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COMBINED BEAMFORMING WITH ALAMOUTI CODING USING DOUBLE ANTENNA ARRAY GROUPS FOR MULTIUSER INTERFERENCE CANCELLATION

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Abstract—This paper propose a new multiple-input multiple-output (MIMO) signal processing scheme that combines optimum transmit and receive beamforming with the Alamouti space-time block code (STBC) transmission and modifies the decoding process. The scheme uses double antenna array groups to achieve stable performance regardless of direction of arrived (DOA) and angular spread (AS). In a multiuser MIMO communications scenario, the beamforming suppresses co-channel interference (CCI) by maximizing the uplink signal-to-noise-plus-interference-ratio (SINR) and suppress CCI independently while preserving orthogonality of the MIMO channel. It is shown that the beamforming process provides array gain by increasing the bit-error-rate (BER) performance and maximizes the available uplink channel capacity for each user in the presence of CCI.

1. INTRODUCTION

In recent years, as the increase demand of transmitting high data rates, the research of Multiple-Input Multiple-Output (MIMO) techniques, which have the ability of achieving extraordinary bit rates, became a potential technique. Among which, smart antenna and spatial diversity are the emerging MIMO techniques.

Smart antenna technique has gained much attention over last few years for its ability to significantly increase the performance of wireless communication systems, in terms of spectrum efficiency, network scalability and reliable operation etc. Smart antenna utilizes the strong spatial correlation to process the received signal by antenna arrays with beamforming technique. It is able to provide high directional beamforming gain and reduce the interference from other direction under high spatial correlated MIMO channel. Under smart antenna configuration, the antennas spacing is small which is usually half wavelength. So that the signal received at or transmitted from all antennas are highly correlated to achieve spatial directivity or beamforming gain.

For spatial diversity (e.g., space-time block coding) technique, it has been studied extensively as a method of combating fading because of its relative simplicity of implementation and feasibility of having multiple antennas at the base station. Spatial diversity requires the antenna spacing being large enough which results in low spatial correlation. In that way, spatial diversity technique is able to perform well to combat channel fading.

Considering the advantages of these various MIMO techniques, there is a need to integrate them so that the whole system can benefit from these technologies. These two techniques have the same feature in the view of requiring the multiple antenna elements, but have the contradictory requirement for antenna element spacing. Because it is conflictive that the smart antenna works under high spatial correlated MIMO channel while the spatial diversity technique work under low spatial correlated MIMO channel. Recently, there are many researches focus on combining beamforming and STBC techniques [1–3]. The combining techniques usually require more than one smart antenna arrays at the transmitter. The transmit signal is encoded by space-time block coding and precoded by beamforming weights independently before transmitting on different antenna arrays. One way to obtain the beamforming weights is through eigen decomposition to the estimated channel covariance matrix [1]; another way is to utilize the array response vector as the beamforming weights [2, 3]. Combining beamforming and STBC is able to achieve both diversity and beamforming gain. It can improve the system performance.

Due to the frequency reuse, multiple access schemes and multiuser communications wireless channels are impaired with co-channel interference (CCI). Beamforming using smart antennas has long been recognized as an effective means for suppressing CCI to improve the spectrum efficiency. In the context of MIMO communications, the meaning of "beamforming" has been extended to include combining independent signals output from diversity array antennas at both transmitters and receivers. Many techniques have been developed for transmitting and receiving beamformer to suppress CCI [4–12]. In [4], multiuser MIMO communications are realized with the precise channel state information (CSI). However, the joint optimization at both the base station (BS) and multiple mobile subscribes (MS) is required and matrix diagonalization and decomposition are involved. Dighe [5] assume that the number of the transmit antennas of multiple users (the desired users and interferers) is larger than the number of the receive antennas. The CCI is suppressed by maximizing the output signal-to-interference-plus-noise ratio (SINR) jointly at MS transmitters and the BS receiver. Capon beamforming is applied in [6], where data streams from a MS's two transmit antennas are treated independently as desired signals. The receiver minimizes the output power and passes each individual data steam with unit gain. The complexity increases when STBCs are transmitted with more transmit antennas at each MS.

To combine the CCI suppression ability of beamforming techniques with the Alamouti STBC [13] transmission and achieve the receiver computational simplicity, we serially concatenate optimum beamforming with the linear Alamouti decoding process. Specially, we consider the scenario where all the MSs transmit Alamouti STBC with two transmit antenna array groups to stable performance even with correlated channels [2] and the CSI of CCI is unknown to the receiver. Receive antennas are grouped into two subsets, A and B, each having K receive antennas. The BS beamformer at each subset maximizes the SINR independently to suppress CCI. The coantenna interference (CAI) is suppressed at the STBC decoder. Since, the technique preserves algebraic structure of the Alamouti STBC and sustain the orthogonally of the virtual MIMO channel in the presence of beamforming, the maximum-likelihood (ML) decoding process is achieved simple linear processing. The concatenation provides significant BER improvement and capacity increasing [14, 15] in comparison with the conventional Alamouti scheme.

2. COMMUNICATION SYSTEM MODEL

In the following mathematical exposition, superscripts $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote transpose, complex conjugate, and conjugate-transpose, respectively. Under the assumption that $N_t = 2R_t$ and $N_r = 2K$, the Rayleigh block fading uplink channel from MS-m to the base station (BS) is modeled as a $2K$-by-$2R_t$ dimensional matrix:

$$\mathbf{H}^m = \begin{bmatrix} h^m_{1,1} & \cdots & h^m_{1,R_t} & h^m_{1,R_t+1} & \cdots & h^m_{1,2R_t} \\ \vdots & & & & & \vdots \\ h^m_{K,1} & \cdots & h^m_{K,R_t} & h^m_{K,R_t+1} & \cdots & h^m_{K,2R_t} \\ \vdots & & & & & \vdots \\ h^m_{2K,1} & \cdots & h^m_{2K,R_t} & h^m_{2K,R_t+1} & \cdots & h^m_{2K,2R_t} \end{bmatrix} = [\mathbf{H}^m_1 \quad \mathbf{H}^m_2] \tag{1}$$

where each entity is modeled as a statistically independent, and identically distributed (i.i.d.) complex Gaussian variable that has zero mean and unit variance. $\mathbf{H}^m_1$ and $\mathbf{H}^m_2$ are $2K$-by-R_t dimensional channel from MS-m two transmit antenna array groups to the receive antennas at the BS. The fading state of the MIMO channel is assumed invariant. The block diagram of the combined scheme of the beamforming and STBC is shown in Fig. 1.

An Alamouti STBC word [13] sent over MS-m two transmit antenna array groups during two symbol epochs which is represented as:

$$\mathbf{S}^m = \begin{bmatrix} s^m_1 & -(s^m_2)^* \\ s^m_2 & (s^m_1)^* \end{bmatrix} \tag{2}$$

The Alamouti code word has the property as:

$$E\left[\mathbf{S}^m\{\mathbf{S}^m\}^H\right] = 2E_s\mathbf{I}_2 \tag{3}$$

where E_s is the symbol energy. $\mathbf{I}_2$ represents a 2-by-2 dimensional identity matrix, and $E[\ ]$ is the expected value operator. The noise sample collected at those $2K$ receive antennas at the BS over two symbol epochs are represented with an $2K$-by-$2R_t$ dimensional matrix $\mathbf{N}$ with each entry modeled as an i.i.d. zero-mean complex Gaussian variable with variance σ^2. The total transmitted signal power at each MS transmitter is fixed at value $2E_s$. The signal-to-noise ratio (SNR) is defined as $2E_s/\sigma^2$. Assuming that there two cochannel MSs and MS-1 is the desired user while MS-2 is CCI. In (2), the first column at nT and the second column at $(n+1)T$, where n is the discrete

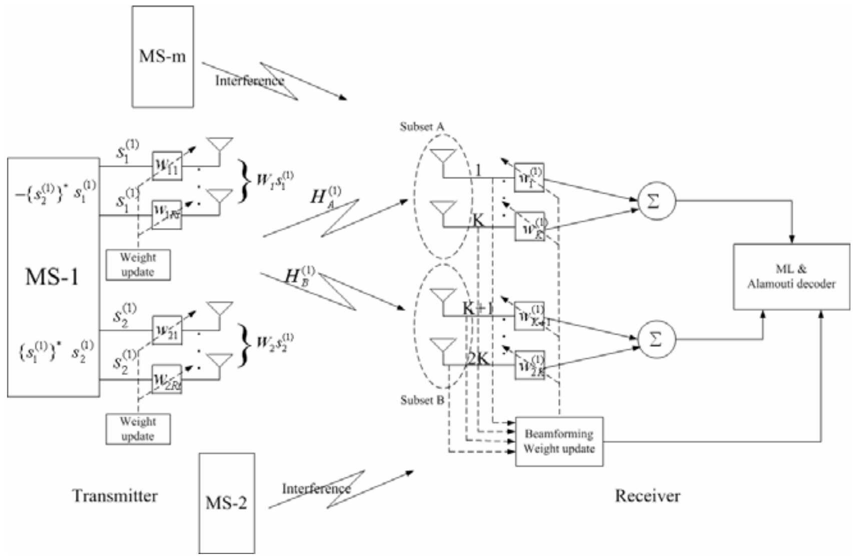

Figure 1. The block diagram of the combined scheme of the beamforming and STBC.

time index and T is the symbol duration. Signal samples on those $2K$ receive antennas at the BS over two symbol epochs are expressed with an $2K$-by-$2R_t$ dimensional matrix:

$$\begin{aligned}\mathbf{r} &= [\mathbf{r}(nT) \quad \mathbf{r}((n+1)T]\\ &= \left[\mathbf{H}_1^1 \quad \mathbf{H}_2^1\right]\begin{bmatrix}\mathbf{w}_{t,1}^1 s_1^1 & \mathbf{w}_{t,1}^1(-s_2^{1*})\\ \mathbf{w}_{t,2}^1 s_2^1 & \mathbf{w}_{t,2}^1 s_1^{1*}\end{bmatrix}\\ &\quad + \left[\mathbf{H}_1^2 \quad \mathbf{H}_2^2\right]\begin{bmatrix}\mathbf{w}_{t,1}^2 s_1^2 & \mathbf{w}_{t,1}^2(-s_2^{2*})\\ \mathbf{w}_{t,2}^2 s_2^2 & \mathbf{w}_{t,2}^2 s_1^{2*}\end{bmatrix} + \mathbf{N}\\ &= \left[\mathbf{H}_1^1\mathbf{w}_{t,1}^1 \quad \mathbf{H}_2^1\mathbf{w}_{t,2}^1\right]\begin{bmatrix}s_1^1 & (-s_2^{1*})\\ s_2^1 & s_1^{1*}\end{bmatrix}\\ &\quad + \left[\mathbf{H}_1^2\mathbf{w}_{t,1}^2 \quad \mathbf{H}_2^2\mathbf{w}_{t,2}^2\right]\begin{bmatrix}s_1^2 & (-s_2^{2*})\\ s_2^2 & s_1^{2*}\end{bmatrix} + \mathbf{N} \qquad (4)\end{aligned}$$

where the vector $\mathbf{w}_{t,i}^m$, $i = 1, 2$ is the MS-m transmit beamforming weight vector applied to the i-th transmit array group. The task of the beanmformers at both transmit and receive antennas is to suppress noise and CCI.

3. GROUPING ALGORITHM AND DECODE

We shall show how to determine $\mathbf{w}_{t,i}^m,\ i = 1, 2$ to maximize the mutual information firstly. For the above system model, instantaneous mutual information is given by [15]

$$C^m = \ln\det\left(\mathbf{I}_{2K\times 2K} + \frac{SNR}{2R_t}\sum_{i=1}^{2}\mathbf{H}_i^m\mathbf{w}_{t,i}^m{\mathbf{w}_{t,i}^m}^H{\mathbf{H}_i^m}^H\right) \tag{5}$$

The mutual information in (5) is lower-bounded as(see Appendix A)

$$C^m \geq \ln\left(1 + \frac{SNR}{2\times 2R_t}\sum_{i=1}^{2}{\mathbf{w}_{t,i}^m}^H{\mathbf{H}_i^m}^H\mathbf{H}_i^m\mathbf{w}_{t,i}^m\right) \tag{6}$$

And therefore the above lower-bound can be maximized by choosing the weight $\mathbf{w}_{t,i}^m,\ i = 1, 2$ as the eigenvector associated with the maximum eigenvalue of ${\mathbf{H}_i^m}^H\mathbf{H}_i^m$.

Since the beamforming processes at BS antennas subset-A and subset-B are equivalent, we focus on the process on subset-A. We set the matrix

$$\begin{bmatrix}\mathbf{H}_1^m\mathbf{w}_{t,1}^m & \mathbf{H}_2^m\mathbf{w}_{t,2}^m\end{bmatrix} = \begin{bmatrix}\mathbf{H}_A^m \\ \mathbf{H}_B^m\end{bmatrix} \tag{7}$$

where $\mathbf{H}_A^m$ and $\mathbf{H}_B^m$ represents a K-by-2 dimensional matrix. The samples at the receive antennas of subset-A over two symbol epochs are represented by K-by-2 dimensional matrix:

$$\begin{aligned}\mathbf{r}_A &= [\mathbf{r}_A(nT) \quad \mathbf{r}_A((n+1)T)] \\ &= \mathbf{H}_A^1\begin{bmatrix}s_1^1 & (-s_2^{1*}) \\ s_2^1 & s_1^{1*}\end{bmatrix} + \mathbf{H}_A^2\begin{bmatrix}s_1^2 & (-s_2^{2*}) \\ s_2^2 & s_1^{2*}\end{bmatrix} + \mathbf{N}_A \\ &= \mathbf{H}_A^1\mathbf{S}^1 + \mathbf{H}_A^2\mathbf{S}^2 + \mathbf{N}_A\end{aligned} \tag{8}$$

The output SINR at the beamformer is defined as:

$$SINR_A^1 = \frac{(\mathbf{w}_{r,A}^1)^H\mathbf{R}_s\mathbf{w}_{r,A}^1}{(\mathbf{w}_{r,A}^1)^H\mathbf{R}_{ni}\mathbf{w}_{r,A}^1} \tag{9}$$

where the k-by-1 beamforming weight vector $\mathbf{w}_{r,A}^1$ at subset-A corresponding to MS-1 is constructed as:

$$\mathbf{w}_{r,A}^1 = \begin{bmatrix}w_{r,1}^1 & w_{r,2}^1 & \cdots & w_{r,K}^1\end{bmatrix}^T \tag{10}$$

Furthermore, we have desired signal covariance matrix and unwanted signals covariance matrix represented as:

$$\mathbf{R}_s = E[\mathbf{H}_A^1 \mathbf{S}^1 \{\mathbf{H}_A^1 \mathbf{S}^1\}^H] = 2E_s \mathbf{H}_A^1 \{\mathbf{H}_A^1\}^H \tag{11}$$

$$\begin{aligned}\mathbf{R}_{ni} &= E[\{\mathbf{H}_A^2 \mathbf{S}^2 + \mathbf{N}_A\}\{\mathbf{H}_A^2 \mathbf{S}^2 + \mathbf{N}_A\}^H] \\ &= 2E_s \mathbf{H}_A^2 \{\mathbf{H}_A^2\}^H + 2\sigma^2 \mathbf{I}_K \end{aligned} \tag{12}$$

from (9), we observe that the beamforming problem is in essence a generalized Rayleigh quotient and its value is bounded by the maximum eigenvalue $\lambda_{\max}$ and the minimum eigenvalue $\lambda_{\min}$ of $\mathbf{R}_{ni}^{-1}\mathbf{R}_s$ [16]. To maximize the output SINR at the beamformer $\mathbf{w}_{r,A}^1$ is chosen as the eigenvector corresponding to the maximum eigenvalue of $\mathbf{R}_{ni}^{-1}\mathbf{R}_s$. In the presence of the beamforming, an equivalent 1-by-2 dimensional vector channel at subset-A is formed for MS-1 as:

$$\mathbf{g}_A^1 = \begin{bmatrix} g_{A,1}^1 & g_{A,2}^1 \end{bmatrix} = (\mathbf{w}_{r,A}^1)^H \mathbf{H}_A^1 \tag{13}$$

Exploiting the algebraic structure of the Alamouti STBC word [13], the virtual MIMO channel over two symbol epochs is constructed as:

$$\mathbf{G}_A^1 = \begin{bmatrix} g_{A,1}^1 & g_{A,2}^1 \\ (g_{A,2}^1)^* & -(g_{A,1}^1)^* \end{bmatrix} \tag{14}$$

It can be seen that the beamforming process dose not destroy the orthogonality of the virtual MIMO channel as expressed in (14). For the cochannel user MS-2, an equivalent 1-by-2 dimensional vector channel is constructed as:

$$\mathbf{g}_A^2 = \begin{bmatrix} g_{A,1}^2 & g_{A,2}^2 \end{bmatrix} = (\mathbf{w}_{r,A}^1)^H \mathbf{H}_A^2 \tag{15}$$

And the Alamouti virtual MIMO channel is:

$$\mathbf{G}_A^2 = \begin{bmatrix} g_{A,1}^2 & g_{A,2}^2 \\ (g_{A,2}^2)^* & -(g_{A,1}^2)^* \end{bmatrix} \tag{16}$$

At antennas subset-B ($\mathbf{G}_B^1$ and $\mathbf{G}_B^2$) is constructed in the same manner for $\mathbf{G}_A^1$ and $\mathbf{G}_A^2$, the overall MIMO channel is constructed as:

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{G}_A^1 & \mathbf{G}_A^2 \\ \mathbf{G}_B^1 & \mathbf{G}_B^2 \end{bmatrix} \tag{17}$$

Subsequently, the decode process is executed with the output signals from both beamformers and ML decode as described below:

1) Construct received signal vector:

$$\mathbf{u} = \left[(\mathbf{w}_{r,A}^1)^H \mathbf{r}_A(nT), (\mathbf{w}_{r,A}^1)^H \mathbf{r}_A^*((n+1)T), \right. \\ \left. (\mathbf{w}_{r,B}^1)^H \mathbf{r}_B(nT), (\mathbf{w}_{r,B}^1)^H \mathbf{r}_B^*((n+1)T)\right]^T \tag{18}$$

2) Get the inverse of the square matrix $\bar{\mathbf{H}}$

$$\bar{\mathbf{H}}_{inv} = (\bar{\mathbf{H}})^{-1} \tag{19}$$

3) Obtain the first row of $\bar{\mathbf{H}}$, $\mathbf{a}_1 = \bar{\mathbf{H}}_{inv}(1, :)$
   Obtain the second row of $\bar{\mathbf{H}}$, $\mathbf{a}_2 = \bar{\mathbf{H}}_{inv}(2, :)$
4) The statistical results for the detection of s_1^1 and s_2^1 are:

$$\tilde{s}_1^1 = \mathbf{a}_1\mathbf{u} \quad \text{and} \quad \tilde{s}_2^1 = \mathbf{a}_2\mathbf{u}$$

5) Maximum-likelihood (ML) decoding process is the same as [13]:

$$\begin{aligned} \widehat{s}_1^{\,1} &= \arg\min_{\hat{s}_1^1 \in s} (sum((\mathbf{H}_1^1 \mathbf{w}_{t,1}^1) \cdot \wedge 2) \\ &\quad + sum((\mathbf{H}_2^1 \mathbf{w}_{t,2}^1) \cdot \wedge 2) - 1) \left| \widehat{s}_1^{\,1} \right| + d^2(\tilde{s}_1^1, \widehat{s}_1^{\,1}) \end{aligned} \tag{20}$$

$$\begin{aligned} \widehat{s}_2^{\,1} &= \arg\min_{\hat{s}_2^1 \in s} (sum((\mathbf{H}_1^1 \mathbf{w}_{t,1}^1) \cdot \wedge 2) \\ &\quad + sum((\mathbf{H}_2^1 \mathbf{w}_{t,2}^1) \cdot \wedge 2) - 1) \left| \widehat{s}_2^{\,1} \right| + d^2(\tilde{s}_2^1, \widehat{s}_2^{\,1}) \end{aligned} \tag{21}$$

4. SIMULATION AND DISCUSSION

We consider a uniform linear array and antenna elements are spaced half wavelength apart. Monte-Carlo simulation is executed to compare with the analytical analysis results. BPSK is employed as the modulation scheme.

We shall consider correlated channel conditions influence on the bit-error-rate (BER) performance firstly. The channel matrix is modeled as [17–19]

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2} \tag{22}$$

where $\mathbf{R}_t$ and $\mathbf{R}_r$ are covariance matrices of transmit antennas and receive antennas, respectively. The $2K$-by-$2R_t$ random matrix $\mathbf{H}_w$ is independent and identically distributed circular symmetric Gaussian with zero-mean and unit-variance.

We shall use the following channel covariance matrix of the transmit antennas [19]:

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho & \cdots & \rho^{2R_t-1} \\ \rho^* & 1 & \cdots & \rho^{2R_t-2} \\ \vdots & \vdots & & \vdots \\ (\rho^{2R_t-1})^* & (\rho^{2R_t-2})^* & \cdots & 1 \end{bmatrix} \tag{23}$$

In contrast to transmit antennas, correlation of receive antennas tends to be negligible since a mobile is likely to be surrounded with more scatters, and therefore it is safe to assume that $\mathbf{R}_r = \mathbf{I}_{2K\times 2K}$ [18, 19].

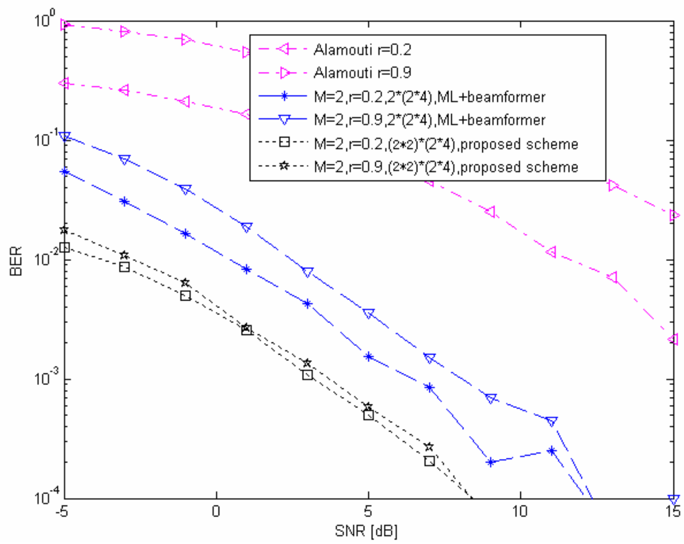

Figure 2. BER with ML decoding under correlated channel.

Figure 2 shows BER performance of Alamouti STBC coding, traditional combined beamforming with STBC, and proposed scheme with ML decoding for $r = 0.2$ and $r = 0.9$. As can be seen, while the Alamouti STBC coding, traditional combined beamforming with STBC have sensitive BER performance depending on the channel correlation r, the proposed scheme has better BER performance than other schemes at any SNR and channel correlation r. This is because we use both STBC and beamforming: when channel correlation is low, we use the advantage of diversity gain; when channel correlation is high, we use the advantage of beamforming.

Figure 3 shows the estimated BERs obtained from Monte-Carlo simulations over different SNRs. The case $M = 1$, Alamouti$2 * 1$

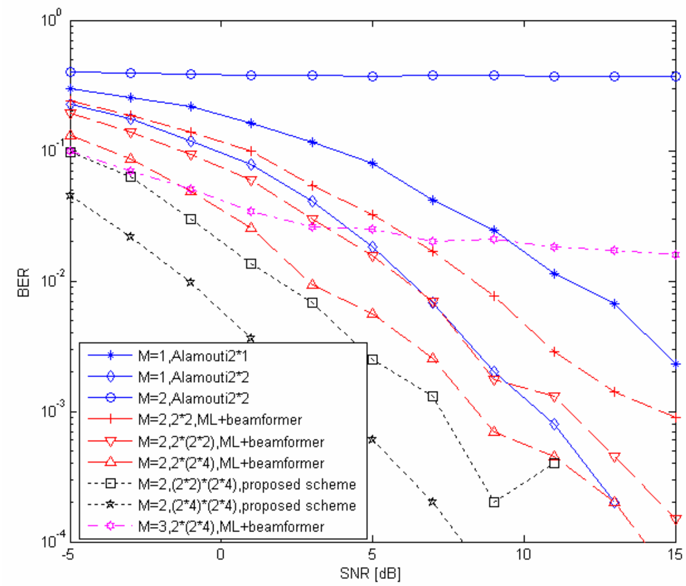

Figure 3. BER performance comparison of various schemes.

represents a conventional single user Alamouti scheme with two transmit and one receive antennas [13]. The curve $M = 2$, $2*2$, ML$+$ beamformer corresponds to the traditional method for beamforming. This suboptimal approach exploits the algebraic structure of the Alamouti STBC and gives a diversity order of two for each MS. When each antennas subset at the BS has two receive antennas $M = 2$, $2*(2*2)$, ML $+$ beamformer, the BER performance for each MS is improved by the SINR gain from the beamformers. The curve is parallel to those for the previous case. The reason is that the extra diversity freedom at the BS is consumed for CCI suppression to obtain a higher SNR. Adding more receive antennas at the BS provides the SINR gain and offers extra diversity freedom. Therefore, the BER performance is further improved. Secondly, we fix receive antennas and increase transmit antennas ($M = 2$, $(2*2)*(2*4)$, proposed scheme). Results show that adding more transmit antennas brings a better performance and a higher CCI tolerance, as we expect. From the above discussions, we conclude that the additional beamforming process brings a higher interference tolerance to improve the BER performance. The technique does not require CSI of cochannel users, thus reducing the receiver computational complexity.

5. CONCLUSIONS

In this paper, we have proposed a MIMO antenna structure that combines transmit and receive beamforming with STBC for multiuser communications. In the proposed structure, the additional beamforming process brings a higher interference tolerance to the multiuser interference cancellation, and thus, improves the BER performance. The two independent beamformers construct an equivalent virtual MIMO channel for each MS with a maximized SINR which achieves much lower BER than traditional technique. We use a grouping algorithm based on the mutual information maximization to transmit beamforming can cope with correlated channel conditions. The simulation results indicate that the proposed scheme has both the advantages of the beamforming technique and STBC diversity gain. It outperforms the traditional beamforming technique and the STBC technique.

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APPENDIX A.

Lemma: for any $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{C}^n$,

$$\det\left(\mathbf{I}_{n\times n} + \mathbf{v}_1\mathbf{v}_1^H\right) \leq \det\left(\mathbf{I}_{n\times n} + \mathbf{v}_1\mathbf{v}_1^H + \mathbf{v}_2\mathbf{v}_2^H\right) \tag{A1}$$

Proof: Let $\lambda_1 \geq \cdots \geq \lambda_n$ be the eigenvalue of $\mathbf{I}_{n\times n} + \mathbf{v}_1\mathbf{v}_1^H$ and $\mu_1 \geq \cdots \geq \mu_n$ be the eigenvalue of $\mathbf{I}_{n\times n} + \mathbf{v}_1\mathbf{v}_1^H + \mathbf{v}_2\mathbf{v}_2^H$. Then it can be shown that [20].

$$\mu_1 \geq \lambda_1 \geq \mu_2 \geq \lambda_2 \geq \cdots \geq \mu_n \geq \lambda_n \tag{A2}$$

Since the determinant of a matrix is the product of its eigenvalues and $\mu_1 \cdots \mu_n \geq \lambda_1 \cdots \lambda_n$, so

$$\det\left(\mathbf{I}_{n\times n} + \mathbf{v}_1\mathbf{v}_1^H\right) \leq \det\left(\mathbf{I}_{n\times n} + \mathbf{v}_1\mathbf{v}_1^H + \mathbf{v}_2\mathbf{v}_2^H\right)$$

Corollary: For any symmetric positive semidefinite matrices $\mathbf{A}$ and $\mathbf{B}$,

$$\det\left(\mathbf{I}_{n\times n} + \mathbf{A}\right) \leq \det\left(\mathbf{I}_{n\times n} + \mathbf{A} + \mathbf{B}\right) \tag{A3}$$

Proof: Since$\mathbf{A}$ and $\mathbf{B}$ are symmetric positive semidefinite,

$$\mathbf{A} = \sum_{i=1}^{n} \lambda_i \mathbf{u}_i \mathbf{u}_i^H, \quad \mathbf{B} = \sum_{i=1}^{n} \mu_i \mathbf{v}_i \mathbf{v}_i^H \tag{A4}$$

with λ_i, $\mu_i \geq 0$, not all zeros if $\mathbf{A} \neq \mathbf{0}$ and $\mathbf{B} \neq \mathbf{0}$. Therefore,

$$\begin{aligned} \det(\mathbf{I}_{n\times n} + \mathbf{A}) &= \det\left(\mathbf{I}_{n\times n} + \sum_{i=1}^{n} \lambda_i \mathbf{u}_i \mathbf{u}_i^H\right) \\ &\leq \det\left(\mathbf{I}_{n\times n} + \sum_{i=1}^{n} \lambda_i \mathbf{u}_i \mathbf{u}_i^H + \sum_{i=1}^{n} \mu_i \mathbf{v}_i \mathbf{v}_i^H\right) \\ &= \det(\mathbf{I}_{n\times n} + \mathbf{A} + \mathbf{B}) \end{aligned} \tag{A5}$$

by Lemma.

Proof of (6): Since $\ln(\cdot)$ is monotonically increasing, maximization of $\ln\det(\cdot)$ is equivalent to the maximization of $\det(\cdot)$. Therefore, by corollary,

$$\begin{aligned} &\det\left(\mathbf{I}_{2K\times 2K} + \frac{SNR}{2R_t} \sum_{i=1}^{P} \mathbf{H}_i \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_i^H\right) \\ &\geq \det\left(\mathbf{I}_{2K\times 2K} + \frac{SNR}{2R_t} \mathbf{H}_j \mathbf{w}_j \mathbf{w}_j^H \mathbf{H}_j^H\right) \\ &= 1 + \frac{SNR}{2R_t} \mathbf{w}_j^H \mathbf{H}_j^H \mathbf{H}_j \mathbf{w}_j \end{aligned} \tag{A6}$$

where P is the number of transmit antenna array groups, for $1 \leq j \leq P$. Adding (A6) to both sides for $1 \leq j \leq P$ and dividing by P,

$$\begin{aligned} &\det\left(\mathbf{I}_{2K\times 2K} + \frac{SNR}{2R_t} \sum_{i=1}^{P} \mathbf{H}_i \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_i^H\right) \\ &\geq 1 + \frac{SNR}{P \times 2R_t} \sum_{j=1}^{P} \mathbf{w}_j^H \mathbf{H}_j^H \mathbf{H}_j \mathbf{w}_j \end{aligned} \tag{A7}$$

Therefore, we can choose $\mathbf{w}_j$ as the eigenvector associated with the maximum eigenvalue of $\mathbf{H}_j^H \mathbf{H}_j$ to maximize the right-hand of (A7).

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