

SPECTRUM OF CHERENKOV RADIATION IN DISPERSIVE METAMATERIALS WITH NEGATIVE REFRACTION INDEX

G. Burlak*

Center for Research on Engineering and Applied Sciences, Autonomous State University of Morelos, Cuernavaca, Mor., Mexico

Abstract—We numerically studied the spectrum of Cherenkov optical radiation by a nonrelativistic anisotropic electron bunch crossing 3D dispersive metamaterial. A practically important case when such a medium is described by Drude model is investigated in details. In our theory only parameters of a metamaterial are fixed. The frequency spectrum of internal excitations is left to be defined as a result of the numerical simulation. It is found that a periodic field structure coupled to plasmonic excitations is arisen when the dispersive refractive index of a metamaterial becomes negative. In this case the reversed Cherenkov radiation is observed.

1. INTRODUCTION

Recently emerging fields of metamaterials promise a variety of applications in nanophotonics and plasmonics with the potential for faster information processing. The investigations of optical negative-index [1] metamaterials (NIM) using the nanostructured metal-dielectric composites already have led to both fundamental and applied achievements that have been realized in various structures [2–21].

The main applications of negative index metamaterials (or lefthanded materials (LHM)) are connected with a remarkable property: the direction of the energy flow and the direction of the phase velocity are opposite in NIM that results unusual properties of electromagnetic waves propagating in these mediums. The negative real part of the refractive index is typically observed together with strong dispersion, so that in general the absorption cannot be disregarded. Recently, authors of [12] have demonstrated that the

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* Corresponding author: Gennadiy Burlak (gburlak@uaem.mx).

incorporation of gain material in a metamaterial allows fabricating very low-loss NIM. Thus, the original loss-limited negative refractive index can be drastically improved with loss compensation in the visible wavelength range [4].

Cherenkov radiation by a charged source that moves in (or in the interface) a left-handed material has been studied in number of works [15–21]. Both experimental and theoretical frameworks are investigated, see review [16] and references therein.

In this paper the Cherenkov optical radiation in 3D metamaterials with a special emphasis on the dispersive properties of the medium is numerically studied. To do that we performed the FDTD simulations with the use of the material parameters, however without references to the operational frequency range.

2. BASIC EQUATIONS

In metamaterials, it is necessary to treat electromagnetic wave (EMW) interactions with a metal ingredient using a dispersive formulation that allows correct description of the internal electron dynamics. In this paper we exploit the Drude model that became widely used for modeling in complex materials where for a range of frequency the negative refraction index n is expected. The Maxwell equations read

$$\nabla \times \mathbf{E} = -\mu_0\mu_h \frac{\partial \mathbf{H}}{\partial t} - \mathbf{J}_m - \sigma_m \mathbf{H}, \quad (1)$$

$$\nabla \times \mathbf{H} = \varepsilon_0\varepsilon_h \frac{\partial \mathbf{E}}{\partial t} + q\mathbf{v}_0 f(r, t) + \mathbf{J}_e + \sigma_e \mathbf{E}, \quad (2)$$

where \mathbf{J}_e is the electrical current and \mathbf{J}_m the magnetic current which obey the following material equations

$$\dot{\mathbf{J}}_e + \gamma_e \mathbf{J}_e = b_e \mathbf{E}, \quad \dot{\mathbf{J}}_m + \gamma_m \mathbf{J}_m = b_m \mathbf{H}, \quad (3)$$

here γ_e and γ_m are the electrical and magnetic collision frequencies respectively. $b_e = \varepsilon_0\omega_{pe}^2$, $b_m = \mu_0\omega_{pm}^2$, ω_{pe} and ω_{pm} are frequencies of electric and magnetic plasmons respectively. σ_e and σ_m are conductivities. ε_h and μ_h are dielectric and magnetic functions of the host medium respectively [22, 23]. For metals such as aluminum, copper, gold, and silver, the density of the free electrons is on the order of 10^{23} cm^{-3} . The typical value $\omega_{pe} \approx 2 \cdot 10^{16} \text{ s}^{-1}$ ([24], p.44). In a metamaterial with fishnet structure [12] we consider the bunch of electrons (charge q) moving with a velocity parallel to x direction: $\mathbf{v}_0 \parallel \hat{\mathbf{e}}_x$ and the density of the bunch is defined by the anisotropic Gaussian as $f(r, t) = W^{-3} \exp\{ -[(x-v_0t)^2 + y^2/q_2^2 + z^2/q_2^2]/W^2 \}$, where $q_2 \geq 1$ is const, W is the bunch width; it is interesting that at $W \rightarrow 0$

such a distribution can be simplified to the isotropic point-source distribution $f(r, t) \rightarrow (\pi)^{3/2} \delta(x - v_0 t) \delta(y) \delta(z)$. Further for simulations we use dimensionless variables, where for renormalization are used the vacuum light velocity $c = (\varepsilon_0 \mu_0)^{-0.5}$ and the typical spatial scale $l_0 = 75$ nm. With such a normalization, e.g., above indicated the metal plasma frequency becomes $\omega_{pe} = 5$. The electrical and magnetic fields are renormalized with the electrical scale $E_0 = ql_0 \varepsilon_0$ and magnetic scale $H_0 = (\varepsilon_0 / \mu_0)^{0.5} E_0$ correspondingly. Some metamaterials exhibit anisotropic properties with tensor permittivity and permeability. To seek for simplicity in this paper we concentrated in the isotropic geometry. Modeling anisotropic medium is a straightforward extension of this model, see details in Refs. [16, 21].

The idea of our simulations is as follows. In optical experiments normally we can refer only to the parameters of material ($\gamma_e, \sigma_e, \omega_{pe}$, and $\gamma_m, \sigma_m, \omega_{pm}$). It is of significant interest to consider Cherenkov radiation in such a system, proceeding from simple principles, using only material parameters and without of references to the operational frequency band. In this situation the frequency spectrum of internal excitations ω must be left as a free parameter that has to be defined from simulations by a self-consistent way. For 3D dispersion material such a problem becomes too difficult for analytical consideration. Therefore in this paper the standard numerical algorithms in the time domain [FDTD and ADE (auxiliary differential equation) [25]] were used. For our numerical simulations was employed the program scheme [26] that we have converted to visual C# code and adapted for our purposes.

We consider a general 3D case in Cartesian coordinates since such a geometry normally is used on the optical investigations [12]. We examine a spatially averaged metamaterial composition: nanostructured metal-dielectric composites (fishnet), similarly that was used in the experiment [12]. In this case the spatial average scale is less then the infrared (IR) and visual wavelengths, so we can deem that the material (dielectric and magnetic) dispersion is allowed by the Drude model and the role of the active dielectric ingredient is reduced to a compensation of losses due to the metal ingredient. In our simulations numerical grid L^3 with $L = 100, 120, 150$ was used. To avoid the no-physical wave reflections in the numerical 3D boundaries the standard analytical absorbing boundary conditions (ABC) were employed (see details in [25] Chapter 6). It is found that already the use of first order ABC allows us to reach the acceptable small level of the field reflection (a few percents) at a reasonable time-consuming computation. We have used PC with Intel Duo CPU with 2.40 GHz and 4 GB RAM. For 3D numerical grid with L^3 , $L = 100$ the typical time of FDTD simulation

was 15 minutes.

In our approach the following steps have been used: (i) In first one we calculated the time-spatial field dynamics that is raised by the crossing electron bunch, Equations (1)–(3). (ii) In second step we apply the Fourier analysis for the time dependencies calculated in the first step in order to reveal the spectrum of internal excitations. The following dimensionless parameters were used in our simulations: $\omega_{pe} = 5$, $\omega_{pm} = 7$, $\varepsilon_h = 1.44$, $\mu_h = 1$, $\gamma_e = \gamma_m = 10^{-4}$, $\sigma_e = \sigma_m = 10^{-7}$, $W = 3$, $q_2 = 2$. We varied the velocity v_0 to study different regimes of

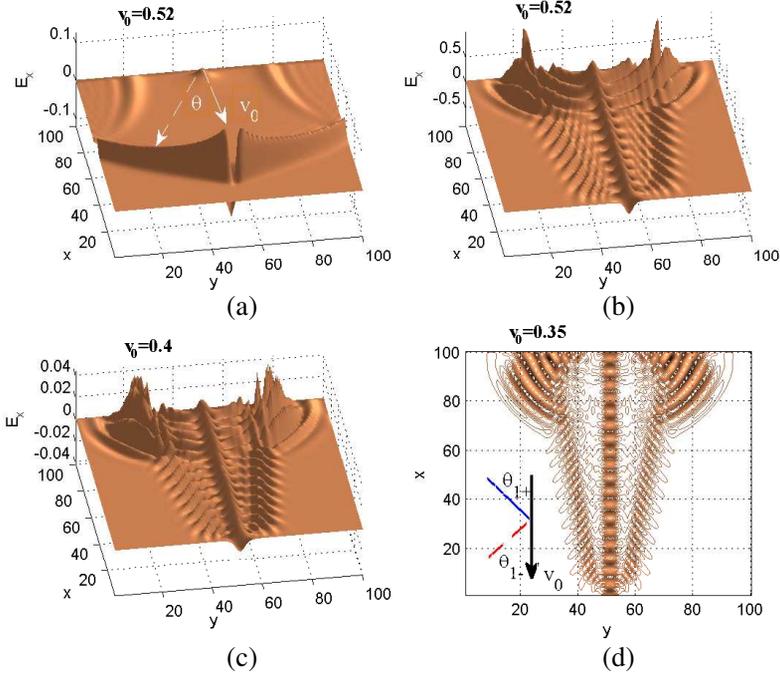


Figure 1. (color online.) Snapshots of the field component $E_x(\mathbf{r}, t)$ in plane $(x, y, z = 80)$ for metamaterial with $\varepsilon_h = 1.44$, $\mu_h = 1$ at time $t = 200$ and various particle velocity v_0 . In (a) shown the case of dispersiveless dielectric (θ is a light emission angle) with $\omega_{pe} = \omega_{pm} = 0$, $v_0 = 0.52$ and $\text{Re}(n) = 1.2$, while in panels (b), (c), (d) are shown the cases of metamaterial with $\omega_{pe} = 5$, $\omega_{pm} = 7$. (b) $v_0 = 0.52$; (c) $v_0 = 0.4$; (d) $v_0 = 0.35$. The inset in (d) shows the complementary angle $\theta_{1\pm}$ for cases $\text{Re}(n) > 0$ and $\text{Re}(n) < 0$ respectively. The oscillations in the top of the figures exhibit the shock waves (bremsstrahlung) arising by charged bunch at the beginning of motion (see [27], Chapter 2).

the Cherenkov radiation. Our results are shown in Figures 1–3.

For a conventional dispersiveless dielectric the emission angle θ (see Figure 1(a) that is the angle between the direction of wave propagation and charge velocity) is given by [22] $\cos(\theta) = c/nv_0$. In what follows we will use other complementary angle $\theta_1 = \pi/2 - \theta$. For such angle we have $\cos(\theta) = \sin(\theta_1) = c/nv_0$, so for conventional material (with $\text{Re}(n) > 0$) θ_1 is positive $\theta_1 = \theta_{1+} > 0$, while for negative refraction index metamaterial NIM (with $\text{Re}(n) < 0$) θ_1 is negative $\theta_1 = \theta_{1-} < 0$, see inset in Figure 1(d).

The normalized numerical velocity of the propagating field in 3D homogeneous numerical grid is $dl/dt = \sqrt{3}dx/dt$ (with $dx = dy = dz$) [25]. In what follows we will refer to $dx/dt = (1/\sqrt{3})dl/dt = c_n \equiv 0.577$ as normalized “*vacuum light velocity*” in the numerical grid, such that $c_n > v_0$. Other important velocity is $c_{cr} = c_n/|n|$ corresponding to the critical velocity of bunch v_0 when the Cherenkov radiation appears in a dispersiveless dielectric with refraction index n . We note that in a dispersive medium c_{cr} is a frequency depending parameter.

First we have simulated the Cherenkov radiations in conventional dispersiveless dielectric, when in Equations (1)–(3) $\omega_{pe} = 0$, $\omega_{pm} = 0$ and $n = 1.2$ are. The result is shown in Figure 1(a) for $c_n > v_0 = 0.52 > c_{cr} = 0.475$. From Figure 1(a) we can observe the sharply defined Cherenkov wave front with the angle $\theta_1 = 67.67^\circ$ that corresponds to specified n and v_0 with the accuracy about 1%.

The field dynamics that emerges at different values velocity v_0 in metamaterials with no-zero ω_{pe} , ω_{pm} is shown in Figures 1(b), (c), (d) and exhibits some representative cases. First we studied the case with $\omega_{pe} < \omega_{pm}$ ($\omega_{pe} = 5$, $\omega_{pm} = 7$), see details in Figures 1(b), (c), (d). Comparison of Figures 1(a) and (b), (c), and (d) shows that the field structures for dispersiveless dielectric and dispersive metamaterial are very different. Figures 1(b), (c), (d) show the formation of a periodic field structures in metamaterial. For case when $\omega_{pe} < \omega_{pm}$ we clearly observe the formation of the wave fronts with negative emission angle in area closely to the output of system.

In order to see what type of internal dynamics occurs in such a system, further we study the time dependence of the field strengths $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{H}(\mathbf{r}, t)$ in some fixed point \mathbf{r}_0 . Figure 2 shows such a dynamics in point $\mathbf{r}_0 = (50, 50, 80)$.

We observe from Figures 2(a), (b) that the field amplitudes still have oscillating behavior, although the charged bunch already has left the system. The Fourier analysis (see Figure 2(d)) shows that the spectrum of such oscillating motion has narrow peak at frequency $\omega = 4.08$ that is in the area of the plasmon frequency $\omega_{pe} = 5$. In case $\omega_{pe} < \omega_{pm}$ such a behavior weakly depends on ω_{pm} value.

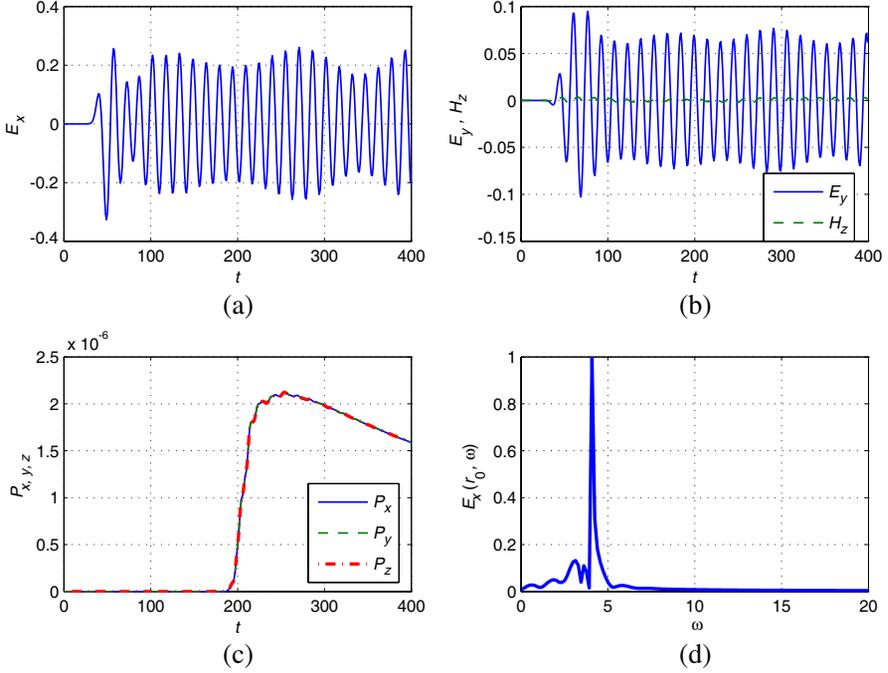


Figure 2. (color online.) Time dynamics of fields in point $\mathbf{r}_0 = (50, 50, 80)$ of metamaterial with L^3 , $L = 100$. (a) $E_x(\mathbf{r}_0, t)$; (b) $E_y(\mathbf{r}_0, t)$ and $H_z(\mathbf{r}_0, t)$; (c) Components of Poynting vector $P_x(t)$, $P_y(t)$, $P_z(t)$ in output at plane $x = 0$; (d) Fourier spectrum of $E_x(\mathbf{r}_0, t)$ having a resonance at $\omega_0 = 4.08$.

Figure 2(c) shows the dynamics of the time averaged Poynting vector \mathbf{P} in the output plane. We observe that such flux (as expected) arrives the output with some retardation time Δt equal to the time motion the bunch through the system $\Delta t = L/v_0$. The amplitudes $P_{x,y,z}$ slowly reduce due to radiation of the excited plasmons out the system. Since $P_i \neq 0$, we conclude that such excitations correspond to propagating plasmons rather than static plasma oscillations at frequency ω_{pe} .

Finally, it is of interest to calculate the value that the refractive index of metamaterial $n(\omega)$ has for the sharp frequency resonance $\omega_0 = 4.08$ in Figure 2(d). In the frequency domain the Drude dispersive permittivity and permeability have the following form ([25], Chapter 9) $\varepsilon(\omega) = \varepsilon_h - \omega_{pe}^2/(\omega^2 + i\gamma_e\omega)$ and $\mu(\omega) = \mu_h - \omega_{pm}^2/(\omega^2 + i\gamma_m\omega)$. For such $\varepsilon(\omega)$ and $\mu(\omega)$ the complex refractive index for the NIM metamaterial

can be written as [28]

$$n(\omega) = \sqrt{|\varepsilon(\omega)\mu(\omega)|}e^{i[\phi_\varepsilon(\omega)+\phi_\mu(\omega)]/2}. \quad (4)$$

As a result of straightforward substituting the resonant frequency $\omega_0 = 4.08$ into $\varepsilon(\omega)$, $\mu(\omega)$ and then in $n(\omega)$ Equation (4) we obtain $n = -0.33 - i10^{-5}$; thus, such a resonance (as well as its vicinity) corresponds to the negative refraction index of the metamaterial (NIM); in this case $\text{Re}(n(\omega)) = -0.33$.

In the above the cases with $\omega_{pe} < \omega_{ph}$ are investigated with details. The generation of the plasmons coupled to free electromagnetic waves (EMW) having negative phase velocity is found, see figures Figures 1 and 2. It was established out that the refractive index n is negative

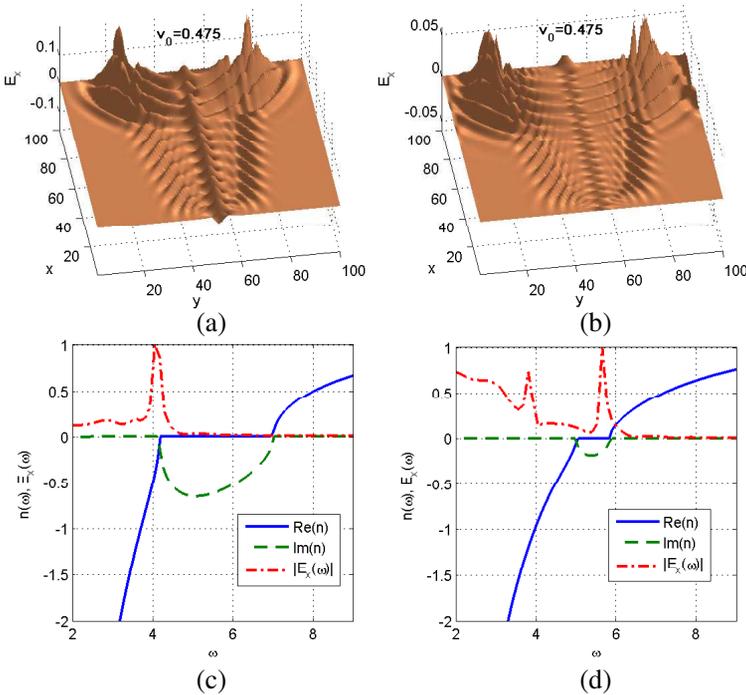


Figure 3. (color online.) (a), (b) Snapshots of the field component $E_x(\mathbf{r}, t)$ in plane $(x, y, z = 80)$ for metamaterial with $\varepsilon_h = 1.44$, $\mu_h = 1$ at time $t = 400$, particle velocity $v_0 = c_{cr} = c_c/n_h = 0.475$: (a) $\omega_{pm} = 7 > \omega_{pe} = 5$; (b) $\omega_{pm} = 5 < \omega_{pe} = 7$. Frequency dependence of the complex reflection index $n(\omega)$ and $E_x(\mathbf{r}_0, \omega)$ for (c) $\omega_{pm} = 7 > \omega_{pe} = 5$; (d) $\omega_{pm} = 5 < \omega_{pe} = 7$, where point $\mathbf{r}_0 = (50, 50, 80)$. Maximal frequency peaks of $E_x(\mathbf{r}_0, \omega)$ are in (c) at $\omega_0 = 0.408$, and (d) at $\omega_0 = 0.565$ respectively. See details in the text.

for the resonant frequency of such excitations. The following question emerges, whether such a state (field + plasmons) will have the same behavior for the alternative relation when $\omega_{pe} > \omega_{ph}$? It is instructively to compare the spectra for indicated situations. The answer can be seen by the inspection of Figure 3 where the case of velocity v_0 having critical value $v_0 = c_{cr} = c_n/n_h$ is shown. We observe from Figures 3(a), (c) that for $\omega_{pe} < \omega_{ph}$ the EMW still are generated closely to the plasmonic frequency area and the emission angle clearly has a negative value. Fourier transformation in Figure 3(c) shows that in this situation the frequency resonance for $E_x(\mathbf{r}_0, \omega)$ again rises up at $\omega_0 \approx 4.08$ where corresponding refraction index is $n(\omega_0) = -0.34 - 0.00056i$, so it is pronounced NIM case. However, as one can see from Figures 3(b), (d) the opposite relation $\omega_{pe} > \omega_{ph}$ leads that in Figure 3(b) EMW have smaller amplitudes regarding to Figure 3(a). Furthermore, $E_x(\mathbf{r}_0, \omega)$ has maximal resonance at $\omega_0 = 5.65$ where the refraction index $n(\omega_0) = 8.1 \cdot 10^{-5} - 0.14i$ is almost pure imaginary one. The latter represents a medium with essential dissipation of EMW rather than NIM. We can conclude that the reversed Cherenkov radiation can be consistently observed in metamaterials with $\omega_{pe} < \omega_{ph}$ relation.

3. CONCLUSION

We numerically study the frequency spectrum of the Cherenkov optical radiation of a charged anisotropic bunch crossing 3D dispersive metamaterial with small losses. A practically important case when such a metallo-dielectric metamaterial is described by Drude model for both dielectric permittivity and magnetic permeability is investigated. In our approach only parameters of a metamaterial are fixed. The spectrum of the internal excitations was defined in a result of numerical simulation. It is found that for $\omega_{pe} < \omega_{ph}$ a periodic field structure coupled to plasmonic excitations is arisen for resonances when the dispersive refractive index of the metamaterial becomes negative. In this case the reversed Cherenkov radiation is observed. This effect opens new interesting possibilities in various applications metamaterials in nanophotonics with the potential for deep control and quick information processing.

REFERENCES

1. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ ," *Sov. Phys. Usp.*, Vol. 10, 509, 1968. [*Usp. Fiz. Nauk.*, Vol. 92, 517, 1967].

2. Shalaev, V. M., "Optical negative-index metamaterials," *Nature Photonics*, Vol. 1, 41–48, 2007.
3. Soukoulis, C. M. and M. Wegener, "Past achievements and future challenges in the development of three-dimensional photonic metamaterials," *Nature Photonics*, Vol. 5, 523–530, 2011.
4. Hess, O., J. B. Pendry, S. A. Maier, R. F. Oulton, J. M. Hamm, and K. L. Tsakmakidis, "Active nanoplasmonic metamaterials," *Nature Materials*, Vol. 11, 573–584, 2012.
5. Chen, H., C. T. Chan, and P. Sheng, "Transformation optics and metamaterials," *Nature Materials*, Vol. 9, 387–396, 2010.
6. Gordon, J. A. and R. W. Ziolkowski, "CNP optical metamaterials," *Opt. Express*, Vol. 16, 6692–6716, 2008.
7. Milton, G. W., "Realizability of metamaterials with prescribed electric permittivity and magnetic permeability tensors," *New Journal of Physics*, Vol. 12, 033035, 2010.
8. Podolskiy, V., A. Sarychev, and V. Shalaev, "Plasmon modes and negative refraction in metal nanowire composites," *Optics Express*, Vol. 11, No. 7, 735–745, 2003.
9. Shalaev, V. M., W. Cai, U. K. Chettiar, H.-K. Yuan, A. K. Sarychev, V. P. Drachev, and A. V. Kildishev, "Negative index of refraction in optical metamaterials," *Opt. Lett.*, Vol. 30, No. 24, 3356–3358, 2005.
10. Burlak, G., A. Díaz-De-Anda, R. S. Salgado, and J. P. Ortega, "Narrow transmittance peaks in a multilayered microsphere with a quasiperiodic left-handed stack," *Optics Commun.*, Vol. 283, No. 19, 3569–3577, 2010.
11. Burlak, G. and A. Díaz-De-Anda, "The field confinement, narrow transmission resonances and Green function of a multilayered microsphere with metamaterial defects," *Journal of Atomic, Molecular, and Optical Physics*, Vol. 2011, 1–13, 2011.
12. Xiao, S., V. P. Drachev, A. V. Kildishev, X. Ni, U. K. Chettiar, H.-K. Yuan, and V. M. Shalaev, "Loss-free and active optical negative-index metamaterials," *Nature*, Vol. 466, 735–738, 2010.
13. Deb, S. and S. Dutta Gupta, "Absorption and dispersion in metamaterials: Feasibility of device applications," *J. Phys.*, Vol. 75, No. 5, 2010.
14. Cherenkov, P. A., "Visible emission of clean liquids by action of γ -radiation," *Dokl. Akad. Nauk.*, Vol. 2, 451–454, 1934.
15. Averkov, Yu. O. and V. M. Yakovenko, "Cherenkov radiation by an electron bunch that moves in a vacuum above a left-handed material," *Phys. Rev. B*, Vol. 79, 193402–193412, 2005.

16. Duan, Z., B.-I. Wu, S. Xi, H. Chen, and M. Chen, "Research progress in reversed Cherenkov radiation in double-negative metamaterials," *Progress In Electromagnetics Research*, Vol. 90, 75–87, 2009.
17. Xi, S., H. Chen, T. Jiang, L. Ran, J. Huangfu, B.-I. Wu, J. A. Kong, and M. Chen, "Experimental verification of reversed cherenkov radiation in left-handed metamaterial," *Phys. Rev. Lett.*, Vol. 103, 194801, 2009.
18. Averkov, Yu. O., A. V. Kats, and V. M. Yakovenko, "Electron beam excitation of left-handed surface electromagnetic waves at artificial interfaces," *Phys. Rev. B*, Vol. 72, 205110–205114, 2005.
19. Zhou, J., Z. Duan, Y. Zhang, M. Hu, W. Liu, P. Zhang, and S. Liu, "Numerical investigation of Cherenkov radiations emitted by an electron beam bunch in isotropic double-negative metamaterials," *Nuclear Instruments and Methods in Physics Research Section A*, Vol. 654, No. 1, 475–480, 2011.
20. Duan, Z., Y. Wang, X. Mao, W.-X. Wang, and M. Chen, "Experimental demonstration of double-negative metamaterials partially filled in a circular waveguide," *Progress In Electromagnetics Research*, Vol. 121, 215–224, 2011.
21. Duan, Z., C. Guo, and M. Chen, "Enhanced reversed Cherenkov radiation in a waveguide with double-negative metamaterials," *Opt. Express*, Vol. 19, 13825–13830, 2011.
22. Jackson, J. D., *Classical Electrodynamics*, John Wiley and Sons, 1975.
23. Oughstun, K. E., *Electromagnetic and Optical Pulse Propagation 2: Temporal Pulse Dynamics in Dispersive, Attenuative Media* (Springer Series in Optical Sciences), Springer, 2009.
24. Yeh, P., *Optical Waves in Layered Media*, John Wiley and Sons, New York, 1988.
25. Taflove, A. and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, Artech House, Boston, 2005.
26. Schneider, J., *Understanding the Finite-Difference Time-Domain Method*, www.eecs.wsu.edu/~schneidj/ufdtd, 2010.
27. Afanasiev, G. N., *Cherenkov Radiation in a Dispersive Medium, Vavilov-Cherenkov and Synchrotron Radiation, Fundamental Theories of Physics*, Kluwer Academic Publishers, 2004.
28. Ziolkowski, R. W., "Superluminal transmission of information through an electromagnetic metamaterial," *Phys. Rev. E*, Vol. 63, 046604, 2001.