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THE POSSIBLE EFFECTS OF THE MAGNETIC DECLINATION ON THE WAVE POLARISATION COEFFCIENTS AT THE CUTOFF POINT

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Abstract–The dispersion relation and the wave polarisation coefficients of the electromagnetic waves in the ionospheric plasma have been obtained by considering the magnetic declination. If the magnetic declination is taken into account, the polarisation coefficients have real and imaginary parts. It is pointed out that the peculiarity of the real parts of the wave polarisation coefficients become more obvious in the vicinity of the frequency $\omega(= \omega_{pe} + \omega_{pi})$ in the ionospheric plasma, while has no effect on the imaginary parts. This result is different from as in the absence of the magnetic declination.

1. INTRODUCTION

Numerous investigators [1–8] developed a theory for the propagation of electromagnetic waves in the ionospheric plasma. Traditionally, they made certain assumptions such as that the ambient magnetic field is vertical and the magnetic declination is zero, which is unrealistic in ionosphere. We assumed that the z-axis of the coordinate system with its origin located on the ground is vertical upwards. The x-axis

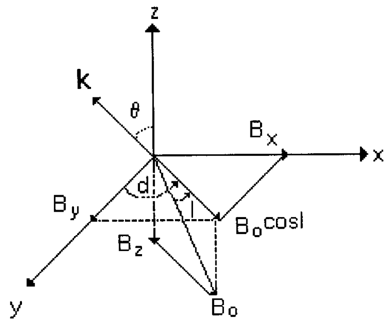

Figure 1. Geometry used for the calculation.

and y-axis are geographic eastward and northward in the northern hemisphere respectively. Hence, the ambient magnetic field in northern hemisphere is

$$\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z \tag{1}$$

where $B_x = B_0\ \cos I \sin d$, $B_y = B_0\ \cos I\ \cos d$ and $B_z = -B_0\ \sin I$. I and d are the magnetic dip and the magnetic declination angles respectively. $\mathbf{a}_x, \mathbf{a}_y$, and $\mathbf{a}_z$ are unit vectors. The yz plane is chosen so that it contains the direction of propagation and wave vector $\mathbf{k}$ makes angle θ with z-axis as shown in Fig. 1.

The present paper studies the effects of the magnetic declination on the refractive indices and the wave polarisation coefficients near the level where $\omega(= \omega_{pe} + \omega_{pi})$ in the ionospheric plasma.

2. CONDUCTIVITY TENSOR

The force acting on the particles in the plasma is given by

$$m_\alpha \frac{d\mathbf{V}_\alpha}{dt} = q_\alpha[\mathbf{E} + \mathbf{V}_\alpha \times \mathbf{B}] - m_\alpha \nu_\alpha \mathbf{V}_\alpha \tag{2}$$

where α denotes e, i and they stand for electron and ion respectively. It is assumed that the velocities and the field vary as $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$. The current density is obtained as

$$\mathbf{J}_\alpha = \sigma_{0\alpha}\mathbf{E} \pm \frac{\omega_{c\alpha}}{\nu_\alpha - i\omega}\mathbf{J}_\alpha \times \mathbf{a} \tag{3}$$

where $\sigma_{0\alpha}\left[= \frac{Ne^2}{m_\alpha(\nu_\alpha - i\omega)}\right]$ is the longitudinal conductivity, $\omega_{c\alpha} = \left|\frac{q_\alpha B}{m_\alpha}\right|$ is the gyrofrequency, $\mathbf{a} = (\cos I \sin d)\mathbf{a}_x + (\cos I \cos d)\mathbf{a}_y + (-\sin I)\mathbf{a}_z$ and e is the charge of electron.

Upper sign in front of $\omega_{c\alpha}$ is for electron and lower sign is for ion. The standard notation of magnetoionic theory is used. The different symbols used stand for

n : Refractive index
N : Electron and ion densities (they are assumed to be equal)
$J_e = -NeV_e$: Electron current density
$J_i = NeV_i$: Ion current density
$J = J_e + J_i$: Total current
ω_{pe} : Angular plasma frequency for electron
ω_{pi} : Angular plasma frequency for ion
ω : Angular frequency of wave
$\nu_e = \nu_{ei} + \nu_{en}$: Electron-ion and electron-neutral collision frequencies
$\nu_i = \nu_{ie} + \nu_{en}$: Ion-electron and ion-neutral collision frequencies

In a cartesian coordinates system, the solution of Eq. (3) can be written in terms of the components of the total current as

$$J_x = \sum_{\alpha=e,i} \sigma_{0\alpha} E_x \pm \sin I \sum_{\alpha=e,i} \frac{\omega_{c\alpha}}{\nu_\alpha - i\omega} J_{\alpha y} \pm \cos I \cos d \sum_{\alpha=e,i} \frac{\omega_{c\alpha}}{\nu_\alpha - i\omega} J_{\alpha z} \quad (4)$$

$$J_y = \sum_{\alpha=e,i} \sigma_{0\alpha} E_y \mp \sin I \sum_{\alpha=e,i} \frac{\omega_{c\alpha}}{\nu_\alpha - i\omega} J_{\alpha x} \mp \cos I \sin d \sum_{\alpha=e,i} \frac{\omega_{c\alpha}}{\nu_\alpha - i\omega} J_{\alpha z} \quad (5)$$

$$J_z = \sum_{\alpha=e,i} \sigma_{0\alpha} E_z \mp \cos I \cos d \sum_{\alpha=e,i} \frac{\omega_{c\alpha}}{\nu_\alpha - i\omega} J_{\alpha x} \pm \cos I \sin d \sum_{\alpha=e,i} \frac{\omega_{c\alpha}}{\nu_\alpha - i\omega} J_{\alpha y} \quad (6)$$

The upper sign are for electron. The total current can be compactly written in terms of the conductivity tensor σ as

$$\mathbf{J} = \sigma \cdot \mathbf{E} \quad (7)$$

with

$$\sigma = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix} \quad (8)$$

where

$$\sigma_{xx} = \sigma_1 + (\sigma_0 - \sigma_1)\cos^2 I \sin^2 d \quad (9)$$

$$\sigma_{yx} = -\sigma_2 \sin I + (\sigma_0 - \sigma_1)\cos^2 I \cos d \sin d \tag{10}$$

$$\sigma_{xz} = -\sigma_2 \cos I \cos d - (\sigma_0 - \sigma_1) \cos I \sin I \sin d \tag{11}$$

$$\sigma_{yx} = \sigma_2 \sin I + (\sigma_0 - \sigma_1)\cos^2 I \cos d \sin d \tag{12}$$

$$\sigma_{yy} = \sigma_1 + (\sigma_0 - \sigma_1)\cos^2 I \cos^2 d \tag{13}$$

$$\sigma_{yz} = \sigma_2 \cos I \sin d - (\sigma_0 - \sigma_1) \cos I \sin I \cos d \tag{14}$$

$$\sigma_{zx} = \sigma_2 \cos I \cos d - (\sigma_0 - \sigma_1) \cos I \sin I \sin d \tag{15}$$

$$\sigma_{zy} = -\sigma_2 \cos I \sin d - (\sigma_0 - \sigma_1) \cos I \sin I \cos d \tag{16}$$

$$\sigma_{zz} = \sigma_0 \sin^2 I + \sigma_1 \cos^2 I \tag{17}$$

in which longitudinal (σ_0), Pedersen (σ_1) and Hall (σ_2) conductivities are

$$\sigma_0 = Ne^2 \left[\frac{1}{m_e (\nu_e - i\omega)} + \frac{1}{m_i (\nu_i - i\omega)} \right] \tag{18}$$

$$\sigma_1 = Ne^2 \left[\frac{\nu_e - i\omega}{m_e \left[\omega_{ce}^2 + (\nu_e - i\omega)^2 \right]} + \frac{\nu_i - i\omega}{m_i \left[\omega_{ci}^2 + (\nu_i - i\omega)^2 \right]} \right] \tag{19}$$

$$\sigma_2 = Ne^2 \left[-\frac{\omega_{ce}}{m_e \left[\omega_{ce}^2 + (\nu_e - i\omega)^2 \right]} + \frac{\omega_{ci}}{m_i \left[\omega_{ci}^2 + (\nu_i - i\omega)^2 \right]} \right] \tag{20}$$

The conductivity tensor given in Eq. (8) is a realistic conductivity tensor for ionospheric plasma with no assumption.

3. DISPERSION RELATION

Maxwell equations can be written as

$$\nabla \times \mathbf{E} = i\omega \mathbf{B} \tag{21}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} - i\omega\mu_0\varepsilon_0 \mathbf{E} \tag{22}$$

where $\mathbf{J} = \sigma \cdot \mathbf{E}$. From these equations, the following equation can be obtained,

$$n^2 \mathbf{E} - \mathbf{n}(\mathbf{n} \cdot \mathbf{E}) - \left[I + \frac{i}{\varepsilon_0 \omega} \sigma \right] \cdot \mathbf{E} = 0 \tag{23}$$

in which $\mathbf{n} = \frac{c}{\omega}\mathbf{k}$ and σ is given in Eq. (8) and I is unit matrix. In the present work, it is assumed that the ionospheric plasma is collisionless.

By using the geometry in Fig. 1, Eq. (23) can be written as.

$$\begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \tag{24}$$

where

$$M_{xx} = n^2 - S - (P - S)\cos^2 I \sin^2 d \tag{25}$$
$$M_{xy} = -iD \sin I - (P - S)\cos^2 I \cos d \sin d \tag{26}$$
$$M_{xz} = -iD \cos I \cos d + (P - S)\cos I \sin I \sin d \tag{27}$$
$$M_{yx} = iD \sin I - (P - S)\cos^2 I \cos d \sin d \tag{28}$$
$$M_{yy} = n^2 \cos^2\theta - S - (P - S)\cos^2 I \cos^2 d \tag{29}$$
$$M_{yz} = -n^2 \cos\theta \sin\theta + iD \cos I \sin d + (P - S)\cos I \sin I \cos d \tag{30}$$
$$M_{zx} = iD \cos I \cos d + (P - S)\cos I \sin I \sin d \tag{31}$$
$$M_{zy} = -n^2 \cos\theta \sin\theta - iD \cos I \sin d + (P - S)\cos I \sin I \cos d \tag{32}$$
$$M_{zz} = n^2 \sin^2\theta - P \sin^2 I - S \cos^2 I \tag{33}$$

in which

$$P = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2} \tag{34}$$
$$R = 1 - \left[\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}\omega} + \frac{\omega_{pi}^2}{\omega^2 + \omega_{ci}\omega}\right] \tag{35}$$
$$L = 1 - \left[\frac{\omega_{pe}^2}{\omega^2 + \omega_{ce}\omega} + \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}\omega}\right] \tag{36}$$
$$S = \frac{1}{2}(R + L) \qquad \text{and} \qquad D = \frac{1}{2}(R - L) \tag{37}$$

The normal modes of the system are accordingly given by

$$\det(\mathbf{M}) = 0 \tag{38}$$

Eq. (24) is the basic dispersion relation. The refractive index n and the polarisation coefficients can be obtained in terms of plasma parameters. From this determinant one can obtain two n^2. Each root is associated with a distinctive wave mode, one of them is the slow wave and the

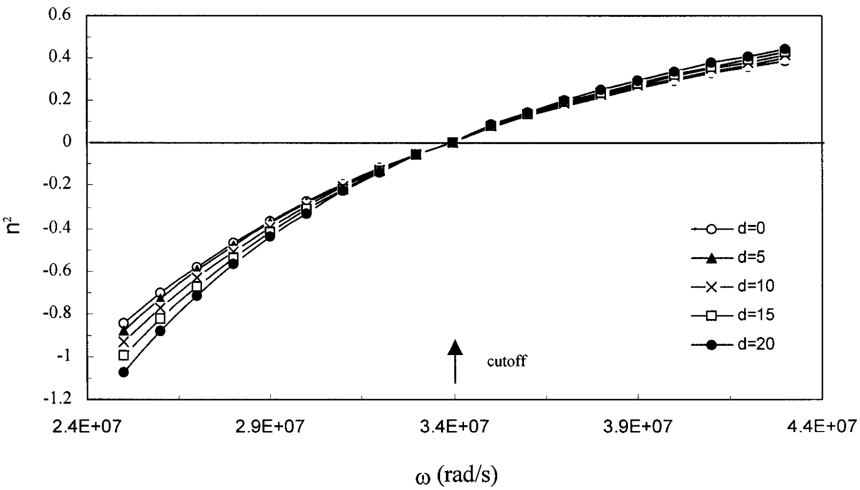

Figure 2. The variations of n^2 with frequency ω and declination angle d.

other is the fast wave. Fig. 2 shows the variation of the one of the n^2 with the wave frequency and the magnetic declination. It is noted that the magnetic declination has no effect on n, near the level where $\omega(=\omega_{pe}+\omega_{pi})$ (cutoff point) in the ionospheric plasma.

4. WAVE POLARISATION COEFFICIENTS

Each of the two values of n^2 may be substituted in to Eq. (24) to obtain the ratios of the cartesian components of the complex electric field vector $E_x : E_y : E_z$ namely,

$$\rho_1 = \frac{E_x}{E_y} = a + i\gamma \tag{39}$$

$$\rho_2 = \frac{E_x}{E_z} = b + i\beta \tag{40}$$

$$\rho_3 = \frac{E_y}{E_z} = c + if \tag{41}$$

where

$$a = \frac{F_1}{A_1} \qquad \text{and} \qquad \gamma = \frac{T_1}{A_1} \tag{42}$$

$$b = \frac{F_2}{A_2} \qquad \text{and} \qquad \beta = \frac{T_2}{A_2} \tag{43}$$

$$c = \frac{F_3}{A_3} \qquad \text{and} \qquad f = \frac{T_3}{A_3} \tag{44}$$

in which

$$\begin{aligned} A_1 =& PS-(PS-RL)\cos^2 I\cos^2 d-n^2\left[\left(S-n^2\right)\sin^2\theta+P\sin^2 I\right.\\ &\left.+S\cos^2 I\right]-n^2(P-S)\cos^2 I\sin^2 d\sin^2\theta \end{aligned} \tag{45}$$

$$\begin{aligned} F_1 =& \sin d\cos I\left[(RL-PS)\cos I\cos d+n^2(P-S)\left(\cos I\sin^2\theta\cos d\right.\right.\\ &\left.\left.-\sin I\sin\theta\cos\theta\right)\right] \end{aligned} \tag{46}$$

$$T_1 = D\left[-P\sin I+n^2\left(\sin I\sin^2\theta+\cos I\cos\theta\sin\theta\cos d\right)\right] \tag{47}$$

$$\begin{aligned} A_2 =& -RL\sin^2 I-PS\cos^2 I+n^2\left[S\sin^2 I+P\cos^2 I\right.\\ &\left.-\left(n^2-S\right)\cos^2\theta\right]-n^2(P-S)\cos^2 I\sin^2 d\sin^2\theta \end{aligned} \tag{48}$$

$$\begin{aligned} F_2 =& \sin d\cos I\left[(RL-PS)\sin I+n^2(P-S)\left(\sin I\cos^2\theta\right.\right.\\ &\left.\left.-\cos I\cos d\cos\theta\sin\theta\right)\right] \end{aligned} \tag{49}$$

$$T_2 = D\left[P\cos I\cos d-n^2\left(\cos I\cos^2\theta\cos d+\sin I\sin\theta\cos\theta\right)\right] \tag{50}$$

$$\begin{aligned} A_3 =& -RL\sin^2 I-PS\cos^2 I-n^2\left[\left(n^2-S\right)\cos^2\theta-S\right.\\ &\left.-(P-S)\cos^2 I\cos^2 d\right]+n^2(P-S)\cos^2 I\sin^2 d\cos^2\theta \end{aligned} \tag{51}$$

$$\begin{aligned} F_3 =& (RL-PS)\cos I\sin I\cos d+n^2\left[\left(S-n^2\right)\cos\theta\sin\theta\right.\\ &\left.+(P-S)\cos I\sin I\cos d\right]+n^2(P-S)\cos^2 I\sin^2 d\cos\theta\sin\theta \end{aligned} \tag{52}$$

$$T_3 = D\left(n^2-P\right)\cos I\sin d \tag{53}$$

5. NUMERICAL SOLUTIONS AND DISCUSSIONS

The calculations of the polarisation coefficients have been done for geographic coordinates of $(39°N,\ 40°E,\ I = 55.6)$ at h_mF2 height. The used plasma parameters have been obtained by using IRI for June at $1200LT$. The angle θ and the sunspot number R are taken as $30°$ and 10 respectively. The wave frequencies are selected around the cutoff frequency $\omega(=\omega_{pe}+\omega_{pi})$. The magnetic declination angles are taken as $0°$, $5°$, $10°$, $15°$ and $20°$. If the magnetic declination is taken into account, it is found from Eqs. (39)–(41), the polarisation coefficients ρ_1, ρ_2 and ρ_3 have real and imaginary parts. If the magnetic declination is not considered, Eqs. (46), (49) and (53) vanish, then the polarisation coefficients, ρ_1 and ρ_2 become pure imaginary and ρ_3 is real. These means that there are phase differences between $\mathbf{E}_x$ and $\mathbf{E}_y$, $\mathbf{E}_x$ and $\mathbf{E}_z$ while $\mathbf{E}_y$ and $\mathbf{E}_z$ are in phase.

The variations of a, b, c, γ, β, and f with frequency and the magnetic declination angle are given in Fig. 3–5. It is noted that the effect of the magnetic declination on ρ_1, ρ_2 and ρ_3 is evident. The peculiarity of the real parts of ρ_1, ρ_2 and ρ_3 become more obvious in the vicinity of the frequency $\omega(=\omega_{pe}+\omega_{pi})$. The values of a, b and c show a sharply pronounced maximum as seen in Figs. 3a, 4a and 5a. However, the imaginary parts of ρ_1, ρ_2 and ρ_3 are not affected by the declination angle around the frequency $\omega(=\omega_{pe}+\omega_{pi})$ (Figs. 3b, 4b and 5b).

As shown in Fig. 2, the cutoff of the wave occurs at the frequency 3.4×10^7 rad/sec. This frequency is equal to $\omega(\omega_{pe}+\omega_{pi})$. This means that P is zero at this frequency, then Eqs. (47), (50) and (53) vanish. The real and imaginary parts of the ρ_1, ρ_2 and ρ_3 at the cutoff frequency becomes as follows:

	If d is not considered	If d is considered
ρ_1	$a=0$	$a=\tan d$
	$\gamma=0$	$\gamma=0$
ρ_2	$b=0$	$b=-\cot I\sin d$
	$\beta=0$	$\beta=0$
ρ_3	$c=-\cot I$	$c=-\cot I\cos d$
	$f=0$	$f=0$

These results show that if one neglects the magnetic declination angle in the wave equations, there is no E_x field at the cutoff point. However, if the magnetic declination is taken into account, then E_x field exists and the magnitudes of E_x, E_y, E_z fields depend on the declination angle d (Fig. 6). The imaginary part of the ρ_3 is zero. So that E_y and E_z have the same phase and linear polarisation.

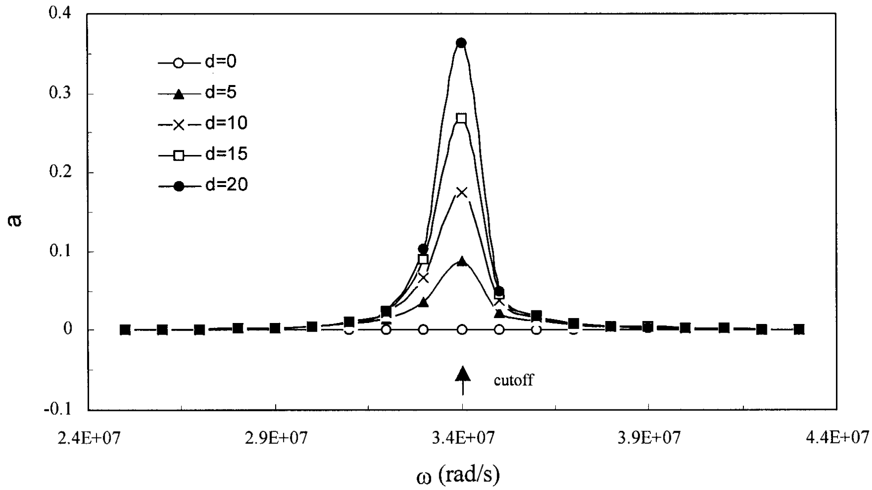

Figure 3a. The variations of a with frequency ω and declination angle d.

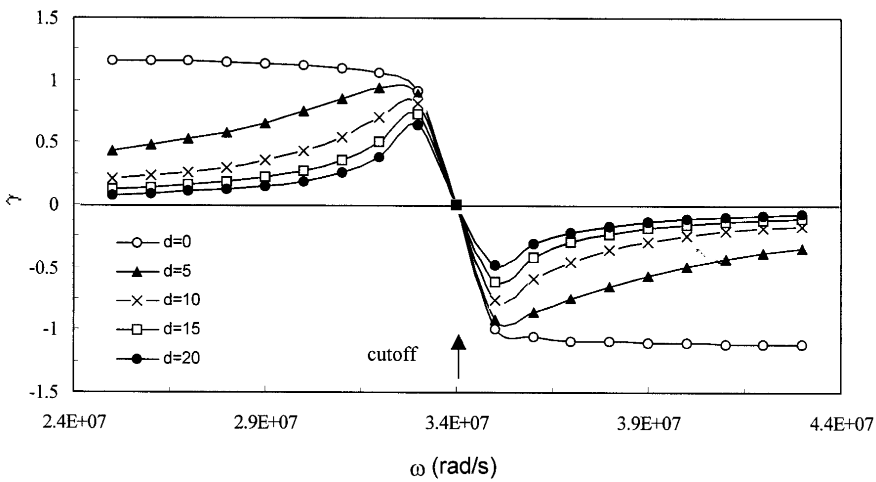

Figure 3b. The variations of γ with frequency ω and declination angle d.

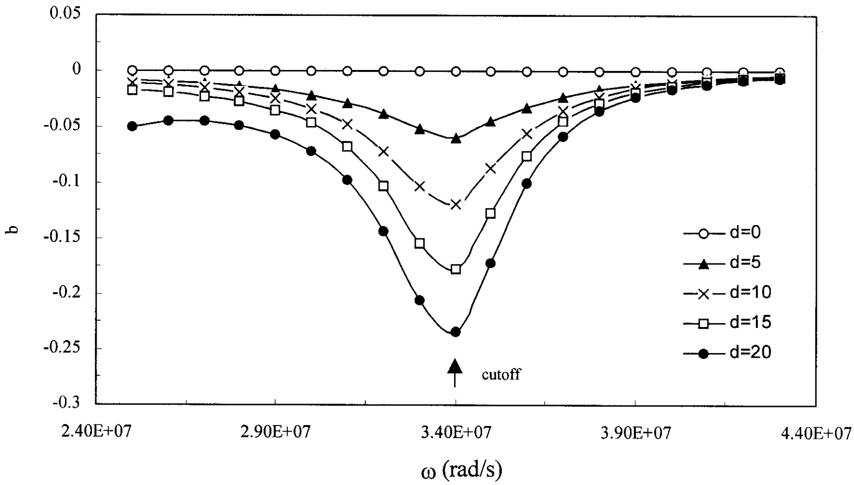

Figure 4a. The variations of b with frequency ω and declination angle d.

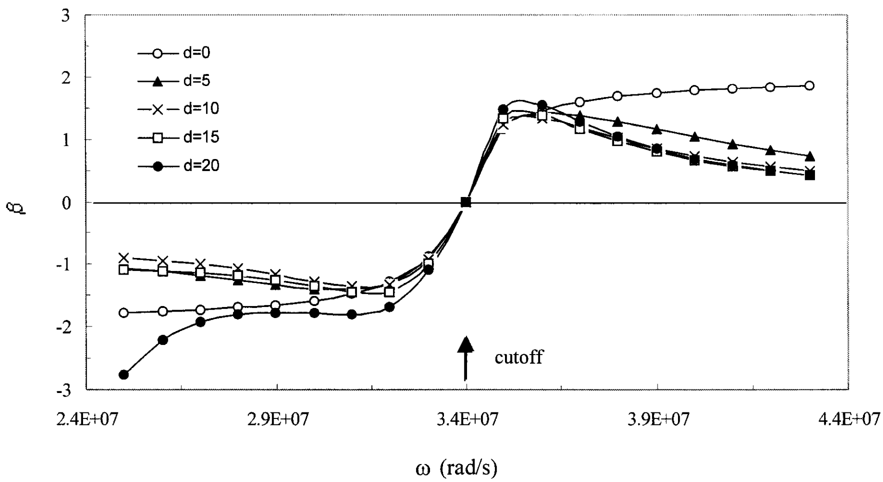

Figure 4b. The variations of β with frequency ω and declination angle d.

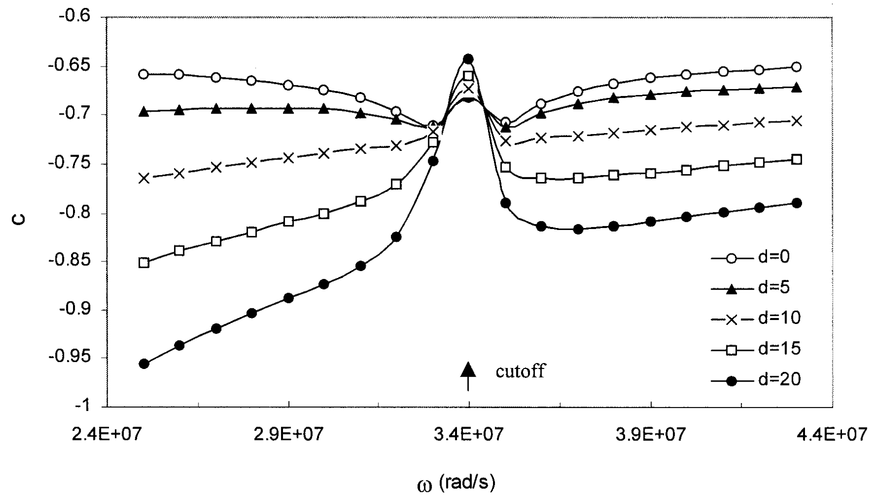

Figure 5a. The variations of c with frequency ω and declination angle d.

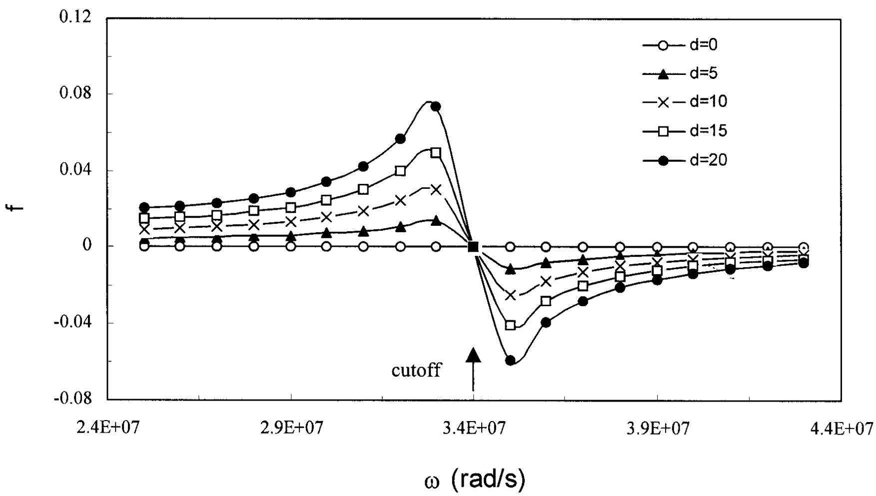

Figure 5b. The variations of f with frequency ω and declination angle d.

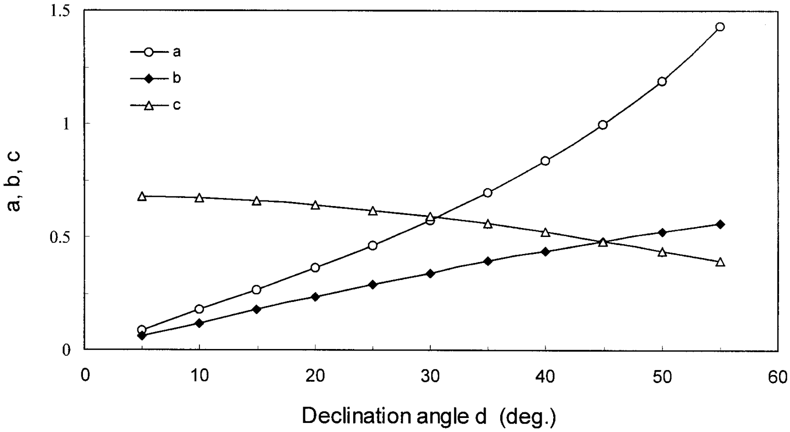

Figure 6. The variations of a, b, c with the declination angle d at cutoff.

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