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ELECTROMAGNETIC SCATTERING FROM PARALLEL CHIRAL CYLINDERS OF CIRCULAR CROSS-SECTIONS USING AN ITERATIVE PROCEDURE

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Abstract—The electromagnetic scattering from a 2D chiral circular cylinders, illuminated by either a TE_z or a TM_z plane wave, is investigated using an iterative scattering procedure. The developed formulation and the implemented code simulate different types of cylinders, where the cylinders can be made of anisotropic chiral material with uniform or non-uniform chiral admittance distribution, homogeneous isotropic dielectric material, perfectly conducting material or a combination of all of them. The technique applies the boundary conditions on the surface of each cylinder in an iterative procedure in order to solve for the field expansion coefficients. Numerical verifications are presented to prove the validity of the formulation before presenting the scattering from an array of chiral cylinders showing significant RCS reduction in forward or backward directions based on the selection of the chirality parameter.

1. INTRODUCTION

The interaction of electromagnetic fields with chiral matters has been studied over the years. Chiral media were used in many applications involving antennas and arrays, antenna radomes, microstrip substrates and waveguides. A chiral object is, by definition, a body that lacks bilateral symmetry, which means that it cannot be superimposed on its mirror image neither by translation nor rotation. This can also be known as handedness. Objects that have the property of handedness are said to be either right-handed or left-handed. Chiral media are optically active — a property caused by asymmetrical molecular structure that enables a substance to rotate the plane of incident polarized light, where the amount of rotation in the plane of polarization is proportional to the thickness of the medium traversed as well as to the light wavelength [1–5]. Thus chiral medium has an effect on the attenuation rate of the right hand and left hand circularly polarized waves. Unlike dielectric or conducting cylinders, chiral scatterers produce both co-polarized and cross-polarized scattered fields. Coating with chiral material is therefore attempted for reducing radar cross-section of targets.

In this paper, an iterative solution to the problem of electromagnetic scattering from an incident plane wave on M different circular cylinders is derived. This solution is then used for presenting simple configurations of chiral cylinders that can be used to enhance or reduce the radar cross-section of two dimensional targets. The cylinders are made of either lossy or lossless anisotropic chiral matters, dielectric, conducting or a combination of all of them. The iterative procedure starts by calculating the initial scattered field from each cylinder due to the incident plane wave, where these fields are zero order-scattered fields. After calculating the initial scattered fields, interaction between the cylinders is to be considered assuming that this interaction is due to mutual scattering among the cylinders. The initial first order-scattered fields from $M-1$ cylinders are considered as the incident field on the remaining cylinder inducing the second order scattered fields from all $M-1$ cylinders after applying the appropriate boundary conditions on the surface of each cylinder is applied in the first order of interaction [6–8]. This iterative scattering procedure between the cylinders yields, after infinite, theoretically, number of interactions, the total scattered field that is the summation of all interactions. Numerical verifications are presented to prove the validity of this developed formulation for chiral cylinders. New configurations of an array of cylinders having a uniform chiralitys distribution is considered to examine the chiralitys effect on the RCS reduction.

2. FORMULATION

Consider a number of parallel circular cylinders excited by an incident plane wave as shown in Fig. 1. The cylinders are numbered from 1 to M, while each cylinder defined by its radius, material type (conductor, dielectric or chiral) and its center coordinate with respect to the global cylindrical coordinates system (ρ, ϕ).

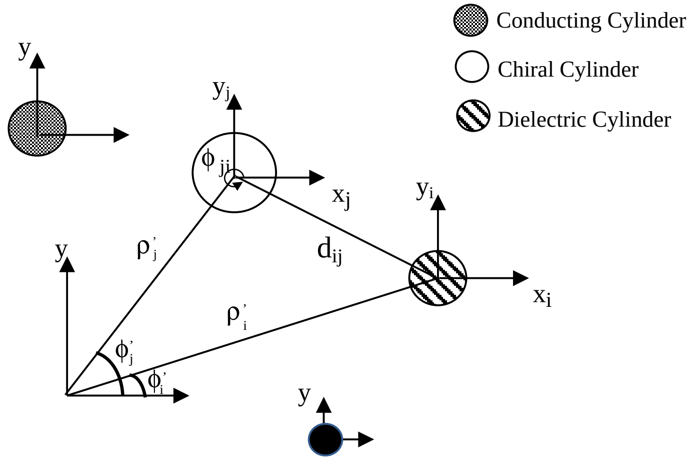


Figure 1. Cross section of parallel circular cylinders with arbitrary locations and radii.

Chiral medium is characterized by the following constitutive relations for electromagnetic field with $e^{j\omega t}$ time-harmonic dependence [9]

$$\underline{D} = \varepsilon \underline{E} - j\xi_c \underline{B} \tag{1}$$

$$\underline{H} = \frac{1}{\mu} \underline{B} - j\xi_c \underline{E}. \tag{2}$$

The chiral media has two different phase velocities for right-hand circularly polarized waves (RCP) and left-hand circularly polarized waves (LCP) leading to two different bulk wave numbers k_+ and k_-, which are given by

$$k_{\pm} = k\left[\sqrt{1+x^2} \pm x\right] \tag{3}$$

where $k = \omega\sqrt{\mu\varepsilon}$ and chirality parameter $x = \sqrt{\mu/\varepsilon}\xi_c$ and ξ_c is the chiral admittance [9].

Considering an E-polarized incident wave (TM_z), the incident electric field of a plane wave on cylinder "i" is expressed in the (ρ_i, ϕ_i) cylindrical coordinates system as

$$\begin{aligned} E_z^{inc}(\rho_i,\phi_i) &= E_0 e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} e^{jk_0\rho_i\cos(\phi_i-\phi_0)} \\ &= E_0 e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} \sum_{n=-\infty}^{\infty} j^n J_n(k_0\rho_i) e^{jn(\phi_i-\phi_0)} \end{aligned} \tag{4}$$

where k_0 is the free space wave number, $J_n(x)$ is the Bessel function of the first kind and ϕ_0 is the angle of incidence of the plane wave with respect to the negative x-axis. This incident field expression is in terms of the cylindrical coordinates of the ith whose center is located at (ρ_i', ϕ_i') of the global coordinates (ρ, ϕ).

The corresponding ϕ component of the magnetic field is given by

$$H_{\phi i}^{inc}(\rho_i,\phi_i) = \frac{E_0}{j\eta_0} e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} \sum_{n=-\infty}^{\infty} j^n J_n'(k_0\rho_i) e^{jn(\phi_i-\phi_0)}. \tag{5}$$

The resulting z component of the scattered electric field from the ith cylinder and the transmitted z component of the field inside the chiral material of this cylinder can be expressed, respectively, as

$$E_{zi}^s(\rho_i,\phi_i) = E_0 \sum_{n=-\infty}^{\infty} C_{in} H_n^{(2)}(k_0\rho_i) e^{jn\phi_i} \tag{6}$$

$$E_{zi}^c(\rho_i,\phi_i) = E_0 \sum_{n=-\infty}^{\infty} \left[A_{in} J_n(k_+\rho_i) + B_{in} J_n(k_-\rho_i)\right] e^{jn\phi_i} \tag{7}$$

where C_{in} is the unknown expansion coefficients for the scattered field, while A_{in} and B_{in} are the unknown expansion coefficients for the fields inside the chiral cylinder. The Hankel functions of the second kind are used here, to satisfy the radiation conditions at infinity. The field components inside the cylinder are expressed in a Fourier series form in terms of Bessel functions of the first kind, as the field is finite at the origin of the cylinder. The corresponding ϕ components of the magnetic fields are obtained as,

$$H_{\phi i}^s(\rho_i,\phi_i) = \frac{1}{j\eta_0 k_0} \frac{\partial E_z^s(\rho_i,\phi_i)}{\partial \rho_i}$$

$$= \frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} C_{in} H_n^{(2)'}(k_0\rho_i) e^{jn\phi_i} \tag{8}$$

$$\begin{aligned} H_{\phi i}^c(\rho_i, \phi_i) &= \frac{1}{j\eta_{ci}k_i} \frac{\partial E_z^c(\rho_i, \phi_i)}{\partial \rho_i} \\ &= \frac{E_0}{j\eta_{ci}} \sum_{n=-\infty}^{\infty} \left[A_{in} J_n'(k_+\rho_i) + B_{in} J_n'(k_-\rho_i) \right] e^{jn\phi_i}. \end{aligned} \tag{9}$$

Unlike nonchiral cylinders, the scattered and the internal fields will contain TE_z fields in addition to the TM_z fields. The z component of the scattered magnetic field and the transmitted z component inside the chiral material of the ith cylinder can be expressed as [3],

$$H_{zi}^s(\rho_i, \phi_i) = j\frac{E_0}{\eta_0} \sum_{n=-\infty}^{\infty} D_{in} H_n^{(2)}(k_0\rho_i) e^{jn\phi_i} \tag{10}$$

$$H_{zi}^c(\rho_i, \phi_i) = j\frac{E_0}{\eta_{ci}} \sum_{n=-\infty}^{\infty} \left[A_{in} J_n(k_+\rho_i) - B_{in} J_n(k_-\rho_i) \right] e^{jn\phi_i} \tag{11}$$

while the corresponding ϕ components of the electric fields are obtained as,

$$\begin{aligned} E_{\phi i}^s(\rho_i, \phi_i) &= \frac{E_0}{j\eta_0 k_0} \frac{\partial H_z^s(\rho_i, \phi_i)}{\partial \rho_i} \\ &= E_0 \sum_{n=-\infty}^{\infty} D_{in} H_n^{(2)'}(k_0\rho_i) e^{jn\phi_i} \end{aligned} \tag{12}$$

$$\begin{aligned} E_{\phi i}^c(\rho_i, \phi_i) &= \frac{E_0}{j\eta_{ci} k_i} \frac{\partial H_z^c(\rho_i, \phi_i)}{\partial \rho_i} \\ &= E_0 \sum_{n=-\infty}^{\infty} \left[A_{in} J_n'(k_+\rho_i) - B_{in} J_n'(k_-\rho_i) \right] e^{jn\phi_i}. \end{aligned} \tag{13}$$

3. SOLUTION OF THE UNKNOWN COEFFICIENTS

The application of the boundary conditions on the surface of the ith cylinder, which will enforce the tangential components of both electric and magnetic fields to be continuous on the surface of the cylinder, leads to

$$E_{zi}^{inc} + E_{zi}^s = E_{zi}^c \quad \text{at} \quad \rho_i = a_i, \quad 0 \le \phi_i \le 2\pi \tag{14}$$

$$H_{\phi i}^{inc} + H_{\phi i}^s = H_{\phi i}^c \quad \text{at} \quad \rho_i = a_i, \quad 0 \le \phi_i \le 2\pi \tag{15}$$

$$H_{zi}^s = H_{zi}^c \quad \text{at} \quad \rho_i = a_i, \quad 0 \le \phi_i \le 2\pi \tag{16}$$

$$E_{\phi i}^s = E_{\phi i}^c \quad \text{at} \quad \rho_i = a_i, \quad 0 \le \phi_i \le 2\pi \tag{17}$$

where a_i is the radius of the ith cylinder. Then the boundary condition of the electric and magnetic fields for both the co and cross-polarized fields respectively, can be written for the ith cylinder in the following form,

$$E_0 e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} \sum_{n=-\infty}^{\infty} j^n J_n(k_0\rho_i) e^{jn(\phi_i-\phi_0)} + E_0 \sum_{n=-\infty}^{\infty} C_{in}^0 H_n^{(2)}(k_0\rho_i) e^{jn\phi_i}$$

$$= E_0 \sum_{n=-\infty}^{\infty} \left[A_{in}^0 J_n(k_+\rho_i) + B_{in}^0 J_n(k_-\rho_i)\right] e^{jn\phi_i} \tag{18}$$

$$\frac{E_0}{j\eta_0} e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} \sum_{n=-\infty}^{\infty} j^n J_n'(k_0\rho_i) e^{jn(\phi_i-\phi_0)} + \frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} C_{in}^0 H_n^{(2)\prime}(k_0\rho_i) e^{jn\phi_i}$$

$$= \frac{E_0}{j\eta_{ci}} \sum_{n=-\infty}^{\infty} \left[A_{in}^0 J_n'(k_+\rho_i) + B_{in}^0 J_n'(k_-\rho_i)\right] e^{jn\phi_i} \tag{19}$$

$$j\frac{E_0}{\eta_0} \sum_{n=-\infty}^{\infty} D_{in}^0 H_n^{(2)}(k_0\rho_i) e^{jn\phi_i} = j\frac{E_0}{\eta_{ci}} \sum_{n=-\infty}^{\infty} \left[A_{in}^0 J_n(k_+\rho_i) - B_{in}^0 J_n(k_-\rho_i)\right] e^{jn\phi_i} \tag{20}$$

$$E_0 \sum_{n=-\infty}^{\infty} D_{in}^0 H_n^{(2)\prime}(k_0\rho_i) e^{jn\phi_i} = E_0 \sum_{n=-\infty}^{\infty} \left[A_{in}^0 J_n'(k_+\rho_i) - B_{in}^0 J_n'(k_-\rho_i)\right] e^{jn\phi_i}. \tag{21}$$

Adding a superscript "0" to the unknown expansion coefficients C_{in}^0, D_{in}^0, A_{in}^0 and B_{in}^0 is to indicate that for these four equations only the scattering from the ith cylinder due to the incident wave is to be considered and no interactions between the other $M-1$ cylinders are assumed. Where C_{in}^0, D_{in}^0 are the initial unknown expansion coefficients for the scattered field, while A_{in}^0, B_{in}^0 are the initial unknown expansion coefficients for the internal fields. After some mathematical manipulations the expressions for the four unknown coefficients C_{in}^0, D_{in}^0, A_{in}^0 and B_{in}^0 can be written as follow

$$C_{in}^0 = \frac{\left[V_\ell^i J_{\ell+}' - v_i V_\ell^{i\prime} J_{\ell+}\right]\left[h_\ell' J_{\ell-} - v_i h_\ell J_{\ell-}'\right] - \left[V_\ell^i J_{\ell-}' - v_i V_\ell^{i\prime} J_{\ell-}\right]\left[v_i h_\ell J_{\ell+}' - h_\ell' J_{\ell+}\right]}{\left[h_\ell J_{\ell-}' - v_i h_\ell' J_{\ell-}\right]\left[v_i h_\ell J_{\ell+}' - h_\ell' J_{\ell+}\right] - \left[h_\ell J_{\ell+}' - v_i h_\ell' J_{\ell+}\right]\left[h_\ell' J_{\ell-} - v_i h_\ell J_{\ell-}'\right]} \tag{22}$$

$$D_{in}^0 = \frac{\left[V_\ell^i J_{\ell+}' - v_i V_\ell^{i\prime} J_{\ell+}\right]\left[h_\ell J_{\ell-}' - v_i h_\ell' J_{\ell-}\right] - \left[V_\ell^i J_{\ell-}' - v_i V_\ell^{i\prime} J_{\ell-}\right]\left[h_\ell J_{\ell+}' - v_i h_\ell' J_{\ell+}\right]}{\left[h_\ell' J_{\ell-} - v_i h_\ell J_{\ell-}'\right]\left[h_\ell J_{\ell+}' - v_i h_\ell' J_{\ell+}\right] - \left[v_i h_\ell J_{\ell+}' - h_\ell' J_{\ell+}\right]\left[h_\ell J_{\ell-}' - v_i h_\ell' J_{\ell-}\right]} \tag{23}$$

$$A_{in}^0 = v_i V_\ell^i \left[\frac{\left(h'_\ell - h_\ell\right)\left(v_i h_\ell J'_{\ell-} - h'_\ell J_{\ell-}\right)}{\left(v_i h'_\ell J_{\ell+} - h_\ell J'_{\ell+}\right)\left(v_i h_\ell J'_{\ell-} - h'_\ell J_{\ell-}\right) - \left(h'_\ell J_{\ell+} - v_i h_\ell J'_{\ell+}\right)\left(v_i h'_\ell J_{\ell-} - h_\ell J'_{\ell-}\right)} \right] \tag{24}$$

$$B_{in}^0 = v_i V_\ell^i \left[\frac{\left(h'_\ell - h_\ell\right)\left(h'_\ell J_{\ell+} - v_i h_\ell J'_{\ell+}\right)}{\left(v_i h'_\ell J_{\ell-} - h_\ell J'_{\ell-}\right)\left(h'_\ell J_{\ell+} - v_i h_\ell J'_{\ell+}\right) - \left(v_i h_\ell J'_{\ell-} - h'_\ell J_{\ell-}\right)\left(v_i h'_\ell J_{\ell+} - h_\ell J'_{\ell+}\right)} \right]. \tag{25}$$

where

$$V_\ell^i = e^{jk_0\rho'_i \cos(\phi'_i - \phi_0)} j^\ell e^{-j\ell\phi_0} \quad h_\ell = \frac{H_\ell^{(2)}(k_0 a_i)}{J_\ell(k_0 a_i)} \quad J_{\ell+} = \frac{J_\ell(k_+ a_i)}{J_\ell(k_0 a_i)}$$

$$J_{\ell-} = \frac{J_\ell(k_- a_i)}{J_\ell(k_0 a_i)} \qquad h'_\ell = \frac{H_\ell^{(2)'}(k_0 a_i)}{J'_\ell(k_0 a_i)}$$

$$J'_{\ell+} = \frac{J'_\ell(k_+ a_i)}{J'_\ell(k_0 a_i)} \qquad J'_{\ell-} = \frac{J'_\ell(k_- a_i)}{J'_\ell(k_0 a_i)}$$

$$v_i = \frac{\eta_{ci}}{\eta_0}.$$

After evaluating the initial scattered field from all cylinders interaction between the cylinders is considered assuming the interaction is due to mutual scattering among the cylinders. For cylinder i, the initial scattered fields from all other cylinders are to be considered as incident fields on it.These incident fields will induce the first order scattered field from cylinder i, and then the first order scattered fields from all other cylinders will induce the second order scattered field from cylinder i. This iterative scattering procedure between the cylinders yields, after infinite number of interactions, the total scattered field that is the summation of all interactions [6]. The incident copolarized fields on cylinder "i" for the first order of interaction can be written as

$$E_{zi}^s = E_0 \sum_{\substack{j=1 \\ j\neq i}}^{M} \sum_{n=-\infty}^{\infty} C_{jn}^0 H_n^{(2)}(k_0\rho_j) e^{jn\phi_j} \tag{26}$$

while the corresponding magnetic incident field can be written as

$$H_{\phi i}^s = \frac{E_0}{j\eta_0} \sum_{\substack{j=1 \\ j\neq i}}^{M} \sum_{n=-\infty}^{\infty} C_{jn}^0 H_n^{(2)'}(k_0\rho_j) e^{jn\phi_j}. \tag{27}$$

The incident cross polarized fields for the first order of interaction can be written as

$$H^s_{zi} = j\frac{E_0}{\eta_0}\sum_{\substack{j=1\\ j\neq i}}^{M}\sum_{n=-\infty}^{\infty} D^0_{jn}H^{(2)}_n(k_0\rho_j)e^{jn\phi_j} \tag{28}$$

while the corresponding electric incident field can be written as

$$E^s_{\phi i} = E_0\sum_{\substack{j=1\\ j\neq i}}^{M}\sum_{n=-\infty}^{\infty} D^0_{jn}H^{(2)'}_n(k_0\rho_j)e^{jn\phi_j} \tag{29}$$

where M is the total number of cylinders. Applying the boundary conditions on the surface of the ith cylinder, we get

$$\begin{aligned}&\sum_{\substack{j=1\\ j\neq i}}^{M}\sum_{n=-\infty}^{\infty} C^0_{jn}H^{(2)}_n(k_0\rho_j)e^{jn\phi_j} + \sum_{n=-\infty}^{\infty} C^1_{jn}H^{(2)}_n(k_0\rho_i)e^{jn\phi_i}\\ &= \sum_{n=-\infty}^{\infty}\left[A^1_{in}J_n(k_+\rho_i) + B^1_{in}J_n(k_-\rho_i)\right]e^{jn\phi_i}\end{aligned} \tag{30a}$$

$$\begin{aligned}&\sum_{\substack{j=1\\ j\neq i}}^{M}\sum_{n=-\infty}^{\infty} C^0_{jn}H^{(2)'}_n(k_0\rho_j)e^{jn\phi_j} + \sum_{n=-\infty}^{\infty} C^1_{in}H^{(2)'}_n(k_0\rho_i)e^{jn\phi_i}\\ &= \frac{\eta_0}{\eta_{ci}}\sum_{n=-\infty}^{\infty}\left[A^1_{in}J'_n(k_+\rho_i) + B^1_{in}J'_n(k_-\rho_i)\right]e^{jn\phi_i}\end{aligned} \tag{30b}$$

$$\begin{aligned}&\sum_{\substack{j=1\\ j\neq i}}^{M}\sum_{n=-\infty}^{\infty} D^0_{jn}H^{(2)}_n(k_0\rho_j)e^{jn\phi_j} + \sum_{n=-\infty}^{\infty} D^1_{in}H^{(2)}_n(k_0\rho_i)e^{jn\phi_i}\\ &= \frac{\eta_0}{\eta_{ci}}\sum_{n=-\infty}^{\infty}\left[A^1_{in}J_n(k_+\rho_i) - B^1_{in}J_n(k_-\rho_i)\right]e^{jn\phi_i}\end{aligned} \tag{30c}$$

$$\begin{aligned}&\sum_{\substack{j=1\\ j\neq i}}^{M}\sum_{n=-\infty}^{\infty} D^0_{jn}H^{(2)'}_n(k_0\rho_j)e^{jn\phi_j} + \sum_{n=-\infty}^{\infty} D^1_{in}H^{(2)'}_n(k_0\rho_i)e^{jn\phi_i}\\ &= \sum_{n=-\infty}^{\infty}\left[A^1_{in}J'_n(k_+\rho_i) - B^1_{in}J'_n(k_-\rho_i)\right]e^{jn\phi_i}\end{aligned} \tag{30d}$$

where the superscript "1" is the unknown expansion coefficients, which refers to the first order of interaction. C_{in}^1, D_{in}^1, A_{in}^1 and B_{in}^1 cannot be obtained directly from Equations (30), because the scattered field from the other $M-1$ cylinders are based on their local coordinates. In this case the addition theorem for the Hankel function should be used to transfer all the scattered fields to the local coordinates of the ith cylinder. In general, the transformation from the gth coordinate to the ith coordinate can be given as

$$H_n^{(2)}(k\rho_g) = \sum_{m=-\infty}^{\infty} J_m(k_0\rho_i) H_{m-n}^{(2)}(k_0 d_{ig}) e^{j[m\phi_i-(m-n)\phi_{ig}]} \tag{31}$$

where d_{ig} and ϕ_{ig} are given by

$$d_{ig} = \sqrt{\rho_i^2+\rho_g^2-2\rho_i\rho_g\cos(\phi_i-\phi_g)} \tag{32}$$

$$\phi_{ig} = \tan^{-1}\left[\frac{\rho_i\sin\phi_i-\rho_g\sin\phi_g}{\rho_i\cos\phi_i-\rho_g\cos\phi_g}\right]. \tag{33}$$

After some mathematical manipulations the expressions for the four unknown coefficients $C_{i\ell}^1$, $D_{i\ell}^1$, $A_{i\ell}^1$ and $B_{i\ell}^1$ can be written for the ith cylinder as follow

$$C_{i\ell}^1 = \frac{\left[\begin{array}{c} C^{\mathrm{I}} \times \sum\limits_{\substack{g=1\\g\neq i}}^{M} \sum\limits_{n=-\infty}^{\infty} C_{gn}^0 H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \\ +D^{\mathrm{I}} \times \sum\limits_{\substack{g=1\\g\neq i}}^{M} \sum\limits_{n=-\infty}^{\infty} D_{gn}^0 H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \end{array}\right]}{X^{\mathrm{I}}} \tag{34}$$

where

$$C^{\mathrm{I}} = \left[\begin{array}{c} \dfrac{J_\ell(k_0a_i)J_\ell'(k_+a_i)-v_iJ_\ell'(k_0a_i)J_\ell(k_+a_i)}{H_\ell^{(2)\prime}(k_0a_i)J_\ell(k_+a_i)-v_iH_\ell^{(2)}(k_0a_i)J_\ell'(k_+a_i)} \\ -\dfrac{J_\ell(k_0a_i)J_\ell'(k_-a_i)-v_iJ_\ell'(k_0a_i)J_\ell(k_-a_i)}{v_iH_\ell^{(2)}(k_0a_i)J_\ell'(k_-a_i)-H_\ell^{(2)\prime}(k_0a_i)J_\ell(k_-a_i)} \end{array}\right] \tag{35}$$

$$D^{\mathrm{I}} = \left[\begin{array}{c} \dfrac{v_iJ_\ell(k_0a_i)J_\ell'(k_+a_i)-J_\ell'(k_0a_i)J_\ell(k_+a_i)}{H_\ell^{(2)\prime}(k_0a_i)J_\ell(k_+a_i)-v_iH_\ell^{(2)}(k_0a_i)J_\ell'(k_+a_i)} \\ -\dfrac{J_\ell'(k_0a_i)J_\ell(k_-a_i)-v_iJ_\ell(k_0a_i)J_\ell'(k_-a_i)}{v_iH_\ell^{(2)}(k_0a_i)J_\ell'(k_-a_i)-H_\ell^{(2)\prime}(k_0a_i)J_\ell(k_-a_i)} \end{array}\right] \tag{36}$$

$$X^{\mathrm{I}} = \left[\begin{array}{c} \dfrac{v_i H_\ell^{(2)'}(k_0 a_i) J_\ell(k_+ a_i) - H_\ell^{(2)}(k_0 a_i) J'_\ell(k_+ a_i)}{H_\ell^{(2)'}(k_0 a_i) J_\ell(k_+ a_i) - v_i H_\ell^{(2)}(k_0 a_i) J'_\ell(k_+ a_i)} \\ -\dfrac{v_i H_\ell^{(2)'}(k_0 a_i) J_\ell(k_- a_i) - H_\ell^{(2)}(k_0 a_i) J'_\ell(k_- a_i)}{v_i H_\ell^{(2)}(k_0 a_i) J'_\ell(k_- a_i) - H_\ell^{(2)'}(k_0 a_i) J_\ell(k_- a_i)} \end{array} \right] \tag{37}$$

also

$$D^1_{i\ell} = \frac{\left[\begin{array}{c} C^{\mathrm{II}} \times \displaystyle\sum_{\substack{g=1 \\ g \neq i}}^{M} \sum_{n=-\infty}^{\infty} C^0_{gn} H^{(2)}_{\ell-n}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \\ +D^{\mathrm{II}} \times \displaystyle\sum_{\substack{g=1 \\ g \neq i}}^{M} \sum_{n=-\infty}^{\infty} D^0_{gn} H^{(2)}_{\ell-n}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \end{array} \right]}{X^{\mathrm{II}}} \tag{38}$$

where

$$C^{\mathrm{II}} = \left[\begin{array}{c} \dfrac{J_\ell(k_0 a_i) J'_\ell(k_+ a_i) - v_i J'_\ell(k_0 a_i) J_\ell(k_+ a_i)}{v_i H_\ell^{(2)'}(k_0 a_i) J_\ell(k_+ a_i) - H_\ell^{(2)}(k_0 a_i) J'_\ell(k_+ a_i)} \\ -\dfrac{J_\ell(k_0 a_i) J'_\ell(k_- a_i) - v_i J'_\ell(k_0 a_i) J_\ell(k_- a_i)}{v_i H_\ell^{(2)'}(k_0 a_i) J_\ell(k_- a_i) - H_\ell^{(2)}(k_0 a_i) J'_\ell(k_- a_i)} \end{array} \right] \tag{39}$$

$$D^{\mathrm{II}} = \left[\begin{array}{c} \dfrac{v_i J_\ell(k_0 a_i) J'_\ell(k_+ a_i) - J'_\ell(k_0 a_i) J_\ell(k_+ a_i)}{v_i H_\ell^{(2)'}(k_0 a_i) J_\ell(k_+ a_i) - H_\ell^{(2)}(k_0 a_i) J'_\ell(k_+ a_i)} \\ -\dfrac{J'_\ell(k_0 a_i) J_\ell(k_- a_i) - v_i J_\ell(k_0 a_i) J'_\ell(k_- a_i)}{v_i H_\ell^{(2)'}(k_0 a_i) J_\ell(k_- a_i) - H_\ell^{(2)}(k_0 a_i) J'_\ell(k_- a_i)} \end{array} \right] \tag{40}$$

$$X^{\mathrm{II}} = \left[\begin{array}{c} \dfrac{H_\ell^{(2)'}(k_0 a_i) J_\ell(k_+ a_i) - v_i H_\ell^{(2)}(k_0 a_i) J'_\ell(k_+ a_i)}{v_i H_\ell^{(2)'}(k_0 a_i) J_\ell(k_+ a_i) - H_\ell^{(2)}(k_0 a_i) J'_\ell(k_+ a_i)} \\ -\dfrac{v_i H_\ell^{(2)}(k_0 a_i) J'_\ell(k_- a_i) - H_\ell^{(2)'}(k_0 a_i) J_\ell(k_- a_i)}{v_i H_\ell^{(2)'}(k_0 a_i) J_\ell(k_- a_i) - H_\ell^{(2)}(k_0 a_i) J'_\ell(k_- a_i)} \end{array} \right] \tag{41}$$

while $A_{i\ell}^2$ and $B_{i\ell}^2$ are given as

$$A_{i\ell}^1 = \frac{\left[\begin{array}{c} C^{\mathrm{III}} \times \sum\limits_{\substack{g=1 \\ g\neq i}}^{M} \sum\limits_{n=-\infty}^{\infty} C_{gn}^0 H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \\ -D^{\mathrm{III}} \times \sum\limits_{\substack{g=1 \\ g\neq i}}^{M} \sum\limits_{n=-\infty}^{\infty} D_{gn}^0 H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \end{array}\right]}{X^{\mathrm{III}}} \tag{42}$$

where

$$C^{\mathrm{III}} = \left[\frac{v_i\left(J_\ell(k_0a_i)H_\ell^{(2)'}(k_0a_i) - J_\ell'(k_0a_i)H_\ell^{(2)}(k_0a_i)\right)}{v_iH_\ell^{(2)'}(k_0a_i)J_\ell(k_-a_i) - H_\ell^{(2)}(k_0a_i)J_\ell'(k_-a_i)}\right] \tag{43}$$

$$D^{\mathrm{III}} = \left[\frac{v_i\left(J_\ell(k_0a_i)H_\ell^{(2)'}(k_0a_i) - J_\ell'(k_0a_i)H_\ell^{(2)}(k_0a_i)\right)}{v_iH_\ell^{(2)}(k_0a_i)J_\ell'(k_-a_i) - H_\ell^{(2)'}(k_0a_i)J_\ell(k_-a_i)}\right] \tag{44}$$

$$X^{\mathrm{III}} = \left[\begin{array}{c} \dfrac{v_iH_\ell^{(2)'}(k_0a_i)J_\ell(k_+a_i) - H_\ell^{(2)}(k_0a_i)J_\ell'(k_+a_i)}{v_iH_\ell^{(2)'}(k_0a_i)J_\ell(k_-a_i) - H_\ell^{(2)}(k_0a_i)J_\ell'(k_-a_i)} \\ -\dfrac{H_\ell^{(2)'}(k_0a_i)J_\ell(k_+a_i) - v_iH_\ell^{(2)}(k_0a_i)J_\ell'(k_+a_i)}{v_iH_\ell^{(2)}(k_0a_i)J_\ell'(k_-a_i) - H_\ell^{(2)'}(k_0a_i)J_\ell(k_-a_i)} \end{array}\right] \tag{45}$$

while

$$B_{i\ell}^1 = \frac{\left[\begin{array}{c} C^{\mathrm{IV}} \times \sum\limits_{\substack{g=1 \\ g\neq i}}^{M} \sum\limits_{n=-\infty}^{\infty} C_{gn}^0 H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \\ -D^{\mathrm{IV}} \times \sum\limits_{\substack{g=1 \\ g\neq i}}^{M} \sum\limits_{n=-\infty}^{\infty} D_{gn}^0 H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \end{array}\right]}{X^{\mathrm{IV}}} \tag{46}$$

where

$$C^{\mathrm{IV}} = \left[\frac{v_i\left(J_\ell(k_0a_i)H_\ell^{(2)'}(k_0a_i) - J_\ell'(k_0a_i)H_\ell^{(2)}(k_0a_i)\right)}{v_iH_\ell^{(2)'}(k_0a_i)J_\ell(k_+a_i) - H_\ell^{(2)}(k_0a_i)J_\ell'(k_+a_i)}\right] \tag{47}$$

$$D^{\mathrm{IV}} = \left[\frac{v_i\left(J_\ell(k_0a_i)H_\ell^{(2)'}(k_0a_i) - J'_\ell(k_0a_i)H_\ell^{(2)}(k_0a_i)\right)}{H_\ell^{(2)'}(k_0a_i)J_\ell(k_+a_i) - v_iH_\ell^{(2)}(k_0a_i)J'_\ell(k_+a_i)}\right] \tag{48}$$

$$X^{\mathrm{IV}} = \left[\begin{array}{c} \dfrac{v_iH_\ell^{(2)'}(k_0a_i)J_\ell(k_-a_i) - H_\ell^{(2)}(k_0a_i)J'_\ell(k_-a_i)}{v_iH_\ell^{(2)'}(k_0a_i)J_\ell(k_+a_i) - H_\ell^{(2)}(k_0a_i)J'_\ell(k_+a_i)} \\ -\dfrac{v_iH_\ell^{(2)}(k_0a_i)J'_\ell(k_-a_i) - H_\ell^{(2)'}(k_0a_i)J_\ell(k_-a_i)}{H_\ell^{(2)'}(k_0a_i)J_\ell(k_+a_i) - v_iH_\ell^{(2)}(k_0a_i)J'_\ell(k_+a_i)} \end{array}\right]. \tag{49}$$

After solving for the first order unknown coefficients $C^1_{i\ell}$, $D^1_{i\ell}$, $A^1_{i\ell}$ and $B^1_{i\ell}$, for all M cylinders we can continue on to find the coefficients of the second order of interaction. A recurrence relation is developed, where the coefficients in the pth interaction depends only on the coefficients of the $(p-1)$th interaction, i.e.,

$$\sum_{\substack{g=1\\g\neq i}}^{M}\sum_{n=-\infty}^{\infty} C^{p-1}_{gn}J_\ell(k_0a_i)H^{(2)}_{\ell-n}(k_0d_{ig})e^{-j(\ell-n)\phi_{ig}} + C^p_{i\ell}H_\ell^{(2)}(k_0a_i)$$
$$= A^p_{i\ell}J_\ell(k_+a_i) + B^p_{i\ell}J_\ell(k_-a_i) \tag{50}$$

$$\sum_{\substack{g=1\\g\neq i}}^{M}\sum_{n=-\infty}^{\infty} C^{p-1}_{gn}J'_\ell(k_0a_i)H^{(2)}_{\ell-n}(k_0d_{ig})e^{-j(\ell-n)\phi_{ig}} + C^p_{i\ell}H_\ell^{(2)'}(k_0a_i)$$
$$= \frac{1}{v_i}\left[A^p_{i\ell}J'_\ell(k_+a_i) + B^p_{i\ell}J'_\ell(k_-a_i)\right] \tag{51}$$

$$\sum_{\substack{g=1\\g\neq i}}^{M}\sum_{n=-\infty}^{\infty} D^{p-1}_{gn}J_\ell(k_0a_i)H^{(2)}_{\ell-n}(k_0d_{ig})e^{-j(\ell-n)\phi_{ig}} + D^p_{i\ell}H_\ell^{(2)}(k_0a_i)$$
$$= \frac{1}{v_i}\left[A^p_{i\ell}J_\ell(k_+a_i) - B^p_{i\ell}J_\ell(k_-a_i)\right] \tag{52}$$

$$\sum_{\substack{g=1\\g\neq i}}^{M}\sum_{n=-\infty}^{\infty} D^{p-1}_{gn}J'_\ell(k_0a_i)H^{(2)}_{\ell-n}(k_0d_{ig})e^{-j(\ell-n)\phi_{ig}} + D^p_{i\ell}H_\ell^{(2)'}(k_0a_i)$$
$$= A^p_{i\ell}J'_\ell(k_+a_i) - B^p_{i\ell}J'_\ell(k_-a_i) \tag{53}$$

and thus the unknown expansion coefficients reduce to,

$$C_{i\ell}^{p} = \frac{\left[\begin{array}{l} C^{\mathrm{I}} \times \displaystyle\sum_{\substack{g=1\\g\neq i}}^{M} \sum_{n=-\infty}^{\infty} C_{gn}^{p-1} H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \\ +D^{\mathrm{I}} \times \displaystyle\sum_{\substack{g=1\\g\neq i}}^{M} \sum_{n=-\infty}^{\infty} D_{gn}^{p-1} H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \end{array}\right]}{X^{\mathrm{I}}} \tag{54}$$

$$D_{i\ell}^{p} = \frac{\left[\begin{array}{l} C^{\mathrm{II}} \times \displaystyle\sum_{\substack{g=1\\g\neq i}}^{M} \sum_{n=-\infty}^{\infty} C_{gn}^{p-1} H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \\ +D^{\mathrm{II}} \times \displaystyle\sum_{\substack{g=1\\g\neq i}}^{M} \sum_{n=-\infty}^{\infty} D_{gn}^{p-1} H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \end{array}\right]}{X^{\mathrm{II}}} \tag{55}$$

$$A_{i\ell}^{p} = \frac{\left[\begin{array}{l} C^{\mathrm{III}} \times \displaystyle\sum_{\substack{g=1\\g\neq i}}^{M} \sum_{n=-\infty}^{\infty} C_{gn}^{p-1} H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \\ -D^{\mathrm{III}} \times \displaystyle\sum_{\substack{g=1\\g\neq i}}^{M} \sum_{n=-\infty}^{\infty} D_{gn}^{p-1} H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \end{array}\right]}{X^{\mathrm{III}}} \tag{56}$$

$$B_{i\ell}^{p} = \frac{\left[\begin{array}{l} C^{\mathrm{IV}} \times \displaystyle\sum_{\substack{g=1\\g\neq i}}^{M} \sum_{n=-\infty}^{\infty} C_{gn}^{p-1} H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \\ -D^{\mathrm{IV}} \times \displaystyle\sum_{\substack{g=1\\g\neq i}}^{M} \sum_{n=-\infty}^{\infty} D_{gn}^{p-1} H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \end{array}\right]}{X^{\mathrm{IV}}} \tag{57}$$

It is obvious that these coefficients depend on the previous interaction scattering coefficients as well as on the physical parameters of the cylinders as was noticed in [6] for conducting and dielectric cylinders. For all cylinders, it is possible to write the scattering coefficients in a matrix form, such that

$$[C^p] = [TC]\left[C^{p-1}\right] + [RC]\left[D^{p-1}\right] \tag{58}$$

$$[D^p] = [TD]\left[C^{p-1}\right] + [RD]\left[D^{p-1}\right] \tag{59}$$

where the unknown coefficients C and D are presented in the following matrix form

$$[C^p] = \begin{bmatrix} [C_1^p] \\ \cdot \\ [C_i^p] \\ \cdot \\ [C_M^p] \end{bmatrix}, \quad \left[C^{p-1}\right] = \begin{bmatrix} \left[C_1^{p-1}\right] \\ \cdot \\ \left[C_i^{p-1}\right] \\ \cdot \\ \left[C_M^{p-1}\right] \end{bmatrix}, \quad \left[D^{p-1}\right] = \begin{bmatrix} \left[D_1^{p-1}\right] \\ \cdot \\ \left[D_i^{p-1}\right] \\ \cdot \\ \left[D_M^{p-1}\right] \end{bmatrix} \tag{60}$$

$$[D^p] = \begin{bmatrix} [D_1^p] \\ \cdot \\ [D_i^p] \\ \cdot \\ [D_M^p] \end{bmatrix}, \quad \left[C^{p-1}\right] = \begin{bmatrix} \left[C_1^{p-1}\right] \\ \cdot \\ \left[C_i^{p-1}\right] \\ \cdot \\ \left[C_M^{p-1}\right] \end{bmatrix}, \quad \left[D^{p-1}\right] = \begin{bmatrix} \left[D_1^{p-1}\right] \\ \cdot \\ \left[D_i^{p-1}\right] \\ \cdot \\ \left[D_M^{p-1}\right] \end{bmatrix} \tag{61}$$

also all the four TC, RC, TD and RD matrices take the same form which is,

$$[T] = \begin{bmatrix} [0] & \cdot & [T_{1,j}] & \cdot & [T_{1,M}] \\ [T_{2,1}] & \cdot & [T_{2,1}] & \cdot & [T_{2,M}] \\ \cdot & \cdot & \cdot & [T_{i,j}] & \cdot \\ \cdot & \cdot & \cdot & [0] & \cdot \\ [T_{M,1}] & \cdot & [T_{M,j}] & \cdot & [0] \end{bmatrix} \tag{62}$$

where

$$[T_{i,j}] = \begin{bmatrix} T_{i,j}^{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & T_{i,j}^{1,n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & T_{i,j}^{\ell,n} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ T_{i,j}^{m,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & T_{i,j}^{m,n} \end{bmatrix}, \text{ with } \begin{array}{l} i \in (1, M) \\ j \in (1, M) \\ m \in (1, 2N_i + 1) \\ n \in (1, 2N_j + 1) \end{array} \tag{63}$$

and

$$TC_{i,j}^{\ell,n} = \frac{C^{\mathrm{I}}}{X^{\mathrm{I}}} \times H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \tag{64}$$

$$RC_{i,j}^{\ell,n} = \frac{D^{\mathrm{I}}}{X^{\mathrm{I}}} \times H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \tag{65}$$

$$TD_{i,j}^{\ell,n} = \frac{C^{\mathrm{II}}}{X^{\mathrm{II}}} \times H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \tag{66}$$

$$RD_{i,j}^{\ell,n} = \frac{D^{\mathrm{II}}}{X^{\mathrm{II}}} \times H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}}. \tag{67}$$

Matrix $[T]$ is a square matrix with diagonal sub matrices [0] while the sub matrices $[T_{i,j}]$ are not necessarily square matrices, especially if the radii of the cylinders are not identical, if combination of different cylinders is used, or if the parameters characterizing the cylinders is different. The physical interpretation of the sub matrix $[T_{i,j}]$ is the effect of cylinder j on cylinder i [6]. The unknown expansion coefficient of the internal fields can also be written in a matrix form given by

$$[A^p] = [AC]\left[C^{p-1}\right] - [AD]\left[D^{p-1}\right] \tag{68}$$

$$[B^p] = [BC]\left[C^{p-1}\right] - [BD]\left[D^{p-1}\right] \tag{69}$$

where the unknown coefficients A and B are presented in the following matrix form

$$[A^p] = \begin{bmatrix} [A_1^p] \\ \cdot \\ [A_i^p] \\ \cdot \\ [A_M^p] \end{bmatrix}, \quad \left[C^{p-1}\right] = \begin{bmatrix} \left[C_1^{p-1}\right] \\ \cdot \\ \left[C_i^{p-1}\right] \\ \cdot \\ \left[C_M^{p-1}\right] \end{bmatrix}, \quad \left[D^{p-1}\right] = \begin{bmatrix} \left[D_1^{p-1}\right] \\ \cdot \\ \left[D_i^{p-1}\right] \\ \cdot \\ \left[D_M^{p-1}\right] \end{bmatrix} \tag{70}$$

$$[B^p] = \begin{bmatrix} [B_1^p] \\ \cdot \\ [B_i^p] \\ \cdot \\ [B_M^p] \end{bmatrix}, \quad \left[C^{p-1}\right] = \begin{bmatrix} \left[C_1^{p-1}\right] \\ \cdot \\ \left[C_i^{p-1}\right] \\ \cdot \\ \left[C_M^{p-1}\right] \end{bmatrix}, \quad \left[D^{p-1}\right] = \begin{bmatrix} \left[D_1^{p-1}\right] \\ \cdot \\ \left[D_i^{p-1}\right] \\ \cdot \\ \left[D_M^{p-1}\right] \end{bmatrix} \tag{71}$$

also all the four AC, AD, BC and BD matrices take the same form which is,

$$[R] = \begin{bmatrix} [0] & \cdot & [R_{1,j}] & \cdot & [R_{1,M}] \\ [R_{2,1}] & \cdot & [R_{2,1}] & \cdot & [R_{2,M}] \\ \cdot & \cdot & \cdot & [R_{i,j}] & \cdot \\ \cdot & \cdot & \cdot & [0] & \cdot \\ [R_{M,1}] & \cdot & [R_{M,j}] & \cdot & [0] \end{bmatrix} \tag{72}$$

where

$$[R_{i,j}] = \begin{bmatrix} R_{i,j}^{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & R_{i,j}^{1,n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & R_{i,j}^{\ell,n} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ R_{i,j}^{m,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & R_{i,j}^{m,n} \end{bmatrix}, \text{ with } \begin{matrix} i \in (1, M) \\ j \in (1, M) \\ m \in (1, 2N_i + 1) \\ n \in (1, 2N_j + 1) \end{matrix} \tag{73}$$

$$AC_{i,j}^{\ell,n} = \frac{C^{\mathrm{III}}}{X^{\mathrm{III}}} \times H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \tag{74}$$

$$AD_{i,j}^{\ell,n} = \frac{D^{\mathrm{III}}}{X^{\mathrm{III}}} \times H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \tag{75}$$

$$BC_{i,j}^{\ell,n} = \frac{C^{\mathrm{IV}}}{X^{\mathrm{IV}}} \times H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}} \tag{76}$$

$$BD_{i,j}^{\ell,n} = \frac{D^{\mathrm{IV}}}{X^{\mathrm{IV}}} \times H_{\ell-n}^{(2)}(k_0 d_{ig}) e^{-j(\ell-n)\phi_{ig}}. \tag{77}$$

The total scattering and internal unknown expansion coefficients are then given by,

$$C_{in}^{tot} = \sum_{p=0}^{N} C_{in}^{p}, \quad D_{in}^{tot} = \sum_{p=0}^{N} D_{in}^{p}, \quad A_{in}^{tot} = \sum_{p=0}^{N} A_{in}^{p}, \quad B_{in}^{tot} = \sum_{p=0}^{N} B_{in}^{p}, \tag{78}$$

where N the total number of interaction. The cross and co polarized field components of the total scattered electric field and the total electric field inside the cylinder are given, respectively by

$$E_{zi}^{s}(\rho_i, \phi_i) = E_0 \sum_{i=1}^{M} \sum_{n=-\infty}^{\infty} C_{in}^{tot} H_n^{(2)}(k_0 \rho_i) e^{jn\phi_i} \tag{79}$$

$$E_{zi}^{c}(\rho_i, \phi_i) = E_0 \sum_{i=1}^{M} \sum_{n=-\infty}^{\infty} \left[A_{in}^{tot} J_n(k_+ \rho_i) + B_{in}^{tot} J_n(k_- \rho_i) \right] e^{jn\phi_i} \tag{80}$$

$$E_{\phi i}^{s}(\rho_i, \phi_i) = E_0 \sum_{i=1}^{M} \sum_{n=-\infty}^{\infty} D_{in}^{tot} H_n^{(2)'}(k_0 \rho_i) e^{jn\phi_i} \tag{81}$$

$$E_{\phi i}^{c}(\rho_i, \phi_i) = E_0 \sum_{i=1}^{M} \sum_{n=-\infty}^{\infty} \left[A_{in}^{tot} J_n'(k_+ \rho_i) - B_{in}^{tot} J_n'(k_- \rho_i) \right] e^{jn\phi_i}. \tag{82}$$

The well-known far field approximations can be applied to Equations (79) and (81) in order to compute the far field patterns. As for the solution of the TE_z case the only difference is to change the expression of where it should take the following form $v_i = \frac{\eta_0}{\eta_{ci}} = \frac{\sqrt{1+\chi^2}}{\sqrt{\mu_r/\varepsilon_r}}$. One should also note that the expressions of H_z and E_ϕ field components in the TE_z case correspond to the expressions of E_z and H_ϕ in the TM_z case, respectively, with E_0 replaced by H_0.

4. NUMERICAL RESULTS

Sample numerical data are presented to show the radiation patterns for a number of conducting, dielectric and chiral cylinders. The scattering cross section of five perfectly conducting cylinders due to a plane wave incident at $\phi = 180°$ is shown in Fig. 2 for TM_z polarized waves,which is similar to a previously published result [10]. Each cylinder radius is 0.1λ and their centers are separated by 0.5λ. For the same configuration the scattering cross section of the dielectric cylinders having relative permittivity equals to 2.2 can be shown in Fig. 3. While that of chiral cylinders having $\varepsilon_r = 2$, $\mu_r = 3$ and $\xi_c = 0.0005$, is seen in Fig. 4. The main objective of these cases, for simple collection of cylinders, is to show that the implemented code not only present the scattered field from chiral cylinders, but also from conducting and dielectric cylinders with different orientation.

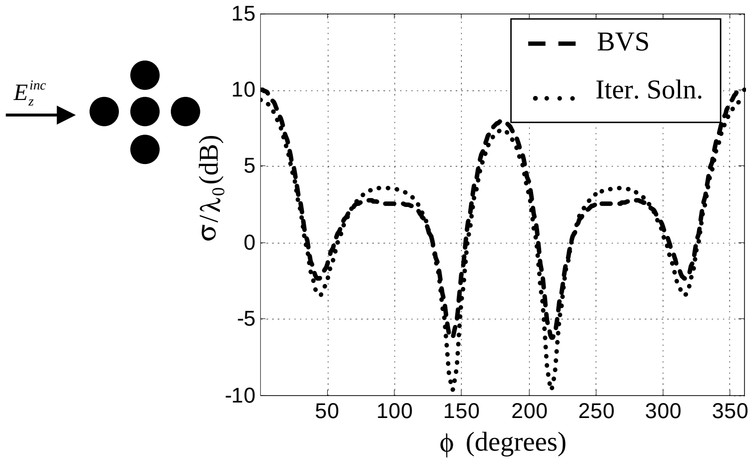


Figure 2. The bistatic scattering cross section of five perfectly conducting cylinders each of radius $= 0.1\lambda$, and their centers are separated by 0.5λ, due to a plane wave incident at $\phi_0 = 180°$ compared to Fig. 2(a) in [10].

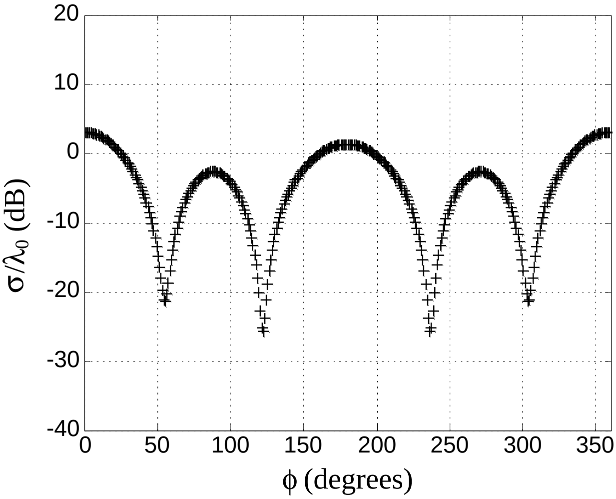


Figure 3. The bistatic scattering cross section of five dielectric cylinders each of radius $= 0.1\lambda$, and $\varepsilon_r = 2.2$ and their centers are separated by 0.5λ, due to a plane wave incident at $\phi_0 = 180°$.

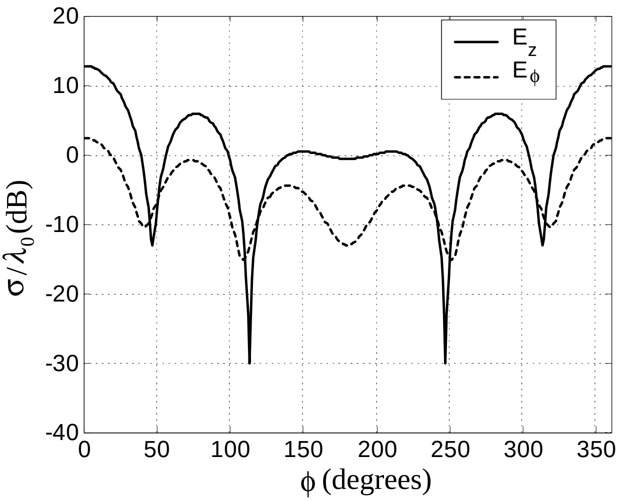


Figure 4. The bistatic scattering cross section of five chiral cylinders each of radius $= 0.1\lambda$, and $\varepsilon_r = 2$, $\mu_r = 3$ and $\xi_c = 0.0005$ and their centers are separated by 0.5λ, due to a plane wave incident at $\phi_0 = 180°$.

The following features characterizes this iterative technique: (1) No matrix inversion is required, (2) The matrices need to be computed once through the entire procedure, and their symmetric feature speeds up the computational time for its generation, (3) The main draw back of this technique is the extra time needed between the iterations to check for the accuracy of the applied boundary conditions in order to terminate the iteration process.

5. RCS REDUCTION

It is observed that the RCS of a target can be drastically affected by chirality but this effect is not predictable by a simple theory. After several investigations for the effect of chirality on an array of dielectric cylinders, we realized that by introducing certain values of chiral admittance; either the backward or forward scattered fields can be significantly reduced as will be shown in the following examples.

Figure 5 shows a comparison between the RCS (in dB) for an array of five dielectric cylinders with the same array of cylinders but after introducing the effect of the chirality, where the chiral admittance is

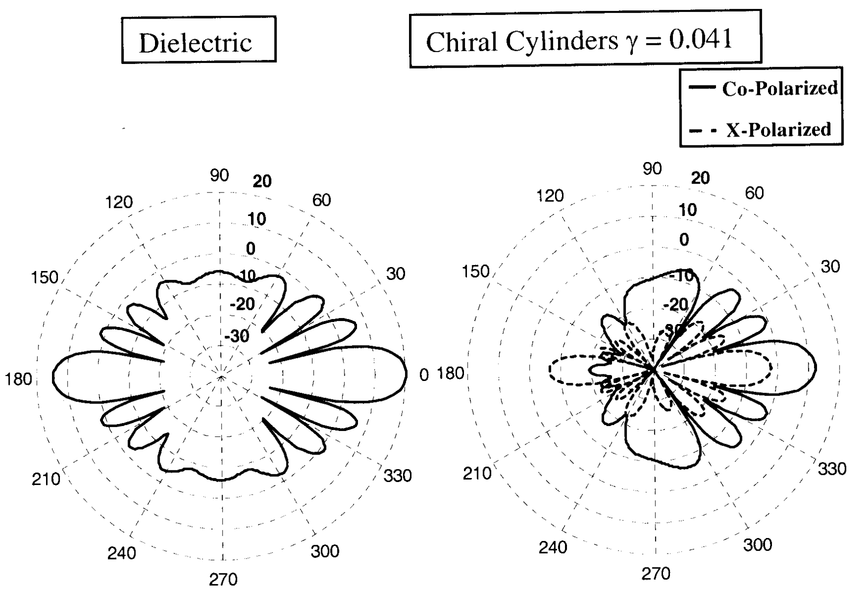
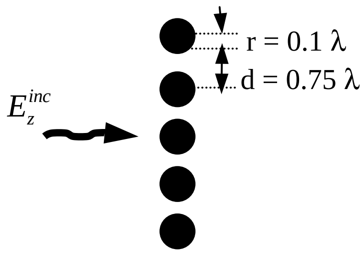


Figure 5. A comparison between the radiation pattern in dB for an array of dielectric cylinders having $\varepsilon_r = 5$ and an array of chiral cylinders of $\gamma = 0.041$.

assigned the value of 0.041. The cylinders is excited by a TM_z plane wave incident at 180λ, the radius of each cylinder is 0.1λ ($\lambda = 1$ m) and the cylinders are apart from each other by a distance 0.75λ from center to center. The computed data shows that after introducing the value of $\gamma = 0.041$ the backward scattered field was significantly reduced in comparison with that of the dielectric cylinders.



It is clear from Table 1 that the value of the backward co-polarized field has been significantly reduced from that of the dielectric cylinders to that of the chiral cylinders by 30 dB, this is due to $\gamma = 0.041$. For the same problem, another investigation was done concerning the value of the chiral admittance and its effect on the RCS. It was noticed that by assigning γ the value of 0.00745, a significant reduction in the forward scattered field has occurred, which can be shown in Fig. 6.

It is clear from Table 2 that the value of the forward co-polarized field has been significantly reduced from that of the dielectric cylinders to that of the chiral cylinders by 24 dB, this is due to $\gamma = 0.00745$.

Table 1. Forward and backward scattered fields for the array of dielectric and chiral cylinders in dB for $\gamma = 0.041$.

	Dielectric Co-Polarized Field	Chiral Co-Polarized Field	Chiral X-Polarized Field
Forward RCS	18.3 (dB)	11 (dB)	-1.35 (dB)
Backward RCS	17 (dB)	-20 (dB)	-8 (dB)

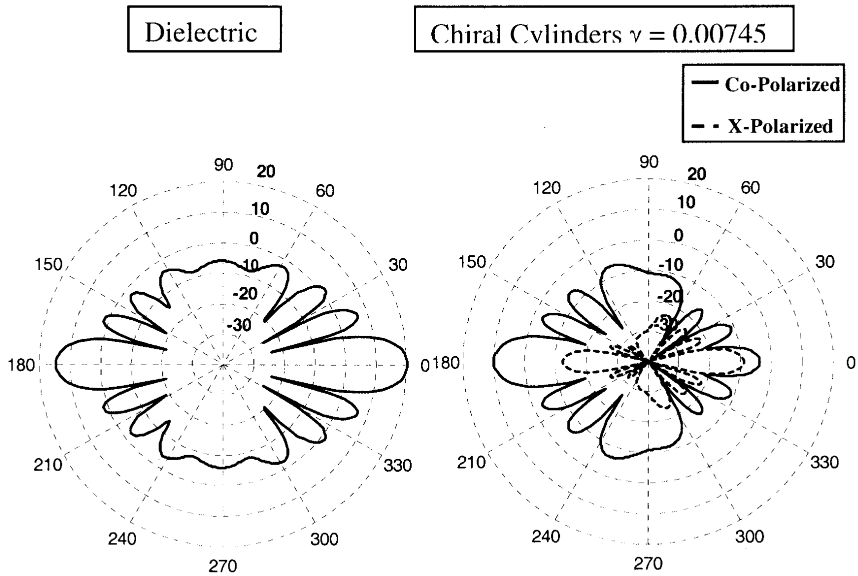


Figure 6. A comparison between the radiation pattern in dB for an array of dielectric cylinders having $\varepsilon_r = 5$ and an array of chiral cylinders of $\gamma = 0.00475$.

Table 2. Forward and backward scattered fields for the array of dielectric and chiral cylinders in dB for $\gamma = 0.00745$.

	Dielectric Co-Polarized Field	Chiral Co-Polarized Field	Chiral X-Polarized Field
Forward RCS	18.3 (dB)	-5 (dB)	-9.5 (dB)
Backward RCS	17 (dB)	10 (dB)	-12 (dB)

6. CONCLUSION

In this paper an iterative technique is presented to the problem of scattering from a collection of parallel chiral cylinders. The cylinders can be excited by either a TE_z or a TM_z plane wave. The main advantage of this technique is that no matrix inversion is required and the matrices are to be computed once through the entire procedure, and their symmetric feature speeds up the computational time for its generation. However, one should point out that the main draw back of this technique is the extra time needed between the iterations to check for the accuracy of the applied boundary conditions in order to terminate the iteration process. However, this process should not be conducted at every iteration and definitely not on the surface of all cylinders. Sample numerical results are presented to show the validity of the formulation as well as some applications showing the effect of assigning different values of the chiral admittance in reducing the backward and forward scattered fields.

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