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DISCRETE OPTIMIZATION PROBLEMS OF LINEAR ARRAY SYNTHESIS BY USING REAL NUMBER PARTICLE SWARM OPTIMIZATION

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Abstract—It is generally believed genetic algorithm (GA) is superior to particle swarm optimization (PSO) while dealing with the discrete optimization problems. In this paper, a suitable mapping method is adopted and the modified PSO can effectively deal with the discrete optimization problems of linear array pattern synthesis. This strategy has been applied in thinned linear array pattern synthesis with minimum sidelobe level, 4-bit digital phase shifter linear array pattern synthesis and unequally spaced thinned array pattern synthesis with minimum sidelobe level. The obtained results are all superior to those in existing literatures with GA, iterative FFT and different versions of binary PSO, that show the effectiveness of this strategy and its potential application to other discrete electromagnetic optimization problems.

1. INTRODUCTION

Linear array pattern synthesis is a classical electromagnetic optimization problem [1]. Its purpose is to find an optimized excitation vector to obtain a prospective far field pattern. The excitation vector of linear array consists of excitation amplitude, phase and position of each array element. As for excitation amplitude and phase of array elements, accurate feed network is required for continuous adjustment, which increases the cost and complexity of the array [2]. In order to simplify the feed network, continuous adjustment for the excitation amplitude and phase of array elements can be adjusted as discrete adjustment,

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such as array thinning technology adopted for pattern synthesis by controlling the on/off status of elements [3–13], phase array technology adopting N-bit digital phase shifter [14–17], and pattern synthesis combining array thinning and unequally spaced array [9, 18] etc.

Particle swarm optimization (PSO) is an evolutionary algorithm based on swarm intelligence [19]. And this algorithm is widely used in pattern synthesis, such as linear array pattern synthesis [20–22], planar array pattern synthesis [7, 9–11, 13, 23], conformal array pattern synthesis [24–26], thinned array pattern synthesis [7, 9–11, 13] etc.

It is generally believed that the genetic algorithm (GA) and ant colony optimization (ACO) algorithm are more suitable to handle discrete optimization problems, because of the GA and ACO using discrete coding method and dealing directly with discrete variables. Unlike GA and ACO, PSO is an effective tool to deal with continuous optimization problems, and it cannot be used to directly deal with discrete optimization problems [7, 13, 27]. How to extend PSO to discrete optimization problems and to maintain its outstanding performance became a research focus.

In order to deal with discrete variables, PSO require mapping real variables to discrete variables or modify algorithm framework. In [27], a basic binary PSO (BBPSO) is proposed for dealing with discrete problems. As for BBPSO and other similar improved optimization, particle's velocity and position are not directly associated any longer, which makes BBPSO fail to inherit the advantages of real number PSO, and this strategy may cause low efficiency and premature convergence of the optimization in dealing with binary discrete optimization problems [7]. In order to improve the optimization effect of binary PSO, a chaotic binary PSO (CBPSO) is proposed in [13] and applied to thinned array antenna pattern synthesis. [28] proposes a quantum-inspired binary PSO (QBPSO) which is based on the concept and principles of quantum computing.

[29] redefines the velocity equation of the particle and particle position update equation based on Boolean algebra instead of the method to create a mapping between real number velocity and discrete particle position. In [23], Boolean PSO proposed in [29] is applied to thinned planar array antenna pattern synthesis. [7] applies a mutation operator to the particle velocities (BPSO-vm) based on [29]. [21] combines the adaptive mutation strategy and Boolean velocity update equation and applies the improved algorithm to linear array pattern synthesis. As Boolean PSO is completely not compatible with the real number PSO, the improved achievements of real number PSO are unavailable.

Apart from 0–1 discrete optimization problem, PSO is also applied in integer optimization and mixed optimization problems [18, 30, 31]. In order to solve these problems, the real number position variable of particle in PSO must be mapped as integer variable. Different ideas are proposed in various literatures. As for binary PSO, several binary variables can be used to institute integer variables and real numbers so that mixed optimization problem can be easily solved. The disadvantage of this method is that it is hard to determine the length for bit string of the real variables. Therefore, the accuracy of optimization results will be affected [18]. In order to deal with the integer and mixed optimization problems more effectively, it is necessary to improve the PSO algorithm with better performance [18, 32].

For these discrete versions of PSO, either the mapping strategy led real number PSO search properties can not be reserved, or the modifications cause incompatibility with real number PSO. Comparing with GA or other methods, the performance of these discrete PSO is poor in dealing with the discrete optimization problems. In order to improve the performance of the PSO to handle discrete optimization problem, more appropriate mapping methods are required, at the same time, the search characteristics of the PSO should be remained as much as possible.

Another effective way is rounding and mapping real number particle position to discrete values, and round-down/round function can be generally adopted. Round-down function is adopted in [31] to round the particle position and PSO can be used for integer programming problem. [33] restricts the chromosomes of genetic algorithm between 0 and 1. By means of combining interval mapping and rounding function, mixed integer optimization of array antenna pattern and microstrip antenna can be achieved. In [34], the particle position is updated after rounding the particle velocity. However, "out-of-boundary solutions" might occur in this algorithm, which will worsen the performance of the algorithm [18].

Rounding strategy and real number PSO are combined in this paper so that PSO can solve 0–1 discrete optimization problem, integer optimization problem and mixed optimization problem. Moreover, pattern synthesis is done to thinned linear array, linear array with 4-bit digital phase shifter and unequally spaced thinned linear array. The most commonly used linearly decreasing weight PSO (LDWPSO) [35] is adopted as the real number PSO for numerical simulation. In all following examples, the control parameters of LDWPSO are same, the inertia weight $\mathbf{w}$ decreases from 0.9 to 0.4, c_1 and c_2 are 1.5.

This paper is organized as follows: Section 2 introduces the optimization strategy proposed in this paper. Section 3 synthesizes the

200-element thinned linear array. Section 4 synthesizes the 100-element linear array with 4-bit discrete phase shifter. Section 5 synthesizes the 20-element unequally spaced thinned linear array. And Section 6 is the conclusion of this paper.

2. OPTIMIZATION STRATEGY

PSO algorithm based on real number variable is adopted in this paper to deal with discrete optimization problems of array pattern synthesis. Therefore the real number particle position in PSO must be mapped as 0–1 discrete value and integer value.

As for PSO whose swarm size is *npop* and the particle position dimension is *nvar*, the particle swarm position is a $npop \times nvar$ real number matrix:

$$P = \begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,nvar} \\ v_{2,1} & v_{2,2} & \cdots & v_{2,nvar} \\ \vdots & \vdots & \ddots & \vdots \\ v_{npop,1} & v_{npop,2} & \cdots & v_{npop,nvar} \end{bmatrix}$$

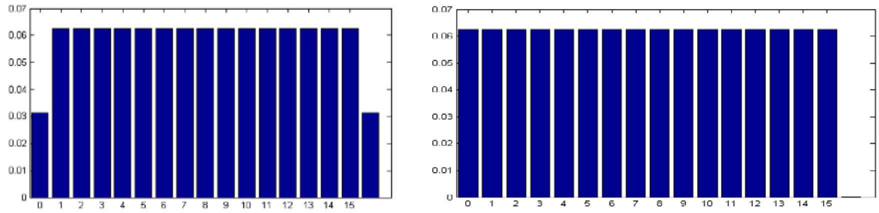
where $v_{n,m}$ is the position of particle n in dimension m and $v_{n,m}$ a real number between 0 and 1. In order to solve the discrete and mixed integer problems, $v_{n,m}$ can be mapped as a real value x_n, an integer I_n or a discrete value b_n [33].

$$\begin{aligned} x_n &= (x_{\max} - x_{\min})v_{n,m} + x_{\min} \\ I_n &= \text{rounddown}\{(I_{\max} - I_{\min} + 1)v_{n,m}\} + I_{\min} \\ b_n &= \text{round}\{v_{n,m}\} \end{aligned} \tag{1}$$

where $x_{\max}/x_{\min}$ and $I_{\max}/I_{\min}$ represent the upper and lower limits of intervals. Rounddown function represents mantissa rounding.

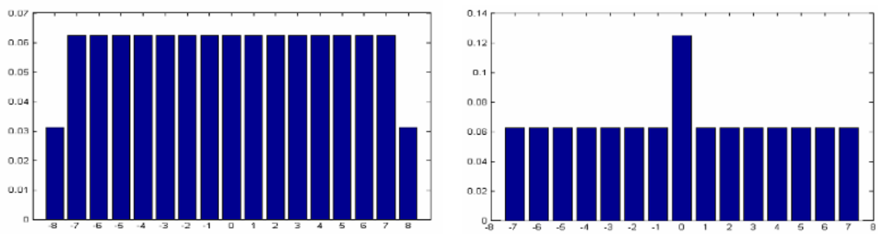
In the mapping between $v_{n,m}$ and I_n, rounddown function rather than round function is adopted in order to guarantee that each integer value can be equally selected. Meanwhile, $I_{\max}/I_{\min}$ cannot be less than 0. If the mapping is an integer less than 0, an integer offset value can be deduced after it is mapped as a positive integer. In order to further explain this problem, the 4-bit phase shifter is illustrated as an example.

As for the 4-bit digital phase shifter, the number of phase shifting angles is 16. The 16 values can be mapped by the value $v_{n,m}$ in the uniform interval $[0, 1]$. Therefore, the probability density of the values should be the same, or the performance of algorithm will be influenced. As the methods for real number rounding in round/rounddown function are different, the distribution probability of each discrete point in different intervals is also varied.



(a) (b)

Figure 1. Probability density of each integer value between 0 and 15. (a) Distribution probability of every discrete point using round function. (b) Distribution probability of every discrete point using the rounddown function.



(a) (b)

Figure 2. Distribution probability of each integer value between -7 and 7. (a) Distribution probability of every discrete point using round function. (b) Distribution probability of every discrete point using the rounddown function.

Figure 1 shows the probability density of each integer values after rounding by adopting round/rounddown function in integer interval $[0, 15]$. It can be concluded from Figure 1 that rounddown function guarantees the distribution probabilities of integer values are the same.

Figure 2 shows the situation where the negative integer is included in the interval, rounding and mapping the real numbers to integer interval $[-7, 7]$. It can be seen from Figure 2 that neither rounding methods can guarantee that the distribution probability at each integer value is the same.

Therefore, in order to map real number positions as different integers by rounding, rounddown function and positive interval must be adopted. If integers includes negative values, the positive interval shall be mapped at first and then translate it along the negative axis.

The rounding process can be done in fitness function. In this way, there is no need to modify the PSO. What we need to do is to set research range of the algorithm swarm in the interval $[0, 1]$. Thus, discrete and mixed integer optimization problems and real number PSO algorithm are related with each other, which make full use of the achievements of the real number PSO.

3. LOW-SIDELOBE PATTERN SYNTHESIS OF LINEAR THINNED ARRAY

Thinning an array means turning off some elements in an antenna array to create a desired radiation pattern [3]. An element connected to the feed network is "on", and an element connected to a matched or dummy load is "off". Generally, the intervals between array elements are the same as $\lambda/2$, and no phase shifter is used between the array element and feed. Compared to the pattern synthesis methods by adjusting amplitude, phase and position of array element, pattern synthesis with array thinning owns several advantages such as low cost and easy implementation [36, 37].

Thinned array is a discrete problem and there are many researches on the synthesis of thinned arrays using discrete coding intelligent algorithms, such as GA [3, 5], ant colony optimization (ACO) algorithm [4], binary PSO [7, 8], Boolean differential evolution algorithm (BDE) [38], iterative FFT [39].

In this example, a 200-element thinned array with structure as Figure 3 is considered. All the elements are uniformly distributed in the X-axis and are symmetric about the origin. In addition, they own the same feed amplitude and equal interval between elements.

If the element interval d is set to $\lambda/2$, its far field pattern expression follows:

$$AF(\theta, I) = \sum_{i=-N}^{-1} I_i e^{j(\pi \cdot (i+0.5) \cdot \sin(\theta))} + \sum_{i=1}^{N} I_i e^{j(\pi \cdot (i-0.5) \cdot \sin(\theta))}, \ \theta \in [0, \pi] \quad (2)$$

where I_i is the excitation amplitude and θ the angle between wave direction and the X-axis. I and θ are all scalar value. For the thinned array, when I_i is "1" means array element is turned on and vice versa.

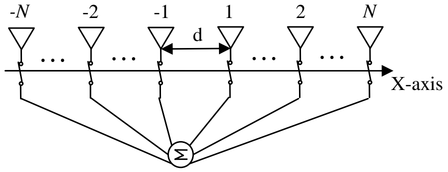


Figure 3. Centro-symmetric thinned linear antenna array.

This example has been studied in [3, 39]. In order to compare with existing research works, the peak sidelobe level (PSLL), 3 dB beamwidth (HPBW) and fill factor η are considered as optimization object. The fitness function is shown as Equation (3).

$$\begin{aligned}\text{fitness}(I) &= F_{\text{PSLL}}(I) + \Gamma_1 \{abs(\text{HPBW} - \text{HPBW}_{set}) > T_1\} \\ &\quad + \Gamma_2 \{abs(\eta - \eta_{set}) > T_2\}\end{aligned} \tag{3}$$

where HPBW_{set} is the desired 3 dB beamwidth, HPBW is the obtained 3 dB beamwidth. η and η_{set} are obtained and desired value of fill factor, $\eta =$ (number of elements turned on)/(total number of elements in array). Γ_1 and Γ_2 are penalty factor, T_1 and T_2 are used as accept threshold. In this example, Γ_1 and Γ_2 are 10 and 10^5, T_1 and T_2 are set as 0.01 and 0.005.

From Equation (3), we can know that the penalty coefficient of fill factor is very large, the algorithm will not converges if there is no filter in population initialization. The purpose of the filter is to ensure that there is at least one particle satisfying the fill factor constraint when initializing swarm positions.

The desirable peak sidelobe level (PSLL) is -23 dB, η_{set} is 77% and 78%, HPBW_{set} is 0.5°. This example is a multi-object optimization, many solutions will be discarded because of the two penalty constraints and the optimization process will be very difficult.

Table 1. Comparisons of the simulation results.

Design parameters	Result 1	Fig. 3(a) in Ref. [3]	Fig. 1(a) in Ref. [39]	Result 2	Fig. 5(a) in Ref. [3]
η	77%	77%	77%	78%	78%
PSLL (dB)	**−23.03**	−22.13	−22.92	−22.8	−22.27
HPBW (°)	0.58	0.58	0.58	0.58	0.58
Directivity (dB)	28.74	28.76	28.90	**28.95**	28.93

Table 2. Switched off elements for each result.

Results	Number of switch off elements
1	$\pm51, \pm52, \pm54, \pm61, \pm64, \pm65, \pm70, \pm71, \pm72, \pm76, \pm77, \pm79, \pm80, \pm82, \pm84, \pm85, \pm87, \pm88, \pm89, \pm91, \pm93, \pm97, \pm99$
2	$\pm53, \pm57, \pm60, \pm64, \pm65, \pm69, \pm70, \pm71, \pm73, \pm75, \pm79$ $\pm81, \pm82, \pm83, \pm84, \pm86, \pm87, \pm89, \pm92, \pm94, \pm95, \pm96$

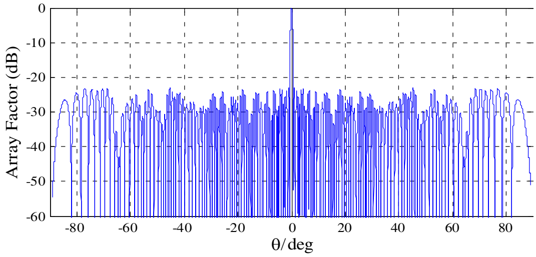


Figure 4. The pattern of result 1 in Table 2. (PSLL = -23.03 dB, fill factor = 77%, HPBW = 0.58°, Directivity = 28.74 dB).

Therefore, the particle size is set as 400, the optimization iteration is 20000, 50 independent runs are repeated.

The results of optimization strategy proposed by this paper and results of existing literatures are listed in Table 1. The best PSLL obtained is -23.03 dB and the fill factor is 77%. If fill factor increases to 78%, the obtained lowest PSLL is -22.8 dB.

The results of BBPSO, CBPSO and BPSO-vm are poor when comparison with Table 1, the best one is close to -21 dB, so these results are not listed.

Table 1 shows that the best results of this paper is superior than results in [3, 39], it indicates that the strategy of this paper can make PSO exhibit the similar performance with GA and IFT method.

Table 2 lists the the switched off element of each result in Table 1.

Figure 4 is the pattern of the best result under the 77% fill factor. Figure 5 is the distribution of PSLLs obtained by this paper and the convergence curves is shown in Figure 6. The distribution of PSLLs shows that most of optimized results are better than -21 dB,and nearly 10% results are close to -22.13 dB,which was the best result in [3].

4. PATTERN SYNTHESIS OF LINEAR ARRAY WITH 4-BIT DIGITAL SHIFTERS

Pattern synthesis for array achieved by controlling the phase of array elements is a problem being widely researched [14, 15, 17]. As for phase array, the excitation amplitudes of each array elements are the same, which decreases the production cost and excitation error.

In application of phase array, digital phase shifter is widely applied for beam forming and interference suppression. Phase shifter can be used to control the phase of array elements. Compared with phase shifter whose phase can be continuously adjusted, the application of

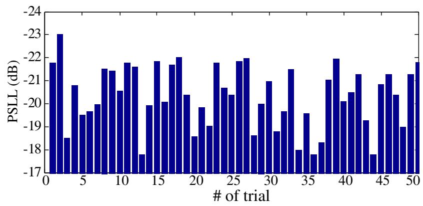


Figure 5. Distribution of PSLLs,77% filled.

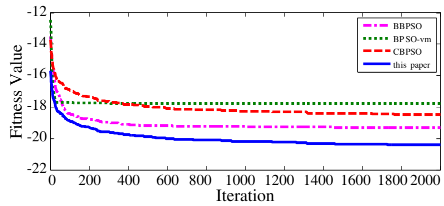


Figure 6. Convergence curves.

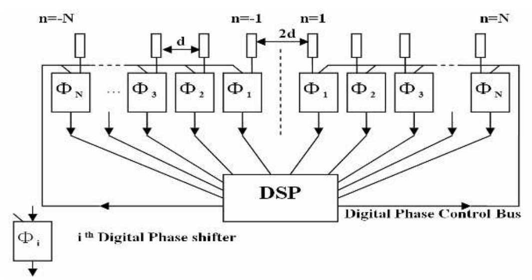


Figure 7. Centro-symmetric antenna array using digital phase shifters.

digital phase shifter is wider [14]. As for digital phase shifter, the number of phase shifting angles is limited.

While as for n bit digital phase shifter, the number of available phase shifting angles is 2^n. Thus, the essence of pattern synthesis for phase array adopting n bit digital phase shifter is an integer optimization problem.

Considering the $2N$-element linear array with 4-bit phase shifter as Figure 7 [14], the excitation amplitudes of all array elements are the same and the phases of array elements are symmetrical to the center of the array. The element spacing d on each side is 0.5λ.

As for this array, its far field pattern is as follows:

$$AF(\theta,\phi)=\sum_{i=-N}^{-1}\mathrm{e}^{\mathrm{j}(\pi\cdot i\cdot\sin(\theta)+\phi_i)}+\sum_{i=1}^{N}\mathrm{e}^{\mathrm{j}(\pi\cdot i\cdot\sin(\theta)+\phi_i)} \tag{4}$$

where ϕ_i is the excitation phase of the ith element.

Consistent with the [14], the phase shifting range of the 4-bit phase shifter adopted in this example is $[0^\circ, 114.5^\circ]$ and the step value is 7.63°.

The optimization target in this example is the pattern synthesis for 100-element array. It is required that sidelobe level shall be lower

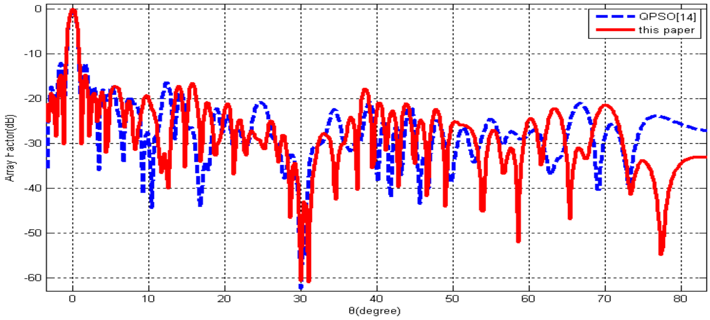


Figure 8. The optimized array pattern with minimum PSLL and null control (solid). 100 elements, null positions are 30° and 31°, PSLL = −14.7 dB, HPBW = 0.8°, directivity = 24.2 dB.

than −12 dB and the null level no lower than −60 dB shall be generated at 30° and 31°.

Therefore, the fitness value function is as shown in formula (5).

$$f(x) = \alpha \cdot \mathrm{PSLL}(AF(\theta, \phi)) + \beta \cdot \sum_{\forall \theta_j \in \psi} AF(\theta_j, \phi) \tag{5}$$

where PSLL is the peak sidelobe level of the pattern, θ_j the specified null angle value, and $\Psi$the set of all the null positions. α and β are the weighting factors of sidelobe level and null, and in this paper they are set to 0.8 and 0.2 respectively. Besides, the angular resolution is 0.1°.

When the particle size and iteration are set as 40 and 400, satisfactory results can be easily obtained by adopting the strategies in this paper and the best results are shown in Figure 8.

As for the arrays in this example, though the initial pattern (all phases of the arrays are the same) can meet the requirements that sidelobe level is not lower than −12 dB, it is quite difficult to meet the requirements for null and sidelobe as the element phase values are limited.

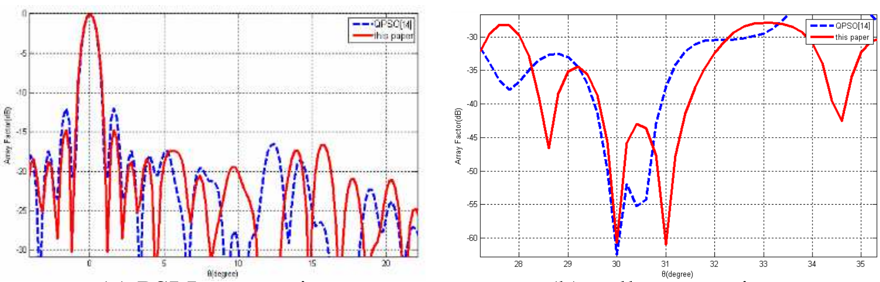
It can be seen from Figure 9(a) that the best result of sidelobe level is −14.7 dB, −2.7 dB higher than −12 dB obtained in [14]. The directivity and HPBW of the best result in this paper are 24.2 dB and 0.8°, same with the result of [14].

It can be seen from Figure 9(b) that the null position of the best result obtained by QPSO in [14] is not accurate. The null depth at 30° is −62 dB, which reaches the optimization target. The null which should appear in the position of 31° appears at 30.5°, so that the null level at 31° is −37.4 dB. Exactly speaking, the optimal solution obtained from QPSO fails to meet the optimization conditions. It can be noticed from Figure 9(b) that the optimal result obtained in this paper meets the requirements on aspects of null string position and depth.

To sum up, the optimal results obtained by strategies in this paper meets the requirements for sidelobe level and null depth, superior to the best result in [14], which shows the high performance of strategies in this paper on discrete phased array pattern synthesis.

Because [14] provides convergence curves comparison between QBPSO and BBPSO, the performance of QBPSO is better than BBPSO. From above we can know strategy proposed in this paper is much superior to QBPSO, so there is no longer available for comparison with other discrete versions of PSO.

The phase values of array elements corresponding to the best result obtained by strategies in this paper are shown in Table 3.



(a) PSLL comparison (b) nulls comparison

Figure 9. The PSLL comparison of two results.

Table 3. Phase data of each element correspond to Figure 8.

Element index	Phase Data(°)	Element index	Phase data(°)	Element index	Phase data(°)	Element index	Phase data(°)	Element index	Phase data(°)
–1/1	**68.67**	–11/11	**76.30**	–21/21	**22.89**	–31/31	**91.56**	–41/41	**99.19**
–2/2	**38.15**	–12/12	**76.30**	–22/22	**7.63**	–32/32	**106.82**	–42/42	**22.89**
–3/3	**61.04**	–13/13	**30.52**	–23/23	**15.26**	–33/33	**99.19**	–43/43	**53.41**
–4/4	**61.04**	–14/14	**76.30**	–24/24	**68.67**	–34/34	**106.82**	–44/44	**53.41**
–5/5	**76.30**	–15/15	**45.78**	–25/25	**76.30**	–35/35	**83.93**	–45/45	**7.63**
–6/6	**76.30**	–16/16	**61.04**	–26/26	**68.67**	–36/36	**7.63**	–46/46	**99.19**
–7/7	**83.93**	–17/17	**91.56**	–27/27	**99.19**	–37/37	**68.67**	–47/47	**45.78**
–8/8	**76.30**	–18/18	**106.82**	–28/28	**68.67**	–38/38	**0**	–48/48	**45.78**
–9/9	**68.67**	–19/19	**83.93**	–29/29	**68.67**	–39/39	**30.52**	–49/49	**68.67**
–10/10	**61.04**	–20/20	**38.15**	–30/30	**83.93**	–40/40	**106.82**	–50/50	**76.30**

5. LOW-SIDELOBE PATTERN SYNTHESIS OF APERIODIC THINNED ARRAY

Recently, it is a research focus to adopt evolutionary algorithm for unequally spaced linear array synthesis with minimum sidelobe level [2, 18, 20, 22]. In [18], a symmetric aperiodic thinned array with a dimension of 9.5λ is studied.

In [18], a hybrid PSO algorithm based on real number and binary variables is adopted. So, the locations and on/off status of array elements are all optimized by hybrid PSO in order to get pattern with PSLL constraint. The optimization application is a combination of thinned array and unequally spaced array and it is obviously a mixed integer optimization problem.

The structure of aperiodic thinned linear array in [18] is shown in Figure 10. The diameter of the array is 9.5λ and the array position is symmetrical on the array center. The positions of both array elements at left and right ends are fixed and remain "on".

Meanwhile, apart from the two array elements at both ends, the positions of other array elements can be adjusted. From another point of view, the example is an application of array pattern synthesis where the number and spacing of array elements are all adjustable. In fact, the pattern synthesis for the array is to find the most suitable number and spacing of array elements to minimize the sidelobe level of the array pattern.

In [18], the maximum number of elements in this array is set as 20. Except for the two constant "on" elements at both ends, the maximum number of adjustable array elements is 18. As the array is symmetrical, it is only required to work out the spaces of 9 array elements and their on/off status. Thus, the solution special dimension of the PSO is set as 18, including nine array positions and nine on/off status of arrays.

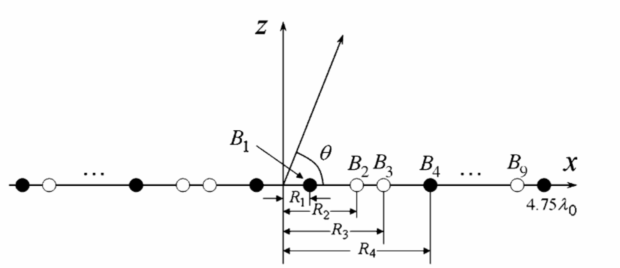


Figure 10. A symmetric aperiodic thinned array with a dimension of 9.5λ and a largest possible element number of 20. The locations and on/off status of isotropic antenna elements are represented by a real vector consisting of 18 real numbers in $[0, 1]$.

As for the single side adjustable array elements, the position interval is $[0, 4.5\lambda]$ and the array space cannot be less than 0.25λ. In all dimensions, the interval of search for PSOs is $[0, 1]$. Particle position can be transferred into array position and array status through real number and 0–1 discrete mapping in Equation (1).

The same as [18], the fitness function in this example is sidelobe level and it is a single target optimization problem. The pattern calculation formula is as follows:

$$AF(\theta, R, B) = \sum_{i=1}^{9} B_{\mathrm{i}} \cos(2\pi R_{\mathrm{i}} \cos\theta) + \cos(4.75 \times 2\pi \cos\theta), \quad \theta \in [0, \pi] \tag{6}$$

where R is the single side array position vector and R_i is the element position of element i. While B is the array status vector and B_i is the on/off status of array element i. The second part in formula (6) represents the radial component of the two fixed array elements at both ends.

In this example, the fitness function is as shown in formula (7), only the PSLL is considered.

$$\text{fitness}(I) = F_{\text{PSLL}}(I) \tag{7}$$

The particle swarm of LDWPSO is 20 and the total optimization iteration is 1000. Experiments shall be done for 20 times, which is the same as that in [18].

The pattern corresponding to the best result in this paper is shown in Figure 11 and Figure 12 is the fitness function curve. The best PSLL obtained in this paper is -23.27 dB with all the 20 array elements turned on, superior than -22.23 dB in [18]. The HPBW and directivity are $5.6°$ and 19.8 dB, similar with the $5.2°$ and 19.5 dB in [18].

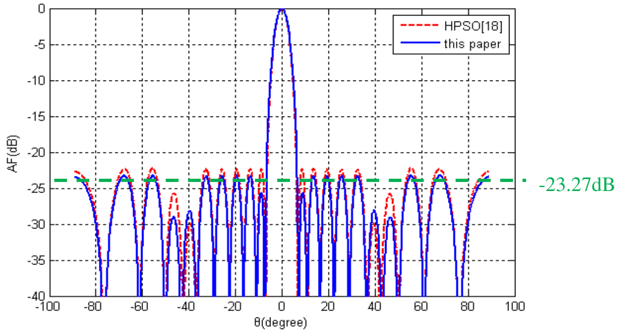
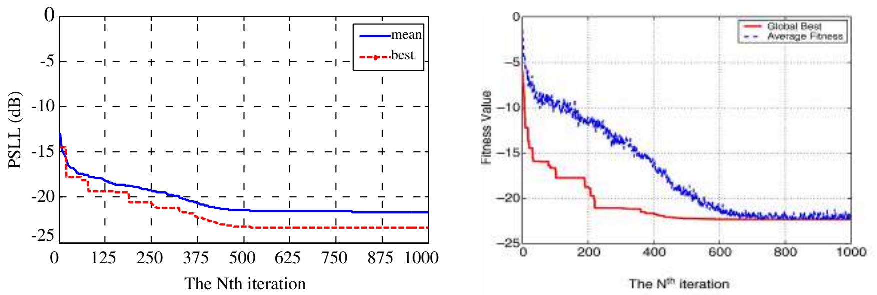


Figure 11. Best result, 20 element, PSLL = -23.27 dB.



(a) Convergence curves of this paper (b) Convergence curves of [18]

Figure 12. Convergence curves comparison.

Convergence curves in this paper and in [18] are shown in Figure 12. The array position corresponding to the best result is shown in formula (8):

$$\begin{aligned} R = \{&\pm 0.1981, \pm 0.5798, \pm 0.9276, \pm 1.3343, \pm 1.7725, \\ &\pm 2.2111, \pm 2.6856, \pm 3.2731, \pm 3.9901, \pm 4.7500\} \end{aligned} \tag{8}$$

Different with the quantity of optimal array elements in this paper, the best quantity of open array elements in [18] is 18. This is to say there are two array elements are closed. At that time, the optimal sidelobe level is -22.3 dB.

Considering that the diameter of this array is 9.5λ. If not all the twenty array elements are open, the average space between array elements are greater than 0.5λ. In order to further verify the conclusion, several simulations are done under the condition that the minimum space is 0.25λ and the maximum number of array elements is set as 40. The simulation result shows that sidelobe level larger than -23 dB can be obtained when the total number of open array elements is 20 or 21. Therefore, the results in this paper is more convective compared with [18].

6. CONCLUSION

PSO is an evolutionary algorithm based on real number variables and it is widely applied in electromagnetic field. In order to solve the discrete optimization problem, different researchers proposed several binary PSO and quantized PSO.

As these improved algorithms make some alteration which is suitable for discrete problems to the particle velocity and position update formula, there is some efficiency difference between modified algorithm and real number PSO. Meanwhile, the existing achievements of real number PSO cannot be directly utilized. It is also an effective way to transfer the real number particles into integer and binary variables with the rounding function.

Rounding and interval mapping strategy are adopted in this paper to solve 0–1 discrete, integer optimization and mixed optimization problem. And pattern synthesis is obtained in different arrays. The results are all superior to the existing research literatures, that show the effectiveness of the strategies proposed in this paper. And it can be further applied in other discrete electromagnetic optimization problems.

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