Progress In Electromagnetics Research C, Vol. 5, 35–43, 2008

SHARP FOCUS AREA OF RADIALLY-POLARIZED GAUSSIAN BEAM PROPAGATION THROUGH AN AXICON

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Abstract—Based upon developed radial FDTD-method, used for solution of Maxwell equations in cylindrical coordinates and implemented in Matlab-7.0 environment, we simulated focusing of the annular Gaussian beam with radial polarization by conical microaxicon with numerical aperture 0.60. It is shown that the area of focal spot (defined as area where intensity exceeds half of its maximum) can be $0.096\lambda^2$, and focal spot diameter equals to 0.35λ.

1. INTRODUCTION

The recent time showed increased interest in sharp focusing of laser radiation and reaching minimal diameter of the focal spot beyond diffraction limits. Decreasing of the focal spot diameter is important for lithography, optical memory and micromanipulation. In [1] with using of microobjective Leika plan apo $100x$ with numerical aperture $\mathrm{NA} = 0.9$ laser beam with radial polarization has been focused in air into a spot with area of half-maximum intensity (area where intensity exceeds half of its maximum, Half-of-Maximum Area, HMA) equal to $\mathrm{HMA} = 0.16\lambda^2$ and with spot diameter (Full Width of Half-Maximum, FWHM) $\mathrm{FWHM} = 0.451\lambda$, where λ is the wavelength in free space. In the experiment were used the fundamental He-Ne laser mode with wavelength of 632.8 nm and annular mask, obstructing the central part (diameter 3 mm) of the incident beam with diameter 3.6 mm. But the record of [1] was recently broken. In [2], also experimentally, with help of parabolic mirror (having diameter 19 mm and numerical aperture $\mathrm{NA} = 0.999$) and radially polarized laser beam with wavelength 632.8 nm, a focal spot with area $\mathrm{HMA} = 0.134\lambda^2$

has been obtained. The intensity distribution in the focal plane has been measured by fluorescent bead with diameter 40 nm. But this value is not limit because by numeric simulation based on Debye theory and Richards-Wolf equations it has been shown in Ref. [3] that with parabolic mirror with numerical aperture NA = 1 it is possible to focus radially polarized hollow Gaussian beam with amplitude $r\exp(-r^2/w^2)$ (where r is the radial coordinate and w is the Gaussian beam waist radius) into a spot with area HMA $= 0.154\lambda^2$ and if the Gaussian beam bounded by narrow annular diaphragm (the energy of light beam will be partially lost), then the area of the focal spot can be decreased to HMA $= 0.101\lambda^2$. For comparison, the area of focal spot of Airy disk in scalar paraxial approximation equals to HMA $= 0.204\lambda^2$.

The finite-difference time-domain (FDTD) method is a universal approach to computer-aided modeling of the electromagnetic waves diffraction [4–11]. Publications dealing with focusing properties of uniaxial crystal lens [12], discrete lens [13, 14], hyperbolic lens [15], metal-plate lens [16], photonic-crystal lens [17], Veselago's lens with a negative refraction [18–20] and other types of optical elements [21, 22] are also numerous.

In this paper, we used numeric calculation of focusing radially polarized annular Gaussian beam $\exp(-(r-r_0)^2/w^2)$ by conical microaxicon with base radius 7 µm and cone height 6 µm. For example, focusing with axicon can be used in organization of microscale objects [23]. It is shown that area of focal spot (defined as area with intensity decreasing to half of its maximum) equals to HMA $= 0.096\lambda^2$. It is the best result in comparison with [1–3].

2. RADIAL SYMMETRIC FDTD

For rigorous calculation of light diffraction by axially-symmetric optical elements in [4] was proposed a variant of FDTD-method, based on solution of Maxwell equations in cylindrical coordinates. However, diffraction of the radially polarized light was not discussed in [4]. Below we consider the variant of FDTD-method, specially designated for calculation of diffraction of radially polarized laser beam by microoptics elements with axial symmetry. Let monochromatic radially polarized electromagnetic wave, propagating along optical axis z, fall normally onto axially-symmetric refraction optical element. In this case, only three components of the electromagnetic field have non-zero values: $E_{r,0}, E_{z,0}, H_{\varphi,0}$. These are radial and longitudinal components of the electric field and azimuthal component of the magnetic field. Zero index means that all three components are independent on azimuthal angle φ. Therefore from six Maxwell equations for radially polarized light only three equations remain:

$$-\frac{\partial H_{\varphi,0}}{\partial z} = \varepsilon\varepsilon_0 \frac{\partial E_{r,0}}{\partial t} + \sigma E_{r,0}, \tag{1}$$

$$\frac{1}{r}\frac{\partial (rH_{\varphi,0})}{\partial r} = \varepsilon\varepsilon_0 \frac{\partial E_{z,0}}{\partial t} + \sigma E_{z,0}, \tag{2}$$

$$\frac{\partial E_{r,0}}{\partial z} - \frac{\partial E_{z,0}}{\partial r} = -\mu\mu_0 \frac{\partial H_{\varphi,0}}{\partial t}, \tag{3}$$

where ε and μ are relative electric and magnetic permeabilities of the material of optical element (further $\mu = 1$), ε_0 and μ_0 are electric and magnetic permeabilities of vacuum, σ is relative conductivity (further $\sigma = 0$). Explicit conditionally stable difference equations, approximating Equations (1)–(3), have the following form:

$$\varepsilon\left(i+\frac{1}{2},j\right)\varepsilon_0 \frac{E_{r,0}^{n}\left(i+\frac{1}{2},j\right) - E_{r,0}^{n-1}\left(i+\frac{1}{2},j\right)}{\Delta t} = \frac{H_{\phi,0}^{n-\frac{1}{2}}\left(i+\frac{1}{2},j+\frac{1}{2}\right) - H_{\phi,0}^{n-\frac{1}{2}}\left(i+\frac{1}{2},j-\frac{1}{2}\right)}{\Delta z}, \tag{4}$$

$$\varepsilon\left(i,j+\frac{1}{2}\right)\varepsilon_0 \frac{E_{z,0}^{n}\left(i,j+\frac{1}{2}\right) - E_{z,0}^{n-1}\left(i,j+\frac{1}{2}\right)}{\Delta t} = \frac{1}{r(i)} \frac{r\left(i+\frac{1}{2}\right) H_{\phi,0}^{n-\frac{1}{2}}\left(i+\frac{1}{2},j+\frac{1}{2}\right) - r\left(i-\frac{1}{2}\right) H_{\phi,0}^{n-\frac{1}{2}}\left(i-\frac{1}{2},j-\frac{1}{2}\right)}{\Delta r}, \tag{5}$$

$$-\mu_0 \frac{H_{\phi,0}^{n+\frac{1}{2}}\left(i+\frac{1}{2},j+\frac{1}{2}\right) - H_{\phi,0}^{n-\frac{1}{2}}\left(i+\frac{1}{2},j+\frac{1}{2}\right)}{\Delta t} = \frac{E_{r,0}^{n}\left(i+\frac{1}{2},j+1\right) - E_{r,0}^{n}\left(i+\frac{1}{2},j\right)}{\Delta z} - \frac{E_{z,0}^{n}\left(i+1,j+\frac{1}{2}\right) - E_{z,0}^{n}\left(i,j+\frac{1}{2}\right)}{\Delta r}. \tag{6}$$

where Δt, Δr, Δz are sampling steps for corresponding axes; n, i, j are integer indexes of nodes in coordinate grid t, r, z. In difference Equations (4)–(6), the Yee scheme (see [5]) with half-integer steps was used. The boundary conditions were chosen as perfectly matched layer [6]. Peculiarities of field calculation on the optical axis ($r = 0$) are described in [4].

3. SIMULATION RESULTS

By using Equations (4)–(6), we simulated focusing of annular Gaussian radially polarized wave, propagating along optical axis and incident onto a microaxicon. Shown on the Fig. 1 is the radial section of the axicon: base radius $R = 7\,\mu\text{m}$ (horizontal), cone height $h = 6\,\mu\text{m}$ (vertical), refractive index $n = 1.5$, numerical aperture of the axicon $\text{NA} = \sin\alpha = 0.60$, where α is half-angle of tilt of cone of rays after traveling through the axicon. This angle can be derived from equation ncos $n\cos\theta = \sin(\alpha+\theta)$, where 2θ is the angle at cone vertex.

The calculation area has the size 20 μm by 8 μm, sampling step for r and z axes is the same and equals $\lambda/50$, and for time axis $t - \lambda/100$. The wavelength was chosen $\lambda = 1\,\mu\text{m}$. If we illuminate the axicon (Fig. 1) by annular Gaussian beam

$$E_{r,0}(r) = \exp(-(r - r_0)^2/w^2), \qquad (7)$$

where $r_0 = 4.5\,\mu\text{m}$, $w = 2.5\,\mu\text{m}$, then we will obtain the diffraction pattern shown on Fig. 2. On the Fig. 2 shown the momentary pattern of the $E_r(a)$ and $E_z(b)$ amplitude distributions for this case. The black horizontal line is the annular Gaussian beam source. Note that radial component of the field always equals to zero on the optical axis. On the Fig. 3a shown (in relative units) radial distribution of intensity $I_E = |E|^2 = |E_r|^2 + |E_z|^2$ (curve 1), $|E_r|^2$ (curve 2) and $|E_z|^2$ (curve 3) immediately beyond the vertex of the cone. On the Fig. 3b shown

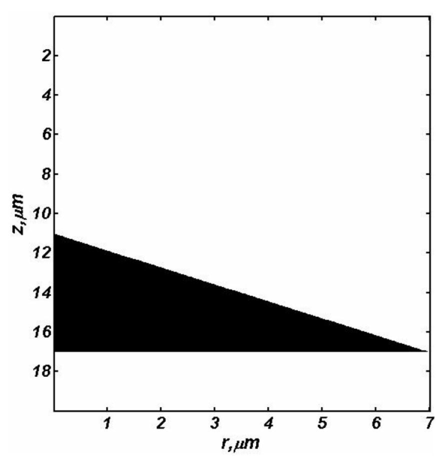
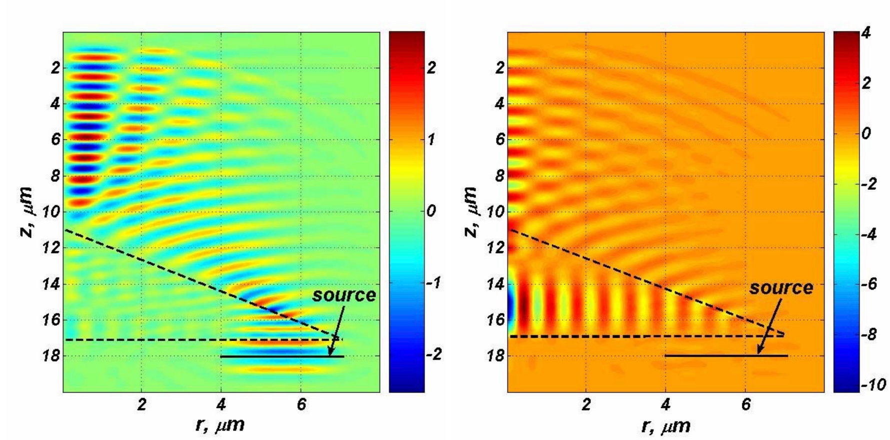
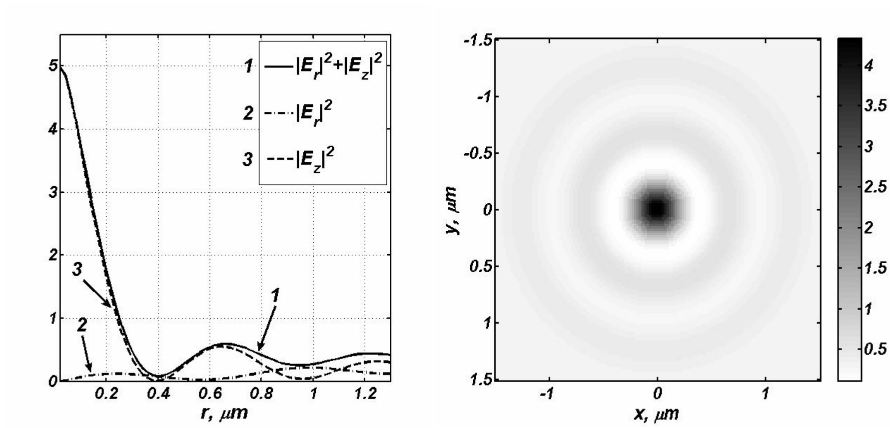


Figure 1. Size of conical microaxicon with axial symmetry (vertical axis — z, horizontal axis — r).



(a) (b)

Figure 2. Momentary distribution of amplitude E_r(a) and E_z(b) for diffraction of the annular Gaussian beam (black solid line shows the waist of the beam and it is marked as "source", black dashed line shows the boundary of the axicon) by the axicon (Fig. 1); the units of axes are μm.



(a) (b)

Figure 3. Radial intensity distribution (a) and 2D diffraction pattern (b) in focal plane (immediately beyond the vertex of the cone on the Fig. 1) of the axicon when annular Gaussian beam with radial polarization falls onto it: $|E_r|^2 + |E_z|^2$ (curve 1); $|E_r|^2$ (curve 2); $|E_z|^2$ (curve 3).

Table 1. Dependence of FWHM and maximal intensity at the center of the focal spot $I_{\max}$ on axicon height h (other parameters are: $R = 7\,\mu\text{m}$, $n = 1.5$, $r_0 = 4.5\,\mu\text{m}$, $w = 2.5\,\mu\text{m}$, $\lambda = 1\,\mu\text{m}$).

h, m	I_{max}, a.u.	*FWHM*,
7	1.1	0.340
6.8	1.7	0.320
6.4	3.3	0.336
6	4.4	0.352
5.8	4.4	0.356
5.6	3.9	0.365
5.4	3.1	0.380
5.2	2.2	0.390
5	1.4	0.400

diffraction pattern in focal plane. The diameter of focal spot equals to FWHM $= 0.35\lambda$, and the area is HMA $= 0.096\lambda^2$.

It is seen from the Table that at $h = 6\,\mu\text{m}$ there will be maximal intensity $I_{\max} = 4.4$ (in arbitrary units) and FWHM $= 0.352\lambda$ in the focus. Although minimal focal spot diameter FWHM $= 0.320\lambda$ will be at $h = 6.8\,\mu\text{m}$, the intensity at the focus will be almost three times less (because of total internal reflection).

Axicon can be fabricated in a glass substrate using, for example, e-beam lithography and ion-beam etching. For a Gaussian beam of radius 1 mm to be focused into a Gaussian beam of waist radius $7\,\mu\text{m}$ a spherical lens of approximate focus 22 mm (for wavelength $1\,\mu\text{m}$) and small numerical aperture NA < 0.1 needs to be used.

Although in electromagnetics a Gaussian beam can have many meanings [24], here a radial component of the electric field (7) is given only in input plane (this plane is marked as "source" in Fig. 2) and is evaluated further by using Eqs. (4)–(6). Note that attempts to realize an exact annular Gaussian beam (7) have failed. In practice, however, there are several alternatives. A thin metallic film with an annular diaphragm can be preliminary sprayed onto an axicon bulk substrate. The simulation we conducted showed that when focusing a Gaussian beam $\exp(-r^2/w^2)$ of waist radius $w = 7\,\mu\text{m}$ bounded by a circular diaphragm of radii $R_1 = 4\,\mu\text{m}$ and $R_2 = 7\,\mu\text{m}$, the FWHM of the resulting focal spot will be almost the same as that in Figs. 2 and 3, with the maximal intensity being two times smaller. Another technique for generating the annular Gaussian beam was presented in [3]. In this work, the modeling was conducted using a beam $r\exp(-r^2/w^2)$, which can be generated with an amplitude mask whose transmittance is changing linearly with radius from zero to unity. In [25], the sharp focusing was modeled using an annular Bessel-Gauss beam in the form $J_1(2r)\exp(-r^2)$. Such a beam can not be implemented with an amplitude mask because the Bessel function can take both positive and negative values.

4. CONCLUSIONS

So, here we considered the radial variant of FDTD-method, designated for solution of Maxwell equations in cylindrical coordinates for radially polarized radiation. Focusing of annular Gaussian beam with radial polarization by microaxicon has been simulated and it was shown that area of the focal spot (defined as area where intensity exceeds half of maximum) can be equal to $0.096\lambda^2$, and spot diameter equals to 0.35λ. These values are less than ones, obtained in [1–3].

ACKNOWLEDGMENT

This work was financially supported by the Russian-American program "Basic Research and Higher Education" (CRDF grant # RUX0-014-SA-06), the Russian Foundation for Basic Research (grant # 08-07-99007) and the Russian Federation Presidential Grant NSH-3086.2008.9.

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