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TRANSFER RELATIONS FOR ELECTROMAGNETIC WAVE SCATTERING FROM PERIODIC DIELECTRIC ONE–DIMENSIONAL INTERFACE: TE POLARIZATION

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1. INTRODUCTION

In Gazaryans's remarkable paper [1] it has been considered a problem of scalar wave propagation in one-dimensional random medium consisted of a stack of homogeneous parallel dielectric layers, which had random positions and did not intersect between them. The layers have been thought as being embedded into a homogeneous background dielectric medium. Gazaryan derived on base of the field superposition principle a mixed system of exact algebraic equations (refereed to further as transfer relations) for wave reflection and transmission coefficients of the medium and the wave amplitudes of waves in splits between layers. Gazayan showed also a possibility to exclude the amplitudes of waves in splits between layers from the transfer relations and to obtain on this way a separate recurrent system of algebraic equations, which describe the incremental change of the wave reflection and transmission coefficients of stack of $n-1$ layers upon attachment of a n-s layer to the stack and refereed to further as recurrent equations with a layer attachment. These recurrent equations manifest the general invariant imbedding principle in applied mathematics [2] discovered due to Ambarzumyan [3]. Applying the derived transfer relations, Gazaryan demonstrated analytically a phenomenon of Anderson localization [4] of classical waves in one-dimensional random medium, i.e., the exponential decay of ensemble averaged wave intensity transmitted through the medium with increasing medium slab thickness.

One should remark here that forth long before the Gazaryan paper, Gertsenshtein and Vasil'ev [5] had demonstrated analytically the Anderson localization of classical waves in onemode waveguide, using a technique of the Fokker-Planck equation (or generalized diffusion equation) in the theory of Brownian motion, described the evolution with increasing waveguide length of the probability distribution function of the wave reflection coefficient of a wave from the waveguide. The written by Gertsenshtein and Vasil'ev Fokker-Planck equation was generalized later by Dorokhov [6] and independently by Mello, Pereyra, and Kumar [7] on the case of disordered multimode wires in terms of the probability distribution function of the eigenvalues of the transmission-matrix product (Dorokhov-Mello-Kumar equation, see review [8] on random matrix theory). The Gertsenshtein and Vasil'ev idea to use the Fokker-Planck equation for probability distribution function of the wave reflection coefficient from a random medium slab was realized by Papanicolau [9] on base of the stochastic Riccati equation for the wave reflection coefficient from an one-dimensional random medium slab with dielectric permittivity fluctuations. Since this Papanicolau paper, the methods of the stochastic Riccati equation in theory of wave propagation in random continuous inhomogeneous media was being elaborated by Klyatskin [10, 11].

As one may see,the Fokker-Planck equation technique allows of to demonstrate the Anderson localization of classical waves in one-dimensional random media more easily compared with the method of Gazaryan's transfer relations. Nevertheless, it does not reduce significance of the Gazaryan transfer relations in general wave multiple scattering theory because of the following reasons.

First, the Fokker-Planck equation technique is restricted by the limit of either a small difference between the dielectric permittivity of layers of a stack and the dielectric permittivity of the background or weak dielectric permittivity fluctuations of an one-dimensional continuous inhomogeneous random medium. Second, a derivation of the Fokker-Planck equation makes use an averaging over fast oscillations of the wave reflection coefficient from a random medium slab with increasing of the slab thickness, an justification for this averaging being a difficult problem (see discussion in [11]). Third, in the case of not small difference between the dielectric permittivity of layers of a stack and the background the Fokker-Planck equation should replaced by more complicative for analytic solution the Kolmogorov-Feller equation (see, e.g., [12]). Fourth, in the case of three-dimensional random medium the wave reflection coefficient from a random medium slab becomes of an integral operator (see, e.g., [11]) and the Fokker-Planck equation gets a functional form [13], that seems much more complicative for analytic solution compared with one-dimensional case.

Perhaps because of these difficulties concerning with the Fokker-Planck equation some authors have applied the numerical methods to solution of the stochastic Riccati equation for the wave reflection coefficient from and an corresponding equation for the wave transmission coefficient through, e.g., an one-dimensional random medium slab [11] to justify the above averaging over fast oscillations of the wave reflection coefficient, and a surface disordered waveguides [14] to study a new regime for coexistence of ballistic, diffuse and localized modes in the transmission of waves.

It would be advantageous to note here the applications of the Riccati equation (or more generally, the imbedding method) also to another,apart from the mentioned, problems such as wave propagation in periodic continuous inhomogeneous one-dimensional medium [11]. Especially interesting for us is to call attention to the imbedding equations [11] for the well known physical problem of wave scattering from rough, and in particular periodic dielectric interface, the imbedding parameter having been chosen for these equations in a direction along the unperturbed interface. Ordinarily this physical problem is studied either by a straightforward approach process using exact integral equations for the boundary values of the field and then obtaining approximative solutions of these equations or by applying to the given boundary problem perturbative analysis. Some of these methods are presented in monographs [15–17] and papers [18–27]. It is worth noting for us paper [28] where the Wood anomalies [29], discovered by light diffraction from a grating, are studied by light diffraction on a rough surface. The problem of wave scattering from periodic dielectric interface are considered using the Waterman [30] extended boundary condition approach in papers [31–35], for the case of one-dimensional interface, and in [36] for the case of two-dimensional interface.

In the last decade a transfer matrix approach has found a wide application in wave multiple scattering theory. This approach as turns out to be closely allied to Gazaryan's transfer relations and is not restricted, in particular, by the case of small difference between the dielectric permittivity of layers of a stack and the background. A convenient property of the transfer matrix is the multiplicative composition rule: the transfer matrix of a stack of layers is the product of the single layer transfer matrices. A basis of the transfer matrix approach was formulated by Barnes and Pendry [37]. They derived with the aid of the field superposition principle the recurrent equations with a layer attachment and showed a possibility to resolve this recurrent equations in terms of the fundamental transfer matrix. In papers [38] and [39] of Pendry with co-authors, the transfer matrix approach was applied to prove a maximal fluctuation theorem for wave propagation in random media, and to consider the effect of coherent backscattering enhancement, respectively.

There is a growing interest in recent years in the studies of the propagation of electromagnetic waves in periodic, both in two and three dimensions, dielectric structures, photonic band structures, based physically on interesting analogy with electron wave propagation in crystals [40, 41]. The existence in such structures of a frequency gap (see, e.g., [42]) where the propagation of electromagnetic waves is forbidden for all wave vectors, can have profound impact on several scientific and technical disciplines (see, e.g., [43, 44]), in particular for understanding of wave localization in dense random media [41, 45]. In paper [46] the transfer matrix technique was used to calculate the transmission coefficient versus the frequency of the incident wave for different polarizations in two-dimensional periodic and/or random arrangement of dielectric cylinders. In paper [47] the photonic band gap effect in a three-dimensional solid state lattice made of closely packed silica spheres was reported together with results simulated by numerical calculations within the framework of a quasicrystalline approximation.

The transfer matrix approach is appeared also from consideration of waves in splits between layers of an inhomogeneous medium as shown by Pendry and Roberts [39] and Pendry [48]. Such system of mutual interaction equation for wave amplitudes in splits between layers was formulated independently by Ngo and Rino [49] using the Foldy-Lax-Twersky formalism with applications [50, 51] to some two-scatterer problems. Earlier a system of mutual interaction equations was obtained, in fact, by Kouznetsov and Budanov [52] with further application by Kamzolov and Kouznetsov [53] to the problem of wave multiple scattering at a dielectric interface.

The aim of our paper is to show, after [54], that the Watson composition rule [55] of the scattering (T-matrix) operator leads to generalization of the Gazaryan transfer relations on the wave propagation case in a three-dimensional inhomogeneous medium as well as to a generalized Riccati equation for the operator wave reflection coefficient and an corresponding equation for the operator wave transmission coefficient of the medium. We demonstrate also in the paper an application of the derived Riccati equation to the problem of electromagnetic wave scattering from a periodic dielectric one-dimensional interface. The choosing the imbedding parameter in direction perpendicular to the unperturbed interface according to Kamzolov and Kouznetsov [53], but not along of this one as in [11], allows of us to treat the problem of interface wave scattering similar to the problem of volume scattering.

The plan of the paper is as follows. In Sec. 2 the Watson composition rule of the scattering operators is adopted to the problem of scalar wave propagation in an inhomogeneous three-dimensional dielectric layered medium. The basis definitions for the operator wave reflection and transmission coefficients of the layered medium as well as the operator wave amplitudes of waves in splits between layers are given in terms of scattering operators. The optical theorem and reciprocity for the operator wave reflection and transmission coefficients are proved in Sec. 3 using the optical theorem and reciprocity for the scattering operator. Sec. 4 includes derivation of the most general statements considered in the paper, that is the transfer relations and the resulting from these ones recurrent equations with a layer attachment; short derivation of the fundamental transfer matrix operator and the mutual interaction equations are given too. The generalized Riccati equation for the operator wave reflection coefficient and an corresponding equation for the operator wave transmission coefficient of the medium are derived in Sec. 5 from the recurrent equations with a layer attachment. In Sec. 6 the generalized Riccati equation for the wave reflection coefficient and a corresponding equation for the wave transmission coefficient of the medium are written in detail for the problem of electromagnetic wave scattering from a periodic dielectric one-dimensional interface in the case of TE polarization, using the Bloch (Floquet) theorem. Results of numerical solution to the Riccati equation for wave reflection from a periodic one-dimensional interface of two dielectric half-spaces are presented in Sec. 7. Sec. 8 gives our conclusions and discussions.

2. BASIC DEFINITIONS FOR WAVE PROPAGATION IN A LAYERED MEDIUM IN TERMS OF SCATTERING OPERATOR

We start with the scalar Helmholtz wave equation for a monochromatic wave field $\Psi(\overline{r})$ in a three-dimensional inhomogeneous dielectric isotropic medium

$$\Delta\Psi(\overline{r}) + \frac{\omega^2}{C^2}\varepsilon(\overline{r})\Psi(\overline{r}) = 0 \tag{1}$$

Here the dielectric permittivity $\varepsilon(\overline{r}) = \varepsilon^{bac} + \delta\varepsilon(\overline{r})$ includes a background constant ε^{bac} and the inhomogeneous part $\delta\varepsilon(\overline{r})$. We suppose both the wave field $\Psi(\overline{r})$ and its normal derivative on any surface to be continuous on this one. Such boundary conditions on any surface together with the radiation conditions in infinite allow one to present a solution to the wave equation (1), using the operator denotations in the three-dimensional domain of the position vector $\overline{r}$, in the form (see, e.g., [56])

$$\Psi = \Psi_\circ + G_\circ T \Psi_\circ \tag{2}$$

where $\Psi_\circ(\overline{r})$ is the incident field, $G_\circ(\overline{r}) = \exp(ik_\circ r)/(-4\pi r)$ is the Green function in a background medium with the wave number $k_\circ = (\omega/C)\sqrt{\varepsilon^{bac}}$ and $T(\overline{r}, \overline{r}')$ is the scattering (T-matrix) operator of the scattering medium. The scattering operator obeys the Lippman-Schwinger equation

$$T = V + V G_\circ T \tag{3}$$

with the effective scattering potential $V(\overline{r})$ of the medium, defined by

$$V(\overline{r}) = -k_\circ^2 \frac{\delta\varepsilon(\overline{r})}{\varepsilon^{bac}} \tag{4}$$

For the case of a two-dimensional scattering medium with the scattering potential $V(\overline{r}) = V(x, z)$ in the cortisone coordinate system x, y, z, where the x, z plane coincides with the incident plane and the electric vector of the incident wave is parallel to the y axis (TE polarization), the scalar Helmholtz equation (1) describes the electromagnetic wave field propagation in the medium.

2.1 Operator Wave Reflection and Transmission Coefficients

Let a wave field $\Psi_\circ(\overline{r})$ be incident upon the left boundary $z = 0$ of a scattering medium occupied the region of the layer $0 < z < L$. We apply to equation (2) the two-dimensional Fourier transform with respect to transverse to the z axis component $\overline{r}_\perp$ of the position vector $\overline{r}$ in the form

$$\Psi(\overline{r}) = \int_{\overline{k}_\perp} \exp(i\overline{k}_\perp \overline{r}_\perp)\Psi(\overline{k}_\perp, z) \tag{5}$$

In the right hand side (r.h.s.) of this equation and henceforth the notation $\int_{\overline{k}_\perp} = (2\pi)^{-2}\int d\overline{k}_\perp$ is used, with $\overline{k}_\perp$ being the transverse to the z axis component of a wave vector $\overline{k}$. Writing similar to (5) the incident field with the transverse, relatively to the z axis, Fourier transform $\Psi_\circ(\overline{k}_\perp, z) = \Psi_\circ(\overline{k}_\perp)\exp(i\sigma_k z)$ where $\sigma_k = \sqrt{k_\circ^2 - k_\perp^2}$ as $k_\perp < k_\circ$ and $\sigma_k = i\sqrt{k_\perp^2 - k_o^2}$ as $k_\perp > k_\circ$, one can find for the transmitted, $z > L$, and reflected, $z < 0$, wave fields the following representations

$$\Psi(\overline{k}_\perp, z) = \exp(i\sigma_k z)\int_{\overline{k}'_\perp} A(\overline{k}_\perp, \overline{k}'_\perp)\Psi_\circ(\overline{k}'_\perp) \tag{6}$$

as $z > L$ and

$$\Psi(\overline{k}_\perp, z) = \Psi_\circ(\overline{k}_\perp, z) + \exp(-i\sigma_k z)\int_{\overline{k}'_\perp} B(\overline{k}_\perp, \overline{k}'_\perp)\Psi_\circ(\overline{k}'_\perp) \tag{7}$$

as $z < 0$. The operator wave transmission $A(\overline{k}_\perp, \overline{k}'_\perp)$ and reflection $B(\overline{k}_\perp, \overline{k}'_\perp)$ coefficients of the layer in the r.h.s. of equations (6) and (7) are given by

$$A(\overline{k}_\perp, \overline{k}'_\perp) = \delta_{\overline{k}_\perp, \overline{k}'_\perp} + \frac{1}{2i\sigma_k} a(\hat{k}^+, \hat{k}^{+'}) \tag{8}$$

and

$$B(\overline{k}_\perp, \overline{k}'_\perp) = \frac{1}{2i\sigma_k} a(\hat{k}^-, \hat{k}^{+'}) \tag{9}$$

Here and henceforth $\delta_{\overline{k}_\perp, \overline{k}'_\perp}$ denotes $(2\pi)^2\delta(\overline{k}_\perp - \overline{k}'_\perp)$, $\hat{k}$ is the unit vector of a vector $\overline{k}, \hat{k}^2 = 1$; the vectors $\overline{k}^\pm$ are defined by $\overline{k}^\pm = \overline{k}_\perp \pm \sigma_k \hat{z}$, respectively, with $\hat{z}$ being the unit vector along the z axis; $a(\hat{k}, \hat{k}')$ is the scattering amplitude of the layer defined as

$$a(\hat{k}, \hat{k}') = T(\overline{k}, \overline{k}') \tag{10}$$

with $T(\overline{k}, \overline{k}')$ being the three-dimensional Fourier transform of the scattering operator $T(\overline{r}, \overline{r}')$ and $k^2 = k'^2 = k_\circ^2$.

Let now a wave field $\tilde{\Psi}_\circ(\overline{r})$ be incident upon the right boundary $z = L$ of the layer with the transverse, relatively to the z axis, Fourier transform $\tilde{\Psi}_\circ(\overline{k}_\perp, z) = \tilde{\Psi}_\circ(\overline{k}_\perp)\exp(-i\sigma_k z)$. In this case one can find for the transmitted, $z < 0$, and reflected, $z > L$, wave fields similar to (6) and (7) the representations

$$\tilde{\Psi}(\overline{k}_\perp, z) = \exp(-i\sigma_k z)\int_{\overline{k}'_\perp} \tilde{A}(\overline{k}_\perp, \overline{k}'_\perp)\tilde{\Psi}_\circ(\overline{k}'_\perp) \tag{11}$$

as $z < 0$ and

$$\tilde{\Psi}(\overline{k}_\perp, z) = \tilde{\Psi}_\circ(\overline{k}_\perp, z) + \exp(i\sigma_k z)\int_{\overline{k}'_\perp} \tilde{B}(\overline{k}_\perp, \overline{k}'_\perp)\tilde{\Psi}_\circ(\overline{k}'_\perp) \tag{12}$$

as $z > L$. Here the operator wave transmission $\tilde{A}(\overline{k}_\perp, \overline{k}'_\perp)$ and reflection $\tilde{B}(\overline{k}_\perp, \overline{k}'_\perp)$ coefficients of the layer are given by

$$\tilde{A}(\overline{k}_\perp, \overline{k}'_\perp) = \delta_{\overline{k}_\perp, \overline{k}'_\perp} + \frac{1}{2i\sigma_k} a(\hat{k}^-, \hat{k}^{-\prime}) \tag{13}$$

and

$$\tilde{B}(\overline{k}_\perp, \overline{k}'_\perp) = \frac{1}{2i\sigma_k} a(\hat{k}^+, \hat{k}^{-\prime}) \tag{14}$$

2.2 Watson Composition Rule of Scattering Operators for Stack of n Layers

Let a scattering medium be a stack of n layers with the scattering potential $V_m(\overline{r})$ of the m's layer, $m = 1, 2, ..., n$ (see Fig. 1).

The scattering potential $V_{1,n}(\overline{r})$ of the medium, occupied again the region $0 < z < L$, is defined by the sum

$$V_{1,n}(\overline{r}) = \sum_{m=1}^{n} V_m(\overline{r}) \tag{15}$$

under condition

$$V_m(\overline{r})V_{m'}(\overline{r}) = 0 \tag{16}$$

as $m \neq m'$ because of the layers are supposed to be not intersected between them.

The scattering operator $T_{1,n}$ of the medium, satisfying the Lippman-Schwinger equation (3) with potential V replaced by (15),

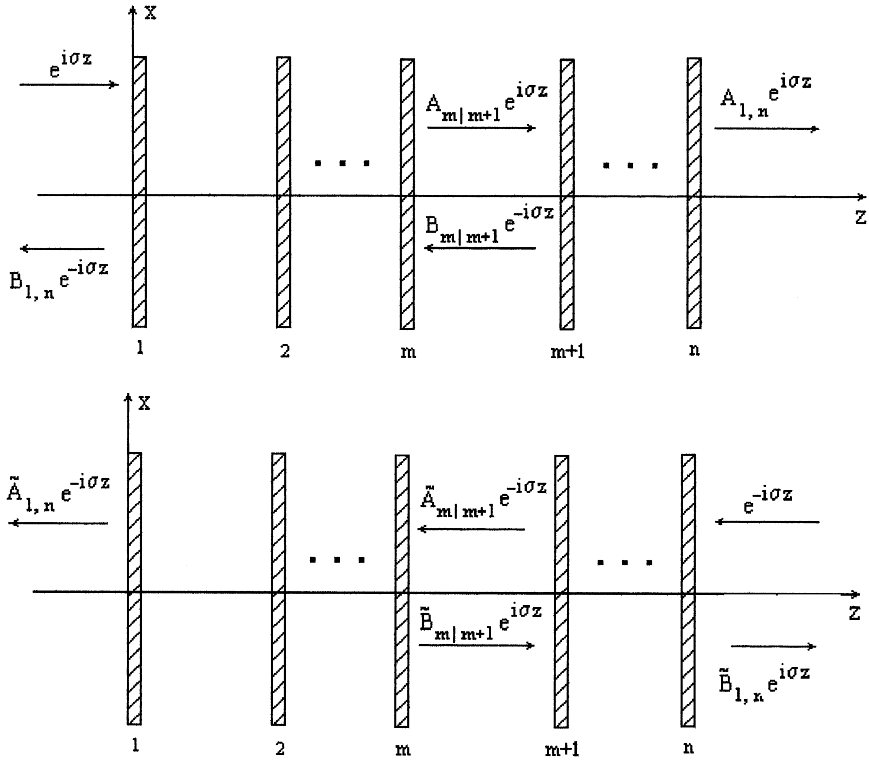

Figure 1. A three-dimensional medium presented as a stack of n layers with splits between them. In the top and bottom pictures the wave be incident upon the 1 st and the n's layer of the stack, respectively.

can be written according to the Watson composition rule [55] as the sum

$$T_{1,n} = \sum_{m=1}^{n} T_{1,n}^{m} \tag{17}$$

where the new set of operators $T_{1,n}^{m}$, $m = 1, 2, ..., n$, are found from the system of n operator equations in the three-dimensional domain of the position vector

$$T_{1,n}^{m} = T_{m,m} + T_{m,m} G_{\circ} \sum_{\substack{m'=1 \\ m \neq m'}}^{n} T_{1,n}^{m'} \tag{18}$$

with $T_{m.m}$ being the single scattering operator of the m's layer, i.e., satisfying the Lippman-Schwinger equation (3) with V replaced by V_m. The quantity $T^m_{1,n}$ may be thought as a scattering operator of the m's layer surrounded by the other $n-1$ layers.

Let a wave field $\Psi_\circ(\overline{r})$ be incident upon the left boundary $z=0$ of the first layer of the stack (see Fig. 1, the top picture). For the wave field $\Psi_{1,n}(\overline{r})$ in splits between layers, defined by equation (2) with T replaced by $T_{1,n}$, one can find, using the two-dimensional Fourier transform (5) and the condition (16) of non-intersection of the layers the following representation

$$\Psi_{1,n}(\overline{k}_\perp, z) = \exp(i\sigma_k z)\int_{\overline{k}'_\perp} A_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp)\Psi_\circ(\overline{k}'_\perp) + \exp(-i\sigma_k z)\int_{\overline{k}'_\perp} B_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp)\Psi_\circ(\overline{k}'_\perp) \tag{19}$$

with z being in the split between the m's and $(m+1)$'s layers, $m = 1, 2, ..., n-1$. The operator wave amplitudes $A_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp)$ and $B_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp)$ of waves in the split between the layers in the r.h.s. of equation (19) are given by

$$A_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp) = \delta_{\overline{k}_\perp, \overline{k}'_\perp} + \frac{1}{2i\sigma_k}\sum_{m'=1}^{m} a^{m'}_{1,n}(\hat{k}^+, \hat{k}^{+'}) \tag{20}$$

and

$$B_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp) = \frac{1}{2i\sigma_k}\sum_{m'=m+1}^{n} a^{m'}_{1,n}(\hat{k}^-, \hat{k}^{+'}) \tag{21}$$

In the r.h.s. of (20) and (21) $a^m_{1,n}(\hat{k}, \hat{k}')$ denotes a scattering amplitude of the m's layer surrounded by the other $n-1$ layers, being defined by equation (10) with $T(\overline{k}, \overline{k}')$ replaced by the three-dimensional Fourier transform $T^m_{1,n}(\overline{k}, \overline{k}')$ of a scattering operator $T^m_{1,n}(\overline{r}, \overline{r}')$ under condition $k^2 = k'^2 = k_\circ^2$.

Let now a wave field $\tilde{\Psi}_\circ(\overline{r})$ be incident upon the right boundary $z = L$ of the n's layer (see Fig. 1, the bottom picture). In this case one can find for the wave field $\tilde{\Psi}_{1,n}(\overline{r})$ in the splits between layers similar to (19) the representation

$$\tilde{\Psi}_{1,n}(\overline{k}_\perp, z) = \exp(-i\sigma_k z)\int_{\overline{k}'_\perp} \tilde{A}_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp)\tilde{\Psi}_\circ(\overline{k}'_\perp) + \exp(i\sigma_k z)\int_{\overline{k}'_\perp} \tilde{B}_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp)\tilde{\Psi}_\circ(\overline{k}'_\perp) \tag{22}$$

with z being in the split between the m's and $(m+1)$'s layers, $m = 1, 2, ..., n-1$. The operator wave amplitudes $\tilde{A}_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp)$ and
$\tilde{B}_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp)$ of waves in the split between the layers in the r.h.s. of equation (22) are given by

$$\tilde{A}_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp) = \delta_{\overline{k}_\perp, \overline{k}'_\perp} + \frac{1}{2i\sigma_k} \sum_{m'=m+1}^{n} a_{1,n}^{m'}(\hat{k}^-, \hat{k}^{-\prime}) \tag{23}$$

and

$$\tilde{B}_{m|m+1}(\overline{k}_\perp, \overline{k}'_\perp) = \frac{1}{2i\sigma_k} \sum_{m'=1}^{m} a_{1,n}^{m'}(\hat{k}^+, \hat{k}^{-\prime}) \tag{24}$$

3. OPTICAL THEOREM AND RECIROCITY FOR WAVE REFLECTION AND TRANSMISSION COEFFICIENTS

The optical theorem for the scattering operator T of a non-absorptive scattering medium has got the form (see, e.g., [56]) $T - T^* = T^*(G_\circ - G_\circ^*)T$ where the asterisk denotes the Hermitian adjoin of a operator. Applying to this optical theorem the two-dimensional Fourier transform like (5) leads to the following relation for the operator wave transmission (13) and reflection (14) coefficients

$$\int_{k_\perp < k_\circ} \sigma_k [\tilde{A}^*(\overline{k}_\perp, \overline{k}'_\perp)\tilde{A}(\overline{k}_\perp, \overline{k}''_\perp) + \tilde{B}^*(\overline{k}_\perp, \overline{k}'_\perp)\tilde{B}(\overline{k}_\perp, \overline{k}''_\perp)] = \sigma_{k'} \delta_{\overline{k}'_\perp, \overline{k}''_\perp} \tag{25}$$

where the asterisk denotes the complex conjugate of a quantity. It is worth noting that integration in the left hand side (l.h.s.) of (25) is performed in $\overline{k}_\perp$-space over the propagating waves, $k_\perp < k_\circ$, only (but not over both the propagating and evanescent, $k_\perp > k_\circ$ waves). Actually, one may see the relation (25) in [48] as a consequence of the current conversation.

The reciprocity for the scattering operator in the form $T(\overline{k}, \overline{k}') = T(-\overline{k}', -\overline{k})$ gives the reciprocity for the operator wave transmission (8), (13) and reflection (9), (14) coefficients written as (compare with reciprocity in [49])

$$\sigma_k \tilde{A}(\overline{k}_\perp, \overline{k}'_\perp) = \sigma_{k'} A(-\overline{k}'_\perp, -\overline{k}_\perp) \tag{26}$$

and

$$\sigma_k B(\overline{k}_\perp, \overline{k}'_\perp) = \sigma_{k'} B(-\overline{k}'_\perp, -\overline{k}_\perp) \tag{27}$$

$$\sigma_k \tilde{B}(\overline{k}_\perp, \overline{k}'_\perp) = \sigma_{k'} \tilde{B}(-\overline{k}'_\perp, -\overline{k}_\perp) \tag{28}$$

The reciprocity (26) relates the operator wave transmission coefficient $\tilde{A}$ to A, whereas the reciprocity (27) and (28) relates the operator wave reflection coefficient B to B and $\tilde{B}$ to $\tilde{B}$, respectively.

4. TRANSFER RELATIONS

Let us return to definition of the operator wave reflection and transmission coefficients of a scattering medium as a stack of n layers, given in terms of the scattering operator $T_{i,n}$ of the medium as well as to definition of the operator amplitudes of waves in splits between layers of the stack, given in terms of the scattering operators $T_{1,n}^m$. The following mixed system of exact operator equations-transfer relations is a direct consequence of the Watson composition rule (15–18) and above definitions

$$B_{1,n} = B_{1,m} + \tilde{A}_{1,m} B_{m|m+1} \tag{29}$$

$$B_{m|m+1} = B_{m+1,n} A_{m|m+1} \tag{30}$$

$$A_{m|m+1} = A_{1,m} + \tilde{B}_{1,m} B_{m|m+1} \tag{31}$$

$$A_{1,n} = A_{m+1,n} A_{m|m+1} \tag{32}$$

and

$$\tilde{B}_{1,n} = \tilde{B}_{m+1,n} + A_{m+1,n} \tilde{B}_{m|m+1} \tag{33}$$

$$\tilde{B}_{m|m+1} = \tilde{B}_{1,m} \tilde{A}_{m|m+1} \tag{34}$$

$$\tilde{A}_{m|m+1} = \tilde{A}_{m+1,n} + B_{m+1,n} \tilde{B}_{m|m+1} \tag{35}$$

$$\tilde{A}_{1,n} = \tilde{A}_{1,m} \tilde{A}_{m|m+1} \tag{36}$$

where $m = 1, 2, ..., n-1$. All products in these transfer relations have got the operator meaning in the two-dimensional $\overline{k}_\perp$-space, e.g., equation (29) is written in details as

$$B_{1,n}(\overline{k}_\perp, \overline{k}'_\perp) = B_{1,m}(\overline{k}_\perp, \overline{k}'_\perp) + \int_{\overline{p}_\perp} \tilde{A}_{1,m}(\overline{k}_\perp, \overline{p}_\perp) B_{m|m+1}(\overline{p}_\perp, \overline{k}'_\perp) \tag{29a}$$

The transfer relations (29–36) allow of a physically transparent interpretation in terms of the field superposition principle.

First the transfer relations (29–36) were obtained by Gazaryan [1] in the case of one-dimensional scattering medium using the field superposition principle. For the case of three-dimensional scattering medium the transfer relations (29–36) were derived in [54] on base of the Watson composition rule (15–18). All transfer relations are derived by the same way which becomes clear from derivation of equation (29) (see Appendix).

4.1 Recurrent Equations with A Layer Attachment

The operator amplitudes of waves in splits between layers of the stack layers may be excluded from the transfer relations (29–36) that leads to the following separate system of recurrent equations with a layer attachment

$$A_{1,n} = A_{n,n}\left(I - \tilde{B}_{1,n-1}B_{n,n}\right)^{-1} A_{1,n-1} \tag{37}$$

$$B_{1,n} = B_{1,n-1} + \tilde{A}_{1,n-1}B_{n,n}\left(I - \tilde{B}_{1,n-1}B_{n,n}\right)^{-1} A_{1,n-1} \tag{38}$$

and

$$\tilde{A}_{1,n} = \tilde{A}_{1,n-1}\left(I - B_{n,n}\tilde{B}_{1,n-1}\right)^{-1} A_{n,n} \tag{39}$$

$$\tilde{B}_{1,n} = \tilde{B}_{n,n} + A_{n,n}\tilde{B}_{1,n-1}\left(I - B_{n,n}\tilde{B}_{1,n-1}\right)^{-1} A_{n,n} \tag{40}$$

where the unit operator I is defined by its kernel $\delta_{\overline{k}_\perp,\overline{k}'_\perp}$. In detail, equations (37) and (38) are obtained by excluding the operator $A_{m|m+1}$ from relations (30) and (31), as well as equations (39) and (40) are obtained by excluding the operator $\tilde{A}_{m|m+1}$ from relations (34) and (35).

First the recurrent equations (37) and (38) were derived by Gazaryan [1] from relations (30) and (31) in the case of one-dimensional scattering medium. Barnes and Pendry [37] obtained a system equations similar to (39) and (40) using the field superposition principle in the case of three-dimensional scattering medium. Total system of the recurrent equations with a layer attachment (37–40) was derived from the transfer relations (30, 31, 34, 35) in [54] for the case of three-dimensional scattering medium.

4.2 Fundamental Transfer Matrix

As was shown by Barnes and Pendry [37] the system of recurrent equations (39) and (40) may be resolved in the form

$$\begin{bmatrix} \tilde{B}_{1,n} \cdot \tilde{A}_{1,n}^{-1} \\ \tilde{A}_{1,n}^{-1} \end{bmatrix} = M_n \begin{bmatrix} \tilde{B}_{1,n-1} \cdot \tilde{A}_{1,n-1}^{-1} \\ \tilde{A}_{1,n-1}^{-1} \end{bmatrix} \tag{41}$$

where the fundamental transfer matrix operator M_n is given by

$$M_n = \begin{bmatrix} A_{n,n} - \tilde{B}_{n,n} \cdot \tilde{A}_{n,n}^{-1} \cdot B_{n,n}, & \tilde{B}_{n,n} \cdot \tilde{A}_{n,n}^{-1} \\ -\tilde{A}_{n,n}^{-1} \cdot B_{n,n}, & \tilde{A}_{n,n}^{-1} \end{bmatrix} \tag{42}$$

4.3 Mutual Interaction Equations

It is difficult (if not impossible) to exclude all operator wave reflection and transmission coefficients from the transfer relations (29–36) getting a separate system equations for the operator amplitudes of waves in splits between layers of the stack. But such system of mutual interaction equations may be obtained from the Watson composition rule (15–18) directly and has the form

$$A_{m|m+1} = A_{m,m} A_{m-1|m} + \tilde{B}_{m,m} B_{m|m+1} \tag{43}$$

$$B_{m-1|m} = B_{m,m} A_{m-1|m} + \tilde{A}_{m,m} B_{m|m+1} \tag{44}$$

where $m = 1, 2, ..., n-1$.

First the system of equations (43) and (44) was derived by Kouznetsov and Budanov [52] in the case of electromagnetic wave propagation in a three-dimensional scattering medium consisting of dipole scatterers on base of the field superposition principle. Pendry and Roberts [39] and Pendry [48] formulated the system of equations (43) and (44) for wave propagation in a three-dimensional scattering medium using the fields superposition principle too. Ngo and Rino [49] derived the mutual interaction equations (43) and (44) in general case of electromagnetic wave propagation in the three-dimensional discrete medium using the Foldy-Lax-Twersky formalism.

According to [39, 48] the system of equations (43) and (44) is resolved in terms of the fundamental transfer matrix operator (42) as follows

$$\begin{bmatrix} A_{m|m+1} \\ B_{m|m+1} \end{bmatrix} = M_m \begin{bmatrix} A_{m-1|m} \\ B_{m-1|m} \end{bmatrix} \tag{45}$$

5. GENERALIZED RICCATI EQUATION

The recurrent equations with a layer attachment (37) and (38) as well as (39) and (40) manifest the invariant imbedding principle for wave reflection and transmission coefficients of a medium as stack of layers.

Let us consider the case of a thin n's layer which attached with the operator wave transmission $A_{n,n}$ and $\tilde{A}_{nn}$ and reflection $B_{n,n}$ and $\tilde{B}_{n,n}$ coefficients subject to the conditions

$$A_{n,n} = I + \Delta A_{n,n} \tag{46}$$

$$\Delta A_{n,n} = \mathrm{O}(\Delta \mathrm{z}) \tag{47}$$

$$B_{n,n} = \mathrm{O}(\Delta \mathrm{z}) \tag{48}$$

$$\tilde{A}_{n,n} = I + \Delta \tilde{A}_{n,n} \tag{49}$$

$$\Delta \tilde{A}_{n,n} = \mathrm{O}(\Delta \mathrm{z}) \tag{50}$$

$$\tilde{B}_{n,n} = \mathrm{O}(\Delta \mathrm{z}) \tag{51}$$

where a thickness Δz of the n's layer tends to zero $\Delta z \to 0$. Substituting (46–51) into the r.h.s. of (40) and ignoring the terms which are less than $\mathrm{O}(\Delta \mathrm{z})$, one can obtain

$$\begin{aligned}\frac{\tilde{B}_{1,n} - \tilde{B}_{1,n-1}}{\Delta z} = \frac{\tilde{B}_{n,n}}{\Delta z} + \frac{\Delta A_{n,n}}{\Delta z}\tilde{B}_{1,n-1}\\ + \tilde{B}_{1,n-1}\frac{\Delta \tilde{A}_{n,n}}{\Delta z} + \tilde{B}_{1,n-1}\frac{B_{n,n}}{\Delta z}\tilde{B}_{1,n-1}\end{aligned} \tag{52}$$

that is a generalized Riccati equation for the operator wave reflection coefficient $\tilde{B}_{1,n}$ of the medium.

In similar manner, substituting (46–51) into the r.h.s. of (39) gives

$$\frac{\tilde{A}_{1,n} - \tilde{A}_{1,n-1}}{\Delta z} = \tilde{A}_{1,n-1}\left(\frac{\Delta \tilde{A}_{n,n}}{\Delta z} + \frac{B_{n,n}}{\Delta z}\tilde{B}_{1,n-1}\right) \tag{53}$$

that is a generalized corresponding equation for the operator wave transmission coefficient $\tilde{A}_{1,n}$ of the medium.

Equations (52) and (53) were derived from the recurrent equations with a layer attachment (39) and (40) in [54].

6. RICCATI EQUATION FOR WAVE SCATTERING FROM PERIODIC ONE-DIMENSIONAL INTERFACE

Let us turn to the case of a two-dimensional scattering medium with the scattering potential $V(\overline{r}) = V(x,z)$. In this case the k_y component of the wave vector $\overline{k}$ is conserved by a wave reflection from or transmission through the scattering medium. This property manifests itself in the following representation for the three-dimensional Fourier transform of the scattering operator of the scattering medium

$$T(\overline{k},\overline{k}') = \delta_{k_y,k'_y} T(k_y;k_x,k_z;k'_x,k'_z) \tag{54}$$

Here and henceforth $\delta_{p,p'}$ denotes $2\pi\delta(p-p')$ and $T(p;k_x,k_z;k'_x,k'_z)$ is the two-dimensional Fourier transform of the scattering operator $T(p;x,z;x',z')$ satisfying the Lippman-Schwinger equation (3) with the two-dimensional scattering potential $V(x,z)$ and one-dimensional Fourier transform $G_\circ(p;x,z,)$ of the Green function in a background with respect to y given by

$$G_\circ(p;x,z) = \int dy \exp(-ipy) G_\circ(x,y,z) \tag{55}$$

In accordance with (54), the operator wave transmission and reflection coefficients of the medium defined by equations (8, 9, 13, 14) take in the case of the two-dimensional scattering medium the form

$$A(\overline{k}_\perp,\overline{k}'_\perp) = \delta_{k_y,k'_y} A(k_x,k'_x) \tag{56}$$

$$B(\overline{k}_\perp,\overline{k}'_\perp) = \delta_{k_y,k'_y} B(k_x,k'_x) \tag{57}$$

$$\tilde{A}(\overline{k}_\perp,\overline{k}'_\perp) = \delta_{k_y,k'_y} \tilde{A}(k_x,k'_x) \tag{58}$$

$$\tilde{B}(\overline{k}_\perp,\overline{k}'_\perp) = \delta_{k_y,k'_y} \tilde{B}(k_x,k'_x) \tag{59}$$

where all quantities in the r.h.s. of these equations are any function of k_y .

In particular, for a thin layer of the two-dimensional medium occupied the region between planes $z = L$ and $z = L + \Delta L$ one can apply the perturbative method to solution of the Lippman-Schwinger equation (3) with result

$$T(p; x, z; x', z') \cong V(x, z)\delta(x - x')\delta(z - z') \tag{60}$$

as $L < z < L + \Delta L$. Substituting (60) into the r.h.s. of equations (56–59) and denoting

$$V(p, z) = \int_{-\infty}^{+\infty} dx \exp(-ipx) V(x, z) \tag{61}$$

gives

$$\frac{\Delta A(k_x, k_x')}{\Delta L} \to \frac{1}{2i\sigma_k} \exp[-i(\sigma_k - \sigma_{k'})L]\ V(k_x - k_x', L) \tag{62}$$

$$\frac{B(k_x, k_x')}{\Delta L} \to \frac{1}{2i\sigma_k} \exp[i(\sigma_k + \sigma_{k'})L]\ V(k_x - k_x', L) \tag{63}$$

$$\frac{\Delta \tilde{A}(k_x, k_x')}{\Delta L} \to \frac{1}{2i\sigma_k} \exp[i(\sigma_k - \sigma_{k'})L]\ V(k_x - k_x', L) \tag{64}$$

$$\frac{\tilde{B}(k_x, k_x')}{\Delta L} \to \frac{1}{2i\sigma_k} \exp[-i(\sigma_k + \sigma_{k'})L]\ V(k_x - k_x', L) \tag{65}$$

as $\Delta L \to 0$.

Using the limits (62–65) in the r.h.s. of the generalized Riccati equation (52) allows one to write for a quantity $R(q, q')$ defined by substitution

$$\tilde{B}(k_x, k_x') = \exp[-i(\sigma_k + \sigma_{k'})L]\ R(k_x, k_x') \tag{66}$$

the following Riccati equation

$$\begin{aligned} \frac{dR(q, q')}{dz} &- i(\sigma_q + \sigma_{q'})R(q, q') \\ &= \frac{1}{2i\sigma_q} V(q - q', z) + \int_{q_1} \frac{1}{2i\sigma_q} V(q - q_1, z) R(q_1, q') \\ &+ \int_{q_1} R(q, q_1) \frac{1}{2i\sigma_{q_1}} V(q_1 - q', z) \\ &+ \int_{q_1} \int_{q_2} R(q, q_1) \frac{1}{2i\sigma_{q_1}} V(q_1 - q_2, z) R(q_2, q') \end{aligned} \tag{67}$$

where $\sigma_q = \sqrt{k_\circ^2 - q^2 - k_y^2}$ and $\int_q = (2\pi)^{-1} \int dq$. The reflected from the two-dimensional scattering medium wave field (12) in the transverse to z axis Fourier transform representation (5) takes due to the substitution (66) the form

$$\tilde{\Psi}(\overline{k}_\perp, z) = \tilde{\Psi}_\circ(\overline{k}_\perp, z) + \exp[i\sigma_k(z - L)] \int_{k'_x} R(k_x, k'_x) \tilde{\tilde{\Psi}}_\circ(k'_x, k_y) \quad (68)$$

as $z > L$, with $\tilde{\tilde{\Psi}}_\circ(\overline{k}_\perp) = \exp(-i\sigma_k L)\tilde{\Psi}_\circ(\overline{k}_\perp)$. First the Riccati equation of type (67) was obtained with the help of another method by Klyatskin [11] for wave reflection from a three-dimensional continuous inhomogeneous medium.

In similar manner, using the limits (62–65) in the r.h.s. of the generalized corresponding equation (53) allows one to write for a quantity $\mathcal{T}(q, q')$ defined by substitution

$$\tilde{A}(k_x, k'_x) = \exp(-i\sigma_{k'} L)\mathcal{T}(k_x, k'_x) \quad (69)$$

the following equation

$$\frac{d\mathcal{T}(q, q')}{dz} - i\sigma_{q'}\mathcal{T}(q, q') = \int_{q_1} \mathcal{T}(q, q_1) \frac{1}{2i\sigma_{q_1}} V(q_1 - q', z) + \int_{q_1} \int_{q_2} \mathcal{T}(q, q_1) \frac{1}{2i\sigma_{q_1}} V(q_1 - q_2, L) R(q_2, q') \quad (70)$$

The transmitted through the two-dimensional scattering medium wave field (11) in the transverse to z axis Fourier transform representation (5) takes due to the substitution (69) the form

$$\tilde{\Psi}(\overline{k}_\perp, z) = \exp(-i\sigma_k z) \int_{k'_x} \mathcal{T}(k_x, k'_x) \tilde{\tilde{\Psi}}_\circ(k'_x, k_y) \quad (71)$$

as $z < 0$.

6.1 Scattering from Periodic One-dimensional Interface

Let us consider a scattering medium with a periodic one-dimensional interface $z = f(x)$ of two dielectric half-spaces $z > f(x)$ and $z < f(x)$ with the dielectric permittivities ε^{bac} and ε, respectively (see Fig.2). One may say about a transition region $0 < z < f(x)$ between two homogeneous dielectric half-spaces $z > h = \max_x f(x)$ and $z < 0$. We thought the lower homogeneous half-space as being a system of two slabs, $-L_\circ < z < 0$ and $z < -L_\circ$, with the dielectric permittivities ε and ε^{bac}, respectively, the plane $z = -L_\circ$ tending to the negative infinitivity to the end, $L_\circ \to 0$.

The dielectric permittivity $\varepsilon(x,z)$ of the transition region may be written in the form

$$\varepsilon(x,z) = \varepsilon^{bac} + (\varepsilon - \varepsilon^{bac}) \sum_{\mu=-\infty}^{\infty} \left\{ H\left[x - x'_\mu(z)\right] - H\left[x - x''_\mu(z)\right] \right\} \quad (72)$$

Here $x'_\mu(z)$ and $x''_\mu(z), x'_\mu(z) < x''_\mu(z)$, are roots of the equation

$$f(x) = z \quad (73)$$

where $0 < z < h$, and the step function $H(x) = 1$ as $x \geq 0$ and $H(x) = 0$ as $x < 0$ The periodic interface is supposed to have a period Λ, $f(x) = f(x + \Lambda)$ and to be symmetrical relatively to the z axis, $f(x) = f(-x)$. This suppositions allow one to write

$$x'_\mu(z) = -x_\circ(z) + \mu\Lambda \quad (74)$$

and

$$x''_\mu(z) = x_\circ(z) + \mu\Lambda \quad (75)$$

where $\mu = 0, \pm1, \pm2$,..... Now on the base of (4), (61), (72), (74), (75) one gets

$$V(q,z) = 2V_\circ \sum_{\mu=-\infty}^{\infty} f_\mu(z) \delta\left(q - \frac{2\pi\mu}{\Lambda}\right) \quad (76)$$

where $V_\circ = -k_\circ^2(\varepsilon - \varepsilon^{bac})/\varepsilon^{bac}$ and

$$f_\mu(z) = \frac{1}{\mu} \sin\left[\frac{2\pi\mu}{\Lambda} x_\circ(z)\right] \quad (77)$$

Turn now to the Riccati equation (67). One could suppose that a solution for this equation in the case of the transition region with the scattering potential (76) in the one-dimensional Fourier transform representation (61) may be sought in the form

$$R(q,q') = \sum_{\mu=-\infty}^{\infty} R_{\mu,0}(q') \delta_{q,q'+2\pi\mu/\Lambda} \quad (78)$$

with a boundary condition

$$R(q,q') \longrightarrow R(q)\delta_{qq'} \tag{79}$$

as $z \to 0$ where

$$R(q) = R_\infty(q)\frac{1-\exp(2i\sigma_{1q}L_\circ)}{1-R_\infty^2(q)\exp(2i\sigma_{1q}L_\circ)} \tag{80}$$

is the reflection coefficient of the homogeneous slab $-L_\circ < z < 0$ with $\sigma_{1_q} = \sqrt{k_1^2 - q^2 - k_y^2}$ and $k_1 = (\omega/C)\sqrt{\varepsilon}$, and

$$R_\infty(q) = \frac{\sigma_q - \sigma_{1_q}}{\sigma_q + \sigma_{1_q}} \tag{81}$$

is the limit of (80) as $L_\circ \to \infty$. the closed system of equations is obtained for the matrix quantities $R_{\mu,\nu}(q')$ defined by

$$R_{\mu\nu}(q') = R_{\mu-\nu,0}\left(q' + \frac{2\pi\nu}{\Lambda}\right) \tag{82}$$

where $\mu, \nu = 0, \pm1, \pm2, \ldots$. This system of equations has got the form

$$\begin{aligned} &\frac{dR_{\mu,\nu}}{dz} - i[\sigma(\mu)+\sigma(\nu)]R_{\mu,\nu} \\ &\quad = \alpha(\mu)f_{\mu-\nu}(z) + \sum_{\mu_1=-\infty}^{\infty}\alpha(\mu)f_{\mu-\mu_1}(z)R_{\mu_1,\nu} \\ &\quad + \sum_{\nu_1=-\infty}^{\infty}R_{\mu,\nu_1}\alpha(\nu_1)f_{\nu_1-\nu}(z) \\ &\quad + \sum_{\mu_1=-\infty}^{\infty}\sum_{\nu_1=-\infty}^{\infty}R_{\mu,\nu_1}\alpha(\nu_1)f_{\nu_1-\mu_1}(z)R_{\mu_1,\nu} \end{aligned} \tag{83}$$

as $0 < z < h$ and

$$R_{\mu,\nu}(q') \to R\left(q' + \frac{2\pi\nu}{\Lambda}\right)\delta_{\mu-\nu,0} \tag{84}$$

as $z \to 0$. In equation (83) $\sigma(\mu)$ denotes σ_q with $q = q' + 2\pi\mu/\Lambda$ and $2\pi\alpha(\mu) = -iV_\circ/\sigma(\mu)$, in equation (84) $\delta_{\mu,\nu}$ is the Kronecker symbol.

The physical meaning of quantities $R_{\mu,0}(q')$ becomes clear after substituting (78) into the r.h.s. of equation (68) for the wave field reflected from the transition region. This substitution gives in the domain of the position vector

$$\tilde{\Psi}(\overline{r}) = \exp[i\overline{k}_\circ(\overline{r} - h\hat{z})] + \sum_{\mu=-\infty}^{\infty} R_{\mu,0}(k_{ox}; h) \exp[i\overline{k}_\mu^+(\overline{r} - h\hat{z})] \quad (85)$$

as $z > h$. Here $\overline{k}_\circ$ is the wave vector of the plane wave incident field with $k_{0_y} = 0; \overline{k}_\mu^+$ is the wave vector of an upgoing reflected from the transition region Bloch wave (mode, see [30]) defined by its components $k_{\mu x}^+ = k_{ox} + 2\pi\mu/\Lambda$ and $k_{\mu z}^+ = \sigma_q$ with $q = k_{\mu x}^+$. We indicate definitely in the r.h.s. of (85) the relation of a solution to the matrix Riccati equation (83) to z at $z = h$. As one can see from (85), the quantity $R_{\mu,0}(k_{0x}; h)$ means a partial wave reflection coefficient into the μ's Bloch mode from the transition region (a Bloch reflection coefficient).

In similar manner with the Riccati equation (67), the corresponding equation (70) may be transformed in the case of the transition region characterized by the scattering potential (76) in the one-dimensional Fourier transform representation (61). One may try to seek a solution to equation (70) in the form

$$\mathcal{T}(q, q') = \sum_{\mu=-\infty}^{\infty} \mathcal{T}_{\mu,0}(q')\delta_{q,q'+2\pi\mu/\Lambda} \quad (86)$$

with a boundary condition

$$\mathcal{T}(q, q') \longrightarrow \mathcal{T}(q)\delta_{qq'} \quad (87)$$

as $z \to 0$ were

$$\mathcal{T}(q) = \frac{4\sigma_q\sigma_{1q}}{(\sigma_q + \sigma_{1q})^2 \exp(-i\sigma_{1q}L_\circ) - (\sigma_q - \sigma_{1q})^2 \exp(i\sigma_{1q}L_\circ)} \quad (88)$$

is the transmission coefficient of the homogeneous slab $L_\circ < z < 0$ corresponding to the reflection coefficient (80). The closed system of equations is obtained for the matrix quantities $\mathcal{T}_{\mu,\nu}(q')$ defined by $\mathcal{T}_{\mu,\nu}(q') = \mathcal{T}_{\mu-\nu,0}(q' + 2\pi\nu/\Lambda)$ where $\mu, \nu = 0, \pm1, \pm2, ...$This system of equations has got the form

$$\frac{d\mathcal{T}_{\mu,\nu}}{dz} - i\sigma(\nu)\mathcal{T}_{\mu,\nu} = \sum_{\nu_1=-\infty}^{\infty} \mathcal{T}_{\mu,\nu_1}\alpha(\nu_1) f_{\nu_1-\nu}(z)$$
$$+ \sum_{\mu_1=-\infty}^{\infty}\sum_{\nu_1=-\infty}^{\infty} \mathcal{T}_{\mu,\nu_1}\alpha(\nu_1) f_{\nu_1-\mu_1}(z) R_{\mu_1,\nu} \quad (89)$$

as $0 < z < h$ and

$$\mathcal{T}_{\mu,\nu}(q') \longrightarrow \mathcal{T}(q_1 + 2\pi\nu/\Lambda)\delta_{\mu-\nu,0} \quad (90)$$

as $z \to 0$.

The physical meaning of quantities $\mathcal{T}_{\mu,0}(q')$ becomes clear after substituting (86) into r.h.s. of equation (71) for the wave field transmitted through the transition region. This substitution gives in the domain of the position vector

$$\tilde{\Psi}(\overline{r}) = \sum_{\mu=-\infty}^{\infty} \mathcal{T}_{\mu,0}(k_{0,x}; h) \exp\left\{ i\overline{k}_{\mu}^{-} \left[\overline{r} + (z + L_{\circ})\hat{z}\right]\right\} \quad (91)$$

as $z + L_{\circ} < 0$. Here $\overline{k}_{\mu}^{-}$ is the wave vector of an downcoming transmitted through the transition region Bloch wave defined by its components $k_{\mu x}^{-} = k_{ox} + 2\pi\nu/\Lambda$ and $k_{\mu z}^{-} = -\sigma_q$ with $q = k_{\mu x}^{-}$. We indicate definitely in the r.h.s. of (91) the relation of a solution to the matrix corresponding equation (89) to z at $z = h$. As one can see from (91) the quantity $\mathcal{T}_{\mu,0}(k_{ox}; h)$ means a partial wave transmission coefficient into the μ's Bloch mode through the transition region (a Bloch transmission coefficient).

6.2 Poynting's Theorem for Bloch's Reflection and Transmission Coefficients

Expansions (85) and (91) of wave fields reflected from and transmitted through the transition region of a periodic one-dimensional interface along the Bloch waves allow of one to evaluate the energy flux in these reflected and transmitted wave fields. Using the Poynting vector $\overline{\Pi} = (1/2)(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)$ one gets

$$\Pi_z(z=h) = k_{0z} + \sum_\mu k^+_{\mu z} |R_{\mu,0}(k_{ox};h)|^2 \tag{92}$$

and

$$\Pi_z(z=-L_\circ) = \sum_\mu k^-_{\mu z} |\mathcal{T}_{\mu,0}(k_{ox};h)|^2 \tag{93}$$

for the energy flux in reflected and transmitted wave fields, respectively; the z components (92) and (93) of the Poynting vector being averaged along the planes $z = h$ and $z = -L_\circ$, respectively. It is worth noting that the sums in the r.h.s. of (92) and (93) are taken over only propagating Bloch modes subjected to the condition

$$|k_{ox} + 2\pi\mu/\Lambda| \leq k_\circ \tag{94}$$

The energy fluxes (92) and (93) should be identical in accordance with the Poynting theorem for the non-absorptive scattering medium. This requirement gives a relation between the Bloch reflection and transmission coefficients which can be obtained also from the optical theorem (25).

7. NUMERICAL SOLUTION TO THE MATRIX RICCATI EQUATION

The matrix Riccati equation (83) is an infinite system of differential equations. To apply a numerical method we truncated this system having considered the 29×29 differential equations (83) for a matrix reflection coefficient $R_{\mu,\nu}(k_{ox};z)$ which describe a mutual transformation of the 29 first Bloch modes with the indices $\mu,\nu = 0, \pm 1, \pm 2, ..., \pm 14$. We did not perform a general estimating the accuracy of the involved truncating to the matrix Riccati equation (83) restricted themselves by a selected comparison of the corresponding results related to solutions to the two truncated systems of the 29×29 and 49×49 equations. A difference between the corresponding results was turned out, in fact, not more than 0.01% for the Bloch reflection coefficients with index $\mu = 0, \pm 1, \pm 2, ..., \pm 13$ and to be 0.03% with $\mu = \pm 14$. Such small difference indicates, as we suppose, a good convergence of the truncating procedure.

The truncated matrix Riccati equation (83) was solved numerically with the aid of the 4th-order Runge-Kutta method in the case of a triangular periodic interface of two half-spaces (see Fig. 2).

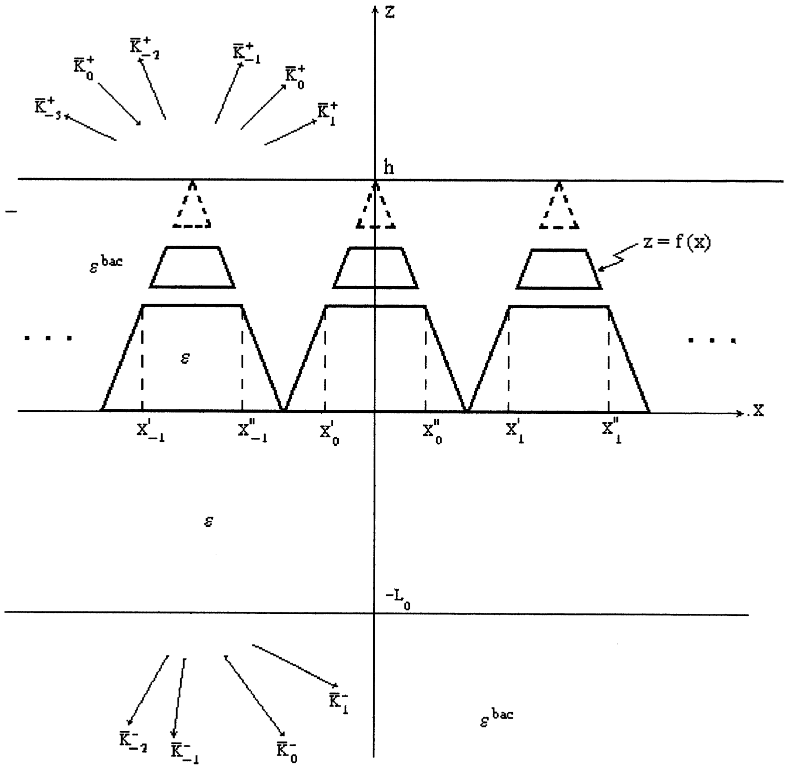

Figure 2. A periodic one-dimensional interface of two dielectric half-spaces with the dielectric permittivities ε^{bac} (upper half-space) and ε (lower half-space). An auxiliary truncated interface is presented in the form of a thin slice separated by a split from the rest truncated interface. The wave vectors of upgoing reflected and downcoming transmitted Bloch waves are depicted.

The time spent on numerical calculation was dependent on the step Δz along the imbedding parameter z in scale of the maximum height h of the interface (see Fig. 2), i.e., on the quantity $\Delta z/h$. The choice of the step was dependent, in its turn, on relative difference between the dielectric permittivities of two half-spaces as well as on proximity of the incident field wave vector to a resonant wave vector satisfying according to [28] the condition (94) with the sign "equal" in the r.h.s. of this one.

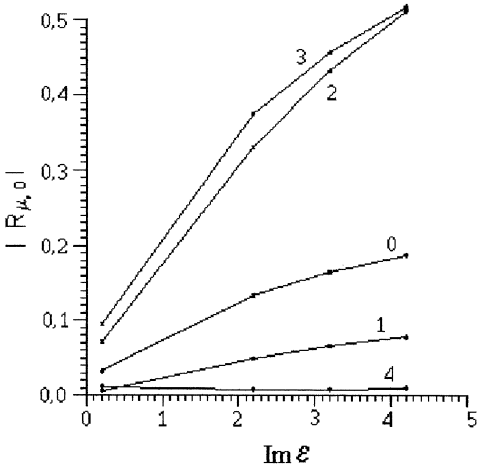

Figure 3. The dependence of the absolute value $|R_{\mu,0}|$ of the Bloch reflection coefficient on the imaginary partx $\Im\epsilon$ of the lower half-space dielectric permittivity for modes $\mu = 0, \pm1, \pm2, \pm3, \pm4$ and under conditions $\Re\epsilon = 1.50$, $\alpha = 0$, $h/\lambda = 0.75$, $\kappa = \lambda/\Lambda = 0.3$.

7.1 Results of Numerical Solution to The Matrix Riccati Equation

Some results of numerical solution to the truncated matrix Riccati equation (83) are plotted in Figs. 3–6. These figures illustrate dependence of the absolute values $|R_{\mu,0}(k_{ox}; h)|$ of the complex Bloch reflection coefficients, corresponding to the first most specific modes with index $\mu = 0, \pm1, \pm2, \pm3, \pm4$ from the 29 studied modes, on the different parameters characterized the incident plane wave and the periodic dielectric interface.

Fig. 3 shows a simple relation of the Bloch reflection coefficients to the imaginary part of the lower half-space dielectric permittivity, the angle α between the incident plane wave direction and the z axis (incidence angle) being supposed to be zero, $\alpha = 0$ (normal incidence). As one can see, the more is the mentioned imaginary part the more becomes the absolute value of a Bloch reflection coefficient.

Fig. 4 depicts the Wood anomalous dependence of the Bloch reflection coefficients, at normal incidence, on the parameter $\kappa = \lambda/\Lambda$ defined by ratio of the incident field wavelength λ to the interface period Λ . Let us remark that only 7 modes with index $\mu = 0, \pm1, \pm2, \pm3$ describe the propagating waves under condition $0.25 < \kappa < 0.33(3)$, from the studied 29 modes. The others 22 modes describe the evanescent waves. There are only 5 modes with index $\mu = 0, \pm 1, \pm 2$ which describe the propagating waves under condition $0.33(3) < \kappa < 0.5$. A boundary point $\kappa = 0.33(3)$ of these two κ intervals (see a vertical dashed line in Fig. 4) corresponds to a resonance which was considered in [28] and may be called the Wood resonance [29]. At this boundary point, the 3-rd mode (with index $\mu = \pm 3$) is transformed from the evanescent wave to propagating wave, the reflection coefficient of the 3-rd mode being sharply changed. Because of connection between modes due to the function (77) in the r.h.s. of the matrix Riccati equation (83), the sharp changing in reflection coefficient of the 3-rd mode gives changing also in reflection coefficients of some others modes (see in Fig. 4 a changing in reflection coefficients of the 0-th and 4-th modes). The curves of $|R_{\mu,0}|$ versus to parameter κ have become smooth since κ has passed through the boundary point $\kappa = 0.33(3)$ and the 3-rd mode has been transformed into the propagating wave (see the interval $0.25 < \kappa < 0.33(3)$ in Fig. 4). By approximating to the second Wood resonance at $\kappa = 0.25$, the 4-th mode with index $\mu = \pm 4$ is transformed to the propagating wave, the behavior of the absolute values of the Bloch reflection coefficients versus κ being similar to that related to the point $\kappa = 0.33(3)$, described above. We did not perform in detail the numerical solution to the truncated matrix Riccati equation (83) in the immediate vicinity of the resonant points $\kappa = 0.25$ and $\kappa = 0.33(3)$ where the step $\Delta z/h$ along the imbedding parameter has to be very small and the time spent on numerical calculation to be very large, respectively.

Figs. 5 and 6 represent the Wood anomalous dependence of the Bloch reflection coefficients, at the parameter $\kappa = 0.3$, on the sine of the incidence angle, sin α. In contrast to Fig. 4, we did not mark here (by vertical dashed line) the immediate vicinities of the Wood resonant points where the numerical solution to the truncated matrix Riccati equation was not performed. In fact, the Wood resonances were observed at the incidence angles defined by: sin $\alpha = 0.1; 0.2; 0.4; 0.5; 0.7; 0.8$ with the corresponding values of the mode index: $\mu = 3; -4; 2; -5; 1; -6$, respectively. Let us remark that on the curve of the 0-th mode one can see the responses on the resonances in the 1-st mode at sin $\alpha = 0.7$ and the 2-nd mode at sin $\alpha = 0.4$, but practically it is difficult to distinguish the responses on resonances in the 3-rd and 4-th modes at sin $\alpha = 0.1$ and 0.2, respectively. Similarly,

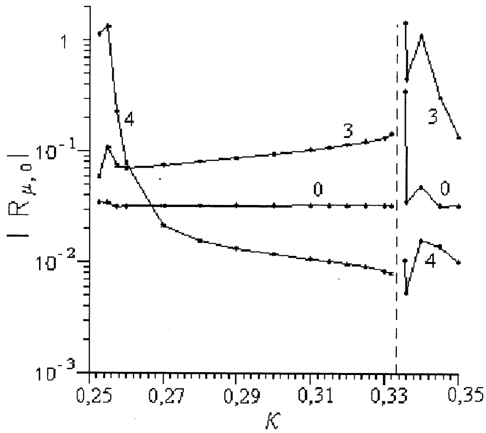

Figure 4. The dependence of the absolute value $|R_{\mu,0}|$ of the Bloch reflection coefficient on the parameter $\kappa = \lambda/\Lambda$ for modes $\mu = 0, \pm 3, \pm 4$ and under conditions $\epsilon = 1.50 + i0.20$, $\alpha = 0$, $h/\lambda = 0.75$.

on the curve of the 1-st mode one can see, besides the eigenresonance at sin $\alpha = 0.7$, the responses on the resonances in the 2-nd and 3-rd modes at sin $\alpha = 0.4$ and 0.1, respectively. Also, on the curve of the 2-nd mode there are seen, besides the eigenresonance at sin $\alpha = 0.4$, the responses on the resonances in the 3-rd and 1-st modes at sin $\alpha = 0.1$ and 0.7, respectively. These responses indicate that the mode with an index μ experiences the most influence from the next modes with indices $\mu \pm 1$, $\mu \pm 2, \dots$.

8. DISCUSSION AND CONCLUSIONS

In this paper a total system of transfer relations for scalar wave propagation in a three-dimensional inhomogeneous layered medium has been derived from the composition rule for scattering T-matrices. The transfer relations led to the separate recurrent systems of equations for wave reflection and transmission coefficients of the medium under consideration with a layer attachment. The recurrent system gave in the case of infinitesimal attached layer the generalized Riccati equation for the wave reflection coefficient and the corresponding equation for the wave transmission coefficient of the medium with taken into account both the propagating and evanescent waves.

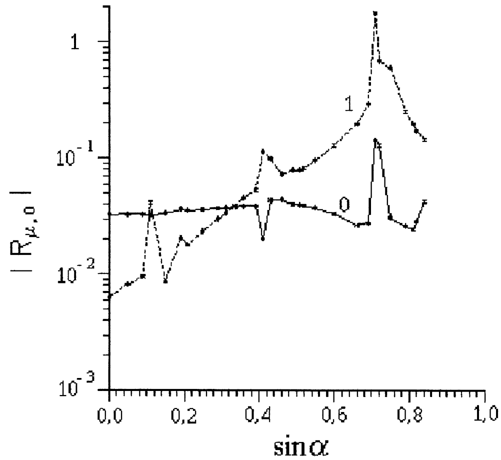

Figure 5. The dependence of the absolute value $|R_{\mu,0}|$ of the Bloch reflection coefficient on the sine of the incidence angle, sin α , for modes $\mu = 0, \pm 1$ and under conditions $\epsilon = 1.50 + i0.20$, $h/\lambda = 0.75$, $\kappa = \lambda/\Lambda = 0.3$.

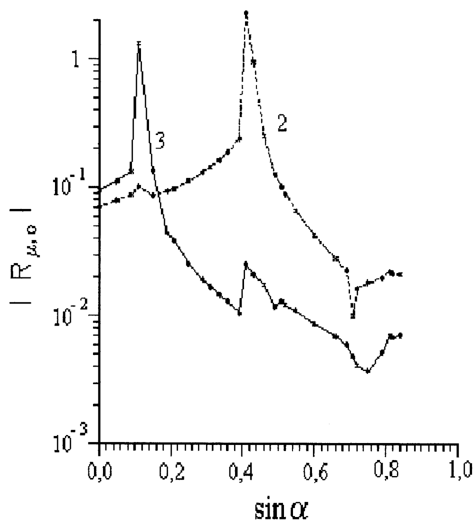

Figure 6. The dependence of the absolute value $|R_{\mu,0}|$ of the Bloch reflection coefficient on the sine of the incidence angle, $\sin \alpha$, for modes $\mu = \pm 2, \pm 3$ and under conditions $\epsilon = 1.50 + i0.20$, $h/\lambda = 0.75$, $\kappa = \lambda/\Lambda = 0.3$.

The generalized Riccati equation was applied to study the Wood anomalies (resonances) by electromagnetic wave reflection from a periodic one-dimensional interface of two dielectric half-spaces in the case of TE polarization. It was shown by numerical solution to the matrix Riccati equation in the representation of the Bloch modes that the Wood resonances are related to a transformation of a Bloch mode from the an evanescent to a propagating wave, and vice versa. The effect of connection between Wood's resonances was considered. Due to this effect, one can see on the curve of the reflection coefficient, for a given Bloch mode versus to, e.g., the incidence angle sine, both the eigenresonance and the response on the resonances in the next modes.

It is useful to make a remark concerning the physical meaning of the Riccati equation (83) applied to the problem of wave reflection from a periodic one-dimensional interface. In accordance with Fig. 2, we introduced an auxiliary set of truncated interfaces $\zeta = f(x,z)$ defined by $f(x,z) = f(x)$ as $f(x) < z$ and $f(x,z) = z$ as $f(x,z) > z$ where z was considered as the imbedding parameter. The truncated interface with the parameter $z + \Delta z$ was presented in the form of the truncated interface with the parameter z and a thin slice of the transition region with the thickness Δz, the slice having been separated from the interface $\zeta = f(x,z)$ by an infinitesimal split. The analysis of wave multiple scattering between the slice and the rest transition region with the interface $\zeta = f(x,z)$, made using the recurrent equation (40) with a layer attachment, led to the matrix Riccati equation (83) for the wave matrix reflection coefficient $R_{\mu,\nu}(z)$ from the truncated periodic interface $\zeta = f(x.z)$. The desired wave matrix reflection coefficient $R_{\mu,\nu}(h)$ was brought by the limit $z \to h$.

To consider electromagnetic wave reflection from or transmission through a periodic one-dimensional interface in the case of TH polarization, when the electric vector of the incident wave lies in the incident x, z plane (see Fig. 2), one should generalize the total system of the transfer relations derived in the paper as well as all consequences from these relations on the case of vector wave electromagnetic field. Such generalization of the transfer relations on the vector wave electromagnetic fields may be performed with the aid of the special decomposing the Green tensor function in a background into a principal part, with an excluded volume (Lorentz cavity) about the singularity at the origin, and a Dirac delta function part according to [35, 57] where the infinitesimal Lorentz cavity is chosen to be of a slab shape perpendicular to the z axis (see Fig. 1). What is more, one can write a Riccati equation like (83) and a corresponding equation like (89) for the vector electromagnetic wave reflection coefficient from and transmission coefficient through of a two-dimensional periodic interface in the cases of both TE and TH polarizations. In contrast to the method of integral equations [36], our approach using the idea of the transfer relations leads to describing the interface in terms of its intersections by planes $z = \text{const}$ similar to the method of the statistical topography [58, 59].

The general Riccati equation (52) and the corresponding equation (53) for the operator wave reflection and transmission coefficients of the medium, after having been written for the case of vector electromagnetic field, may be applied, of course, to the problems of volume wave multiple scattering, for example, to calculate the coefficients of wave reflection from and wave transmission through a periodic (both in two and three dimensions) dielectric structures (photonic band structures). In particular, it is possible to bring the Riccati equation (67) and the corresponding equation (70) in the case of a two-dimensional arrangement of dielectric cylinders to the matrix Bloch representation similar to one in equations (83) and (89) and to solve numerically the obtained system of the matrix differential equations for the dielectric cylinder two-dimensional arrangement wave reflection and transmission coefficients. This problem was solved in [46] by discretizing the separate equations for the electric and the magnetic fields in the space domain and using an equivalence between these discretized equations and the tight-binding model of electronic localization.

In the end, we would like to underline the wide capabilities of the generalized Riccati equation (52) for the operator wave reflection coefficient and the corresponding equation (53) for the operator wave transmission coefficient of a medium due to these equations do not depend on the details of the medium model. We believe, that equations (52) and (53) may be applied, e.g., to study the effect of the Mie resonance on diffuse wave multiple scattering in dense medium for different concentrations of resonant scatterers that has been studied in [60] with the aid of the random matrix theory.

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APPENDIX
Derivation of equation (29)

Let us show how one can derive the transfer relation (29) from the Watson composition rule similar to (15–18).

Using the definition

$$T_{1,n}^{m} = V_m + V_m G_\circ T_{1,n} \tag{A.1}$$

one may compose the scattering operator $T_{1,n}$ of the medium as the sum

$$T_{1,n} = T_{1,n}^{1,m} + T_{1,n}^{m+1,n} \tag{A.2}$$

where the new two operators $T_{1,n}^{1,m}$ and $T_{1,n}^{m+1,n}$ are found from the system of two equations

$$T_{1,n}^{1,m} = T_{1,m} + T_{1,m} G_\circ T^{m+1,n} \tag{A.3}$$

and

$$T_{1,n}^{m+1,n} = T_{m+1,n} + T_{m+1,n} G_\circ T_{1,n}^{1,m} \tag{A.4}$$

In the Fourier transform representation the last three equations take, under condition (16) because of the layers are supposed to be not intersected, the form

$$T_{1,n}(\overline{k}, \overline{k}') = T_{1,n}^{1,m}(\overline{k}, \overline{k}') + T_{1,n}^{m+1,n}(\overline{k}, \overline{k}') \tag{A.5}$$

with

$$T_{1,n}^{1,m}(\overline{k}, \overline{k}') = T_{1,m}(\overline{k}, \overline{k}') + \int_{\overline{p}_\perp} \frac{1}{2i\sigma_p} T_{1,m}(\overline{k}, \overline{p}^-) T_{1,n}^{m+1,n}(\overline{p}^-, \overline{k}') \tag{A.6}$$

and

$$T_{1,n}^{m+1,n}(\overline{k}, \overline{k}') = T_{m+1,n}(\overline{k}, \overline{k}') + \int_{\overline{p}_\perp} \frac{1}{2i\sigma_p} T_{m+1,n}(\overline{k}, \overline{p}^+) T_{1,n}^{1,m}(\overline{p}^+, \overline{k}') \tag{A.7}$$

Substitute now equation (A.6) for $T_{1,n}^{1,m}$ into the r.h.s. of (A.5). Then equation (A.5) being multiplied by factor $1/(2i\sigma_k)$ takes at $\overline{k} = \overline{k}^{-}$ and $\overline{k}' = \overline{k}^{+'}$ the form of the transfer relation (29).

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