is equal to  $\Delta z$  (per-unit length) in our case, the concept of  $\Delta z$  is not needed, and the resonator will be much less than a quarter-wavelength.

$L_L$ ,  $C_L$ ,  $L_R$  and  $C_R$  will be calculated by the equivalence between the ZOR and its corresponding resonator from Table 1. At this time, among a number of possibilities of  $L_L$ ,  $C_L$ ,  $L_R$  and  $C_R$  of one ZOR, care must be taken to meet the ZOR condition that the center frequency



**Figure 3.** Frequency response of the bandpass filter prototype (circuit simulation).



**Figure 4.** Equivalent circuit of a unit CRLH resonator.

should be the ZOR point using the following formulae.

$$\kappa = L_R C_L + L_L C_R, \quad \omega_L = \frac{1}{\sqrt{L_L C_L}}, \quad \omega_R = \frac{1}{\sqrt{L_R C_R}} \quad (1)$$

$$\omega_{sh} = \frac{1}{\sqrt{L_L C_R}}, \quad \omega_{se} = \frac{1}{\sqrt{L_R C_L}}, \quad \omega_0 = \sqrt{\omega_R \omega_L} \quad (2)$$

$$Z_R = \sqrt{\frac{L_R}{C_R}}, \quad Z_L = \sqrt{\frac{L_L}{C_L}} \quad (3)$$

$$\omega_{cL} = \omega_0 \sqrt{\frac{[\kappa + (2/\omega_L)^2] \omega_0^2 - \sqrt{[\kappa + (2/\omega_L)^2]^2 \omega_0^4 - 4}}{2}} \quad (4)$$

$$\omega_{cR} = \omega_0 \sqrt{\frac{[\kappa + (2/\omega_L)^2] \omega_0^2 - \sqrt{[\kappa + (2/\omega_L)^2]^2 \omega_0^4 - 4}}{2}} \quad (5)$$

where  $\omega_{sh}$ ,  $\omega_{se}$ ,  $W_{IDC}$ ,  $l_{IDC}$ ,  $l_{stub}$ , and  $W_{stub}$  are shunt resonance angular frequency, series resonance angular frequency, finger width of interdigital lines, finger length of the interdigital lines, length of the grounded stub, and width of the grounded stub, respectively. Given the important frequency points from the specifications, these equations are used to make  $\omega_{sh}$  and  $\omega_{se}$  identical at the ZOR point, which is the same as the center frequency.

After we have taken the steps above to determine the circuit elements, the design proceeds to the step of figuring out the 3D structure for the target performance. For this, the physical dimensions are found by the 3D EM full-wave simulation, suitable for each element of the circuit, using the initial size by way of the approximate formulae to convert the circuit element to the geometrical element.

### 3. IMPLEMENTATION AND RESULTS

The design and implementation are carried out with the following detailed specifications.

**Table 2.** Design specifications.

Item	Specification	
	GSM band	ISM band
Center frequency	900 MHz	2.4 GHz
Bandwidth	80 MHz	80 MHz
Insertion loss	< 2 dB	< 2 dB
Return loss	< -15 dB	< -15 dB
Stop-band	< -15 dB	

For the physical implementation of the metamaterial resonator, we will choose the interdigital line geometry with the stub short-circuited as in [5], but they will be used in a different way, compared with the structure of [5, 6].

In Fig. 5, there are two versions: Fig. 5(a) does not fit our design, but we choose Fig. 5(b) as the symmetric type (T-shaped) for facilitating coupling elements. Since this structure turns out appropriate for the ZOR, the realization proceeds with the corresponding conversion formulas [9, 10], considering the microstrip line with the 50 mil thick substrate of relative dielectric constant 10.2.

$$L_L = \frac{Z_c}{\omega} \tan(\beta_{\text{eff}} l_{\text{stub}}) \quad (6)$$

$$C_L \approx (\varepsilon_r + 1) l_{DC} [(n-3)A_1 + A_2] \text{ (pF)}$$

with  $n$  the number of the fingers (7)

$$A_1 = 4.409 \tanh [0.55(h/w_{DC})^{0.45}] \cdot 10^{-6} \text{ (pF/}\mu\text{m)} \quad (8)$$

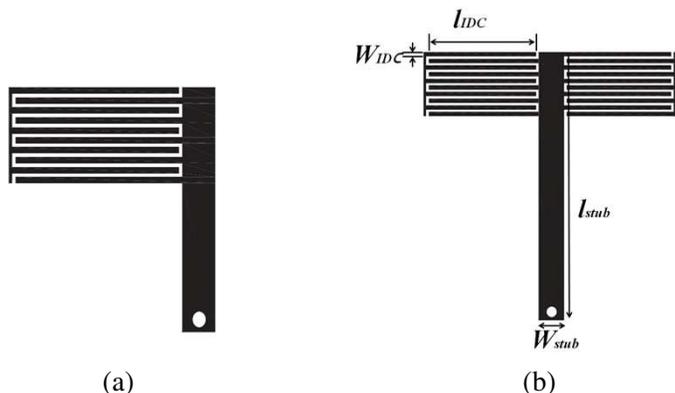
$$A_2 = 9.92 \tanh [0.52(h/w_{DC})^{0.5}] \cdot 10^{-6} \text{ (pF/}\mu\text{m)} \quad (9)$$

And we add two more design parameters to intentionally use parasitic elements of the interdigital line and the stub, which differentiates our approach from the works in [5, 6].

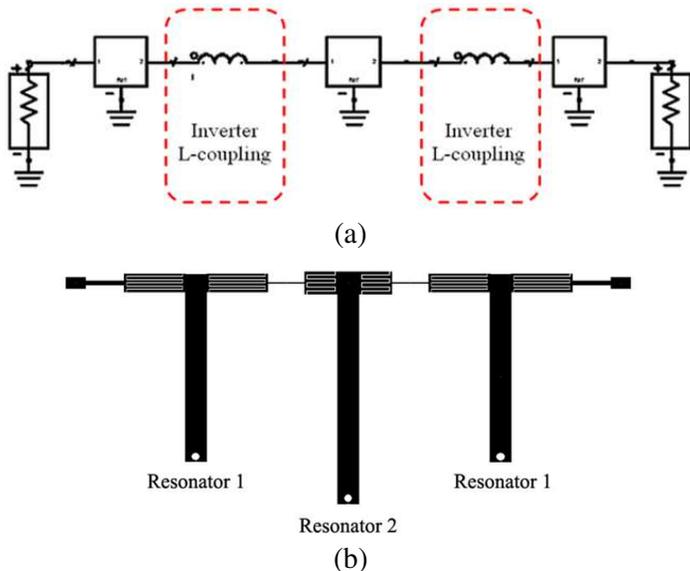
$$L_R = \frac{Z_C \cdot |\tan(\beta_{\text{eff}} l_{DC})|}{n \cdot \omega}, \quad (10)$$

$$C_R = \varepsilon_{\text{eff}} \cdot \frac{w_{\text{stub}} \cdot l_{\text{stub}}}{h} \quad (11)$$

After using these approximate relations, the physical dimensions are varied from the initial values until the frequency response including  $\omega_{se}$ ,  $\omega_{sh}$ ,  $\omega_0$ ,  $\omega_L$  and  $\omega_R$  approaches the ZOR at 900 MHz as the first design goal.



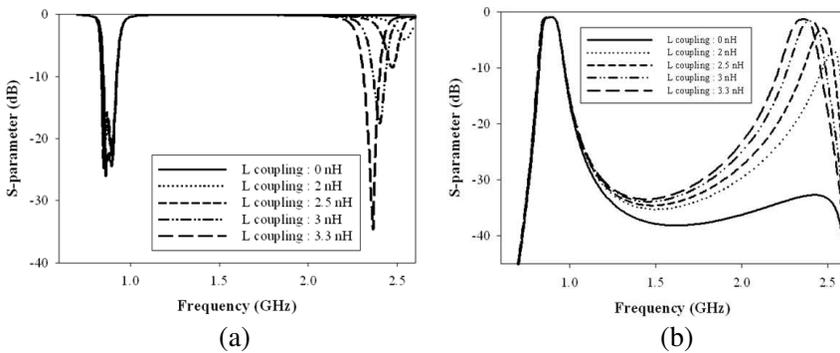
**Figure 5.** Interdigital line and short-circuited stub for the ZOR: (a) Asymmetrical type, (b) symmetrical type for the proposed design.



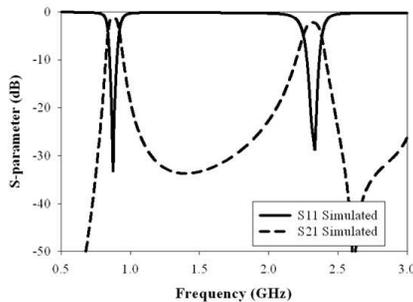
**Figure 6.** Proposed CRLH ZOR bandpass filter with inverter: (a) Conceptual CRLH ZOR resonators in the filter. (b) Actual CRLH ZOR resonators in the filter.

When the design of the ZOR is finished, we go on to propose the coupling between resonators using the inductance as inverter to create the second passband at 2.4 GHz. This proposed coupling element is shown in Fig. 6 where there are the conceptual and actual structures we proposed for a dual-band metamaterial BPF.

Fig. 7 shows that the change depends on inductor values. When the inductive coupling value is zero, one pass band is generated at 900 MHz. As the inductive value increases, the second resonance point begins to appear. Monitoring this behavior, the coupling is adjusted to the two resonance points coinciding at the wanted passbands.



**Figure 7.**  $S$ -parameter depends on inverter values: (a)  $S_{21}$  frequency response, (b)  $S_{11}$  frequency response. (The 3D field simulation includes the dielectric loss).



**Figure 8.** 3D EM Simulation result of the  $S_{11}$  and  $S_{21}$  on the proposed CRLH bandpass filter. (The 3D field simulation includes the dielectric loss).

To see the overall performance, the proposed CRLH bandpass filter should be drawn in 3D EM simulator and go through the EM full-wave field analysis using the CST-MWS. At this moment, the following physical dimensions are provided, which have been finalized by adjusting the initial values corresponding to the elements of Fig. 3 and Fig. 4 to meet the specifications.

**Table 3.** Summary of the physical dimensions.

Item	[mm]	
	Resonator 1 or 3	Resonator 2
$W_{IDC}$	0.2	0.27
$L_{IDC}$	5.8	2.89
Number of Fingers	3	3
Gap between fingers	0.13	0.14
$W_{stub}$	2.22	2.22
$L_{stub}$	17.34	21.45
Width of Coupling part	0.12	
Length of Coupling part	3.8	
$\epsilon_r$ and thickness of Substrate	10.2 and 1.27 (50 mil)	

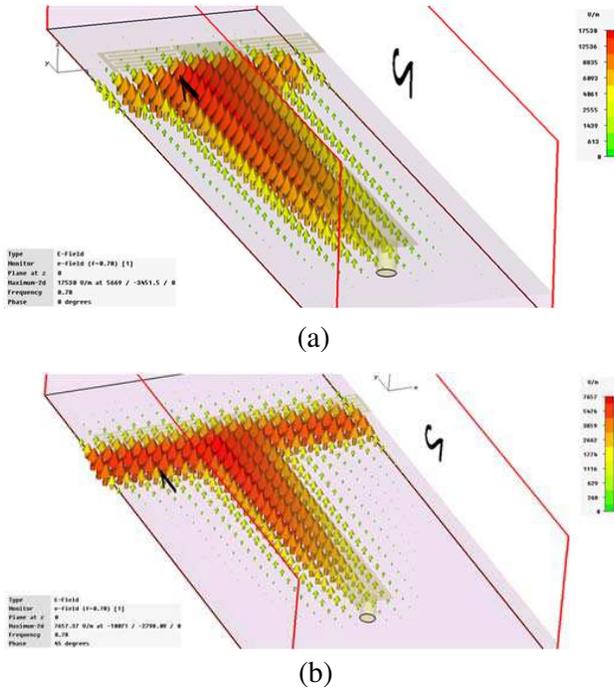
Firstly, considering the dielectric loss, here comes the pair of the scattering parameters of transmission and reflection coefficients of the energy flow from port 1 to port 2.

As expected, passbands are created at 900 MHz and 2.4 GHz. The return loss looks nice. But the loss tangent in the substrate makes the insertion loss degraded, which also happens in others' works [1–4].

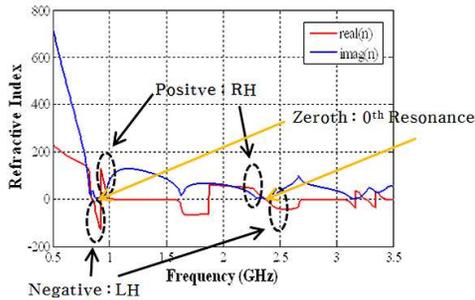
Now we would like to confirm the metamaterial property of the ZOR where the electric field should have the same phase all over the resonator. This characteristic is shown in Fig. 9.

Figure 9 shows the electric field distributions which have no phase variation in resonator 1 and resonator 2. That is to say, they have the same intensity level and the same direction of vectors. Another way to confirm ZOR of the filter is extracting the effective constitutive parameters such as the effective refractive index from the reflection and transmission coefficients of the EM simulation. As the dispersion diagram, the refractive index gives the information on its negative, zero and positive regions as LH, ZOR and RH, respectively.

Figure 10 shows the extracted effective refractive index of the proposed dual-band bandpass filter. Especially, 900 MHz is the ZOR point as the border line between the LH and RH regions.



**Figure 9.** *E*-field distribution of each CRLH resonator at ZOR: (a) Resonator 1 (or resonator 3), (b) Resonator 2.

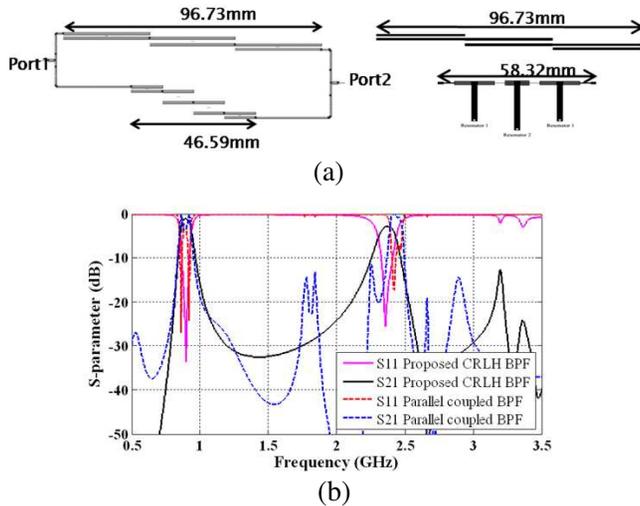


**Figure 10.** Extracted effective refractive index between the ports.

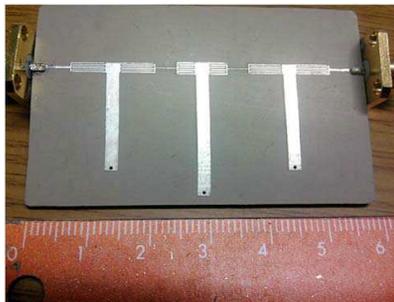
Next, to check how the proposed design has strengths in effective miniaturization of the filter geometry and performances, we compare the conventional parallel coupled bandpass filter and proposed CRLH ZOR dual-band bandpass filter (3rd order), as shown in Fig. 12.

In particular, the conventional filter combines one parallel coupled filtering part for 900 MHz (2nd order) and the other part for 2.4 GHz (3rd order).

For the comparison in Fig. 11(a), the 900 MHz conventional filtering part is taken which has two half-wave long resonators, and it is 96.73 mm long for order 2. If its order becomes 3, its length becomes 128.97 mm. On the contrary, the proposed ZOR filter has  $1/22$  wavelength and  $1/37$  wavelength for resonator 1 and resonator 2, respectively, and its total length is 58.32 mm. So, the proposed ZOR is



**Figure 11.** Comparison between proposed filter (3rd order) and conventional filter (2nd order): (a) size comparison, (b) frequency responses of the two filters.



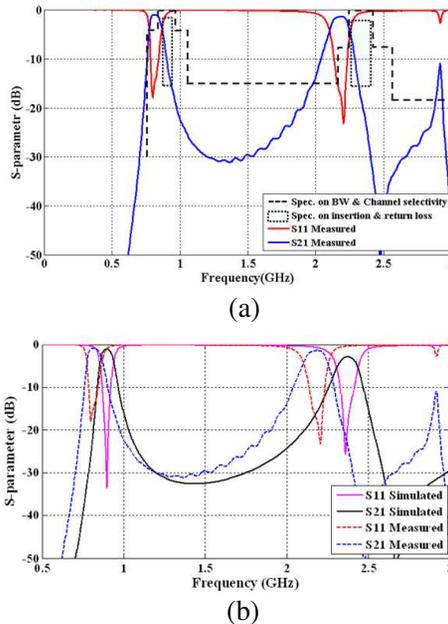
**Figure 12.** Fabricated CRLH ZOR bandpass filter.

as small as  $1/2.21 \sim 1/1.66$  of the conventional filter for orders 3 and 2, respectively. This means 55% and 40% of size reduction for orders 3 and 2, respectively. In addition, in Fig. 11(b), the improvement in the isolation between the two channels is made by the proposed ZOR filter due to the coupling elements implemented to suit the specs.

Furthermore, we show the photo of the realized ZOR dual-band bandpass filter below.

The entire geometry is realized by the simple microstrip manufacturing technology. It shows the effective miniaturization with no lumped element loading as observed previously in Figs. 6 and 12.

Figure 13 shows the measurement results of the proposed ZOR filter. The bands are shifted from the design and simulation where the material property of the substrate is constant vs. frequency. But in reality, the material is frequency-varying, and the dielectric constant is not uniform over the substrate. Also, when we were soldering connectors on the substrate, we had to use a little bit thick lead reel. But, bandwidth, return loss and stopband characteristics are acceptable. To make the frequency shift as small as possible, the



**Figure 13.** Result of fabricated CRLH bandpass filter: (a) specs. vs. measurement, (b) 3D field simulation and measurement.

measured permittivity of the substrate to be used should be considered in the design with the 3D EM field solver, and care must be taken in handling the SMA connectors more skillfully.

#### 4. CONCLUSION

This paper proposes the design of a very compact dual-band bandpass filter for the GSM and ISM band application. To reduce the overall size of the component, we adopted the concept of the ZOR from the LH line combined with the RH line to make the resonators a lot less than a quarter-wavelength. In addition to the basic metamaterial theory, the inverters were suggested and implemented for inductively controlling the interaction between the ZORs to create the dual-band and good isolation between the two bands of interest. The validity of the design is verified by showing the field distributions, effective constitutive parameters of metamaterial ZOR properties, and the frequency response of the 3D EM simulation and measurement. Especially, remarkable size reduction was achieved by 40 % ~ 55 %, when the proposed ZOR dual-band filter was compared to conventional filters of one order less and the same order while the dual-passband performance in the frequency response was observed.

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