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A UNEQUAL COUPLED-LINE WILKINSON POWER DIVIDER FOR ARBITRARY TERMINATED IMPEDANCES

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Abstract—A unequal Wilkinson power divider based on asymmetrical coupled line section is presented in this paper. The proposed unequal Wilkinson power divider topology uses asymmetrical coupled-line section and two transmission-line transformers. For simplification, a section asymmetrical coupled line is applied to obtain front compact matching structure. For generalization, three ports terminated impedances are defined as arbitrary values. In particular, the output ports impedances are arbitrary complex values. Design parameters and analysis equations for scattering parameters are also provided. In addition, the theoretical external performances of several examples are illustrated. Finally, the EM simulated results are validated with the characterization of two microstrip unequal power dividers at 2 GHz.

1. INTRODUCTION

Miniaturization of microwave components can reduce the occupied size. Thus, compact active and passive components are preferred in low-cost systems. Wilkinson power divider [1], as a key device in wireless front-ends, is widely used in array antenna systems and power amplifiers. It can provide effective equal power-dividing function and ideal isolation and ports-matching performances, simultaneously. In recent years, many efforts have been made to develop high-performance power dividers [2–7]. After decreasing the inherent circuit size of the conventional Wilkinson power divider, a coupled-line section is used to design compact power divider in [8]. From the analyzed results of [8], it can be found that the odd-mode impedance of the coupled line is a free design variable. Therefore, the design tradeoffs among

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the output-ports matching, isolation and the odd-mode impedance can be easily achieved. In addition, Ekinge proposed a matched wide-band three-ports in [9]. This kind of power divider consists of n lossless asymmetrical coupled-line sections. In fact, the proposed theory in [9] includes the design approach in [8] conceptually. In addition, three-port 3-dB Wilkinson power dividers terminated in arbitrary impedances are described in [10]. However, the asymmetrical coupled-line sections are not considered. The additional transmission losses are inserted and the occupy size is still large in these modified dividers. Recently, dual-band Wilkinson power dividers are also be investigated by authors [11–13]. It is necessary to point out that one of these dividers uses couple line structures [12]. Obviously, the coupled-line dual-band power divider has compact size. However, the proposed circuit in [12] is symmetrical, only suitable for equal power dividers.

In this paper, asymmetrical coupled-line sections are considered in the usage of unequal power dividers to obtain reduced-size circuit layout. The simplified even- and odd-mode parameters are applied to synthesize and analyze this proposed power divider. Three terminated impedances are defined as arbitrary values (complex values for output ports). Thus, this proposed coupled-line power divider not only has unequal power division, but also satisfies three arbitrary terminated impedances. Furthermore, closed-form equations for synthesizing characteristic impedance and analyzing external performances (without including the output impedance transformers) are provided. Finally, several numeral examples are presented to verify our proposed design theory. According to the design guideline, two 2-GHz microstrip Wilkinson power dividers (with 2-dB and 3-dB power-dividing ratio, respectively) are designed, fabricated, and measured for demonstration. There are good agreements between EM simulation and measurement.

2. ANALYSIS OF THE UNEQUAL COUPLED-LINE POWER DIVIDER

The proposed unequal coupled-line power divider is shown in Figure 1. As shown in this figure, the characteristic impedances and the electrical length of the asymmetric coupled lines are referred to as Z_{e1}, Z_{e2}, Z_{o1}, Z_{o2}, and θ_1, respectively. This definition for electrical parameters of asymmetric coupled lines is simpler than that in the paper [14]. The port impedances of this proposed divider are defined as Z_a, Z_b, and Z_c, which are three arbitrary values. Additional transmission lines with characteristic impedances Z_2, Z_3 and electrical lengths θ_2, θ_3 are used to match equivalent impedances R_2, R_3 to arbitrary terminated

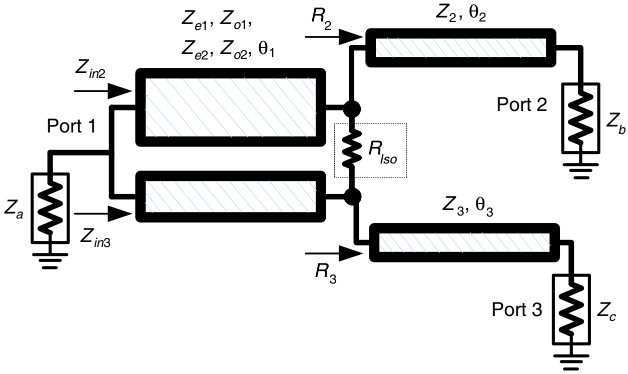


Figure 1. Detail of the proposed single-section unequal coupled-line power divider.

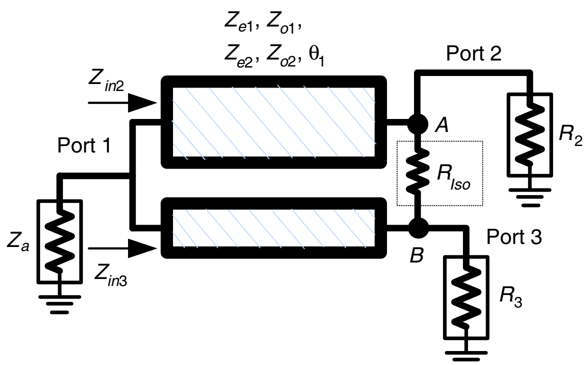


Figure 2. The simplified circuit of the proposed unequal coupled-line power divider.

impedances Z_b, Z_c. In fact, the values of Z_b and Z_c can be arbitrary complex impedances in this proposed unequal (or equal) coupled-line power divider. However, for simplification, the input-port impedance Z_a is considered as real-value impedance for this divider. The isolation structure for this new divider is a single resistor with the resistance value of R_{Iso}.

For analyzing this power divider easily, the simplified circuit shown in Figure 2 is considered. We assume that the power-dividing ratio of the port 2 to the port 3 is expressed as $P_2/P_3 = k^2$. In addition, the equivalent impedances R_2, R_3 in Figure 2 are defined as two real impedances, and their resistance values satisfy the following equation [10, 13]

$$R_3 = k^2 R_2. \tag{1}$$

Under the special assumption of (1), there are no currents through the isolation resistor R_{Iso}. Therefore, when the microwave signals input from the port 1 in Figure 1, they can be divided into two separated unequal-magnitude signals without any losses. To obtain accurate unequal power-dividing performance, the input equivalent impedances Z_{in2} and Z_{in3} can be given by [10, 13]

$$\begin{cases} Z_{in2} = \frac{1+k^2}{k^2} Z_a, \\ Z_{in3} = \left(1 + k^2\right) Z_a. \end{cases} \tag{2}$$

Obviously, the rigorous relationship that $Z_{in3} = k^2 Z_{in2}$ in (2) is satisfied. For analysis, the presented circuit shown in Figure 2 can be simplified as four equivalent parts, as depicted in Figure 3. At first, the even-mode circuits shown in Figures 3(a) and (b) are analyzed. In order to obtain ideal matching and accurate unequal power division when the electrical length θ_1 is 0.5π at the operating frequency f_o, the even-mode characteristic impedances of asymmetrical couple line can be obtained as

$$Z_{e1} = \sqrt{Z_{in2} R_2} = \sqrt{\frac{1+k^2}{k^2} Z_a R_2}, \tag{3a}$$

$$Z_{e2} = \sqrt{Z_{in3} R_3} = \sqrt{\left(1+k^2\right) Z_a k^2 R_2} = k^2 Z_{e1}. \tag{3b}$$

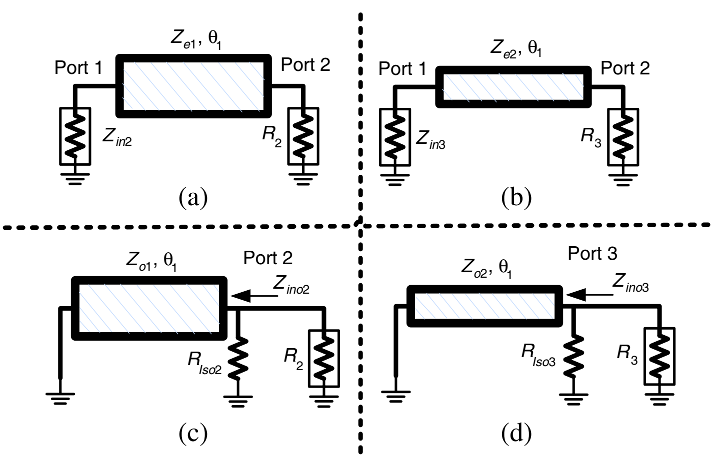


Figure 3. The equivalent simplest circuit for even- and odd-mode analysis: (a), (b) even-mode and (c), (d) odd-mode.

Furthermore, the odd-mode characteristic impedances and isolation resistors can be determined from Figures 3(c) and (d). Because that the electrical length θ of this couple line is 0.5π at the operating frequency, the couple lines in Figures 3(c) and (d) can be regarded as open circuits. Thus, the resistance values of isolation resistors R_{Iso2} and R_{Iso3} can be derived as

$$\begin{cases} R_{Iso2} = R_2, \\ R_{Iso3} = R_3 = k^2 R_2. \end{cases} \tag{4}$$

According to (4), the total isolation resistor R_{Iso} in Figures 1 and 2 can be calculated by

$$R_{Iso} = R_{Iso2} + R_{Iso3} = \left(1 + k^2\right) R_2. \tag{5}$$

It is interesting that Z_{o1} and Z_{o2} in this divider are independent variables for the ideal performance at the operating center frequency f_o. Their values, which can be adjusted according to practical requirements, will influence the operating bandwidth of this kind of divider. In addition, the terminated ports impedances are defined as

$$\begin{cases} Z_b = R_b + jX_b, \\ Z_c = R_c + jX_c. \end{cases} \tag{6}$$

For the output matching networks including Z_2 and Z_3, the characteristic impedances and electrical lengths should be chosen to match R_2 and R_3 to Z_b and Z_c. Their values are expressed as [15, 16]

$$Z_2 = \sqrt{R_2 R_b - \frac{X_b^2 R_2}{(R_2 - R_b)}}, \tag{7a}$$

$$Z_3 = \sqrt{R_3 R_c - \frac{X_c^2 R_3}{(R_3 - R_c)}}, \tag{7b}$$

$$\theta_2 = \arctan\left[\frac{Z_2(R_2 - R_b)}{R_2 X_b}\right], \tag{7c}$$

$$\theta_3 = \arctan\left[\frac{Z_3(R_3 - R_c)}{R_3 X_c}\right], \tag{7d}$$

The practical electrical lengths of the output matching networks should be obtained from the following equation:

$$\theta_{pi} = \begin{cases} \theta_i, & \theta_i > 0 \\ \theta_i + \pi, & \theta_i \leq 0 \end{cases}, \quad i = 2, 3. \tag{8}$$

Here, the design method of the proposed unequal coupled line power divider has been presented. However, the analysis of this power divider is not easy because the asymmetrical coupled-line section does not have electrical parameters-based transmission line models without physical dimensions in common microwave simulation tools. Therefore, the accurate and lossless scattering parameters cannot be simulated from these common simulation tools. For convenient analysis, the analytical mathematical expressions for scattering parameters are given in the following paragraph.

If the odd-mode characteristic impedances satisfy the relationship that $Z_{o2} = k^2 Z_{o1}$, there are two equivalent groups of figures, they are the group of Figures 3(a) and (b) and another group of Figures 3(c) and (d). For Figure 3(a), the $ABCD$ matrix of the equivalent transmission line with characteristic impedance [10] is

$$\begin{bmatrix} A_{ea} & B_{ea} \\ C_{ea} & D_{ea} \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & jZ_{e1}\sin(\theta_1) \\ \frac{j\sin(\theta_1)}{Z_{e1}} & \cos(\theta_1) \end{bmatrix}. \quad (9)$$

According to the relationship between the $ABCD$ matrix and the scattering parameters [10], the scattering parameters of the circuit in Figure 3(a) can be obtained as

$$\begin{bmatrix} S_{11ea} & S_{12ea} \\ S_{21ea} & S_{22ea} \end{bmatrix} = \frac{\begin{bmatrix} R_2\cos(\theta)+jZ_{e1}\sin(\theta) - \frac{jZ_{in2}R_2\sin(\theta)}{Z_{e1}} - Z_{in2}\cos(\theta) & 2\sqrt{R_2 Z_{in2}} \\ 2\sqrt{R_2 Z_{in2}} & -R_2\cos(\theta)+jZ_{e1}\sin(\theta) - \frac{jZ_{in2}R_2\sin(\theta)}{Z_{e1}} + Z_{in2}\cos(\theta) \end{bmatrix}}{R_2\cos(\theta) + jZ_{e1}\sin(\theta) + \frac{jZ_{in2}R_2\sin(\theta)}{Z_{e1}} + Z_{in2}\cos(\theta)}. \quad (10)$$

Since the Figures 3(a) and (b) are equivalent, the scattering parameters of Figure 3(b) can be determined by (10), namely, we can have the following equation:

$$\begin{bmatrix} S_{11eb} & S_{12eb} \\ S_{21eb} & S_{22eb} \end{bmatrix} = \begin{bmatrix} S_{11ea} & S_{12ea} \\ S_{21ea} & S_{22ea} \end{bmatrix}. \quad (11)$$

When the Figures 3(c) and (d) are considered, the scattering parameters of the port 2 can be expressed by

$$S_{22oa} = S_{22ob} = \frac{Z_{ino2} - R_2}{Z_{ino2} + R_2} = \frac{Z_{ino3} - R_3}{Z_{ino3} + R_3}, \quad (12)$$

where the input impedances are

$$Z_{ino2} = \frac{jZ_{o1}R_{Iso2}\tan(\theta_1)}{jZ_{o1}\tan(\theta_1) + R_{Iso2}}, \quad (13a)$$

$$Z_{ino3} = \frac{jZ_{o2}R_{Iso3}\tan(\theta_1)}{jZ_{o2}\tan(\theta_1) + R_{Iso3}}. \quad (13b)$$

Once the even- and odd-mode scattering parameters are known, the external scattering parameters of the Figure 2 can be derived and summarized as

$$S_{11} = \frac{S_{11ea} + S_{11eb}}{2} = S_{11ea}, \tag{14a}$$

$$S_{21} = \frac{kS_{21ea}}{\sqrt{1+k^2}}, \tag{14b}$$

$$S_{31} = \frac{S_{21ea}}{\sqrt{1+k^2}}, \tag{14c}$$

$$S_{22} = \frac{k^2 S_{22ea} + S_{22oa}}{1+k^2}, \tag{14d}$$

$$S_{33} = \frac{k^2 S_{22ob} + S_{22eb}}{1+k^2}, \tag{14e}$$

$$S_{23} = \frac{k(S_{22eb} - S_{22oa})}{1+k^2}. \tag{14f}$$

Note that the scattering parameters of the Figure 1 are not derived since the external scattering parameters of the Figure 2 are enough to present the key performance of this proposed unequal coupled-line power divider.

3. CALCULATED EXAMPLES OF THE PROPOSED POWER DIVIDER SHOWN IN FIGURE 2

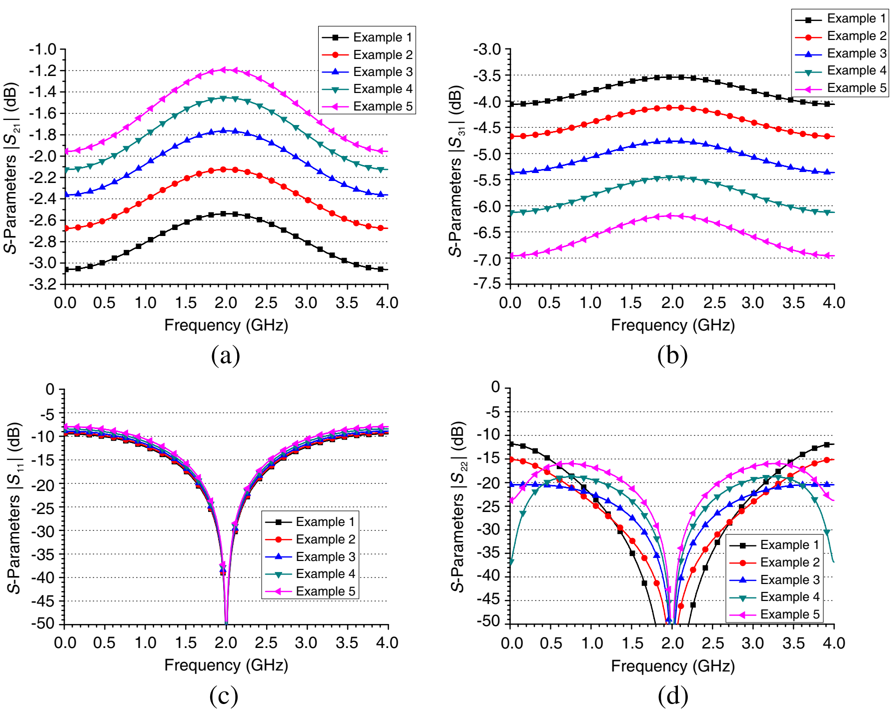
In order to verify the proposed design and analysis theory in the Section 2. After neglecting two output impedance transformers for two arbitrary complex terminated impedances, the simplified equivalent

Table 1. Design parameter values of five typical examples ($Z_a = 50\,\Omega$, $Z_{o1} = 0.8Z_{e1}$).

Examples	**Example 1**	**Example 2**	**Example 3**	**Example 4**	**Example 5**
k	**1.1220**	**1.2589**	**1.4125**	**1.5849**	**1.7783**
Z_{e1} (Ω)	**63.2297**	**56.9104**	**51.5451**	**46.9613**	**43.0165**
Z_{e2} (Ω)	**79.6014**	**90.1968**	**102.8460**	**117.9615**	**136.0303**
Z_{o1} (Ω)	**50.5837**	**45.5283**	**41.2361**	**37.5691**	**34.4132**
Z_{o2} (Ω)	**63.6812**	**72.1575**	**82.2768**	**94.3692**	**108.8242**
R_2 (Ω)	**44.5625**	**39.7164**	**35.3973**	**31.5479**	**28.1171**
R_3 (Ω)	**56.1009**	**62.9463**	**70.6269**	**79.2447**	**88.9140**

circuit shown in Figure 2 is chosen as the discussed power divider in this section. Table 1 lists the design parameters values of five different typical examples. These five power dividers including Examples $1\sim5$ have $1\sim5$ dB power dividing ratios, respectively. Figure 4 shows the compared scattering parameters including ports matching, isolation, and unequal power dividing performances. It can be observed from Figure 4(g) that the unequal power dividing ratios keep constant in the operating frequency band.

For clearly explaining the influence of the odd-mode characteristic impedances Z_{o1} and Z_{o2}, another kind of power divider with 3.5-dB power-dividing ratio is considered here. When different odd-mode characteristic impedances are used, the corresponding transmission property and input port return losses will not change. Thus, the performance about S_{11}, S_{21}, and S_{31} is not given in this section. The accurate parameter values of four typical examples with different odd-mode characteristic impedances are given in Table 2. Figure 5 shows the compared output matching and isolation performance of these four examples. It can be obtained from Figures 5(b) and (c) that the operating bandwidth of this unequal coupled-line power divider will



(a) (b) (c) (d)

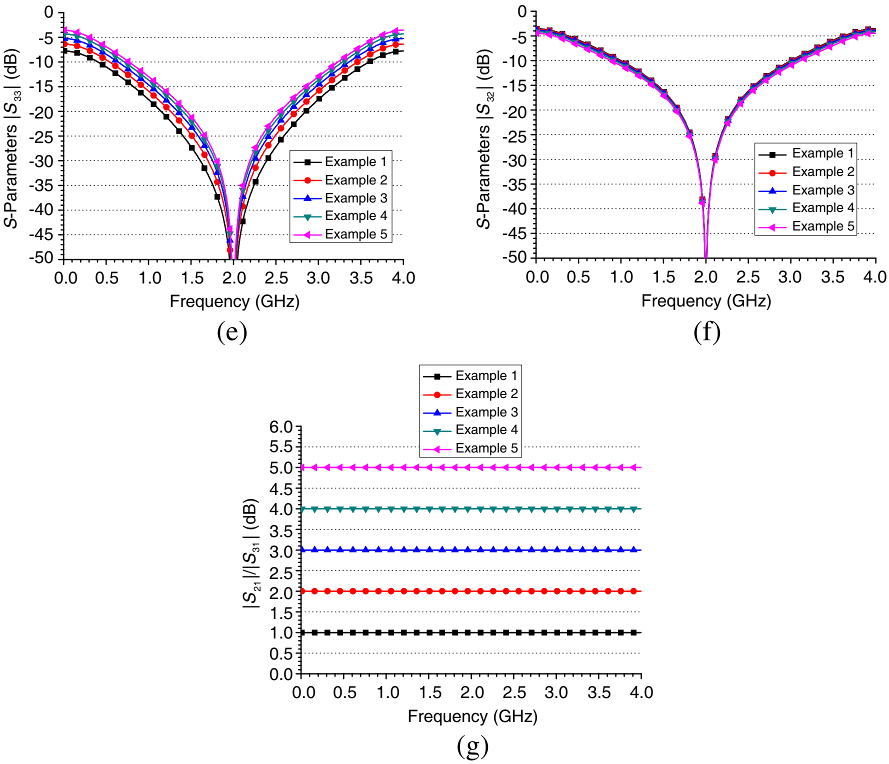
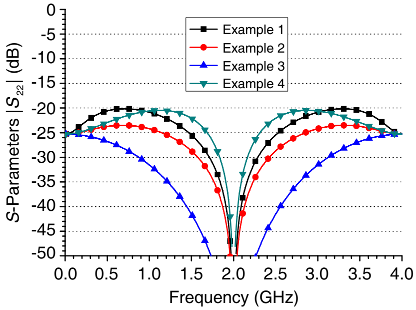


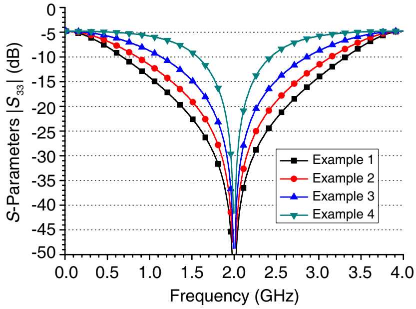
Figure 4. The scattering parameters variation along with different power-dividing ratio: (a) S_{21}; (b) S_{31}; (c) S_{11}; (d) S_{22}; (e) S_{33}; (f) S_{32}; (g) $|S_{21}|/|S_{31}|$.

Table 2. Design parameter values of four typical examples ($Z_a = 50\,\Omega$).

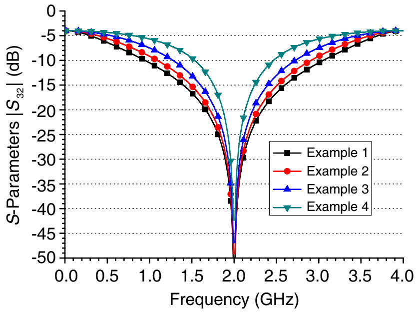
Examples	Example 1	Example 2	Example 3	Example 4
k	1.4962			
Z_{e1} (Ω)	49.1651			
Z_{e2} (Ω)	110.0669			
Z_{o1} (Ω)	39.3321	29.4991	19.6660	9.8330
Z_{o2} (Ω)	88.0535	66.0401	44.0268	22.0134
R_2 (Ω)	33.4172			
R_3 (Ω)	74.8118			



(a)



(b)



(c)

Figure 5. The output matching and isolation performance variation along with different Z_{o1} and Z_{o2} in four examples: (a) S_{22}; (b) S_{33}; (c) S_{32}.

become narrower when the used odd-mode characteristic impedances become smaller.

4. SIMULATION AND MEASUREMENT

As discussed in the above analysis, the asymmetrical coupled-line section can be applied as the main impedance-transforming circuit of the compact unequal Wilkinson power divider. As experimental demonstrations, two coupled-line microstrip Wilkinson power dividers with 2-dB (Example A) and 3-dB (Example B) power-dividing ratio are designed. In electromagnetic simulation and practical fabrication, a Rogers R04350B substrate is used and its relative dielectric constant is 3.48 and the thickness is 0.762 mm. The corresponding physical dimensions definition of the microstrip layout is shown in Figure 6. The 100-Ohm 0805 lumped resistors are used in the Examples A and B, as presented in Figure 7. The main numerical physical values of

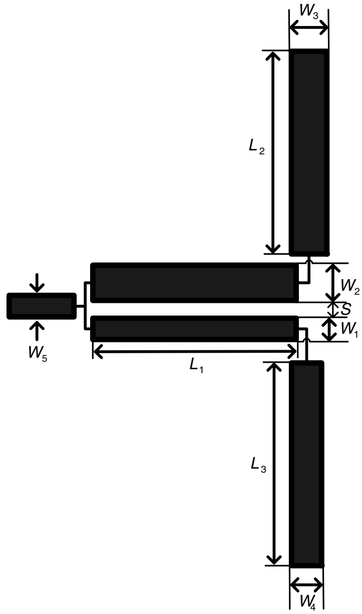
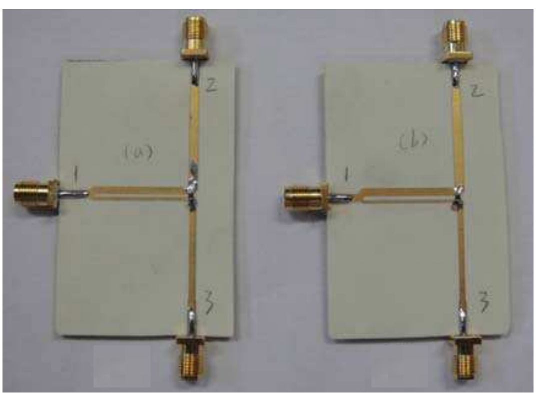


Figure 6. The physical dimensions definition of the practical unequal coupled-line power divider layout.



(a) (b)

Figure 7. The photograph of the Examples (a) A and (b) B.

the Example A are ($W_1 = 0.73\,\text{mm}$, $W_2 = 1.56\,\text{mm}$, $W_3 = 2.06\,\text{mm}$, $W_4 = 1.42\,\text{mm}$, $W_5 = 1.72\,\text{mm}$, $L_1 = L_2 = L_3 = 23\,\text{mm}$, $S = 0.8\,\text{mm}$) while the main numerical physical values of the Example B

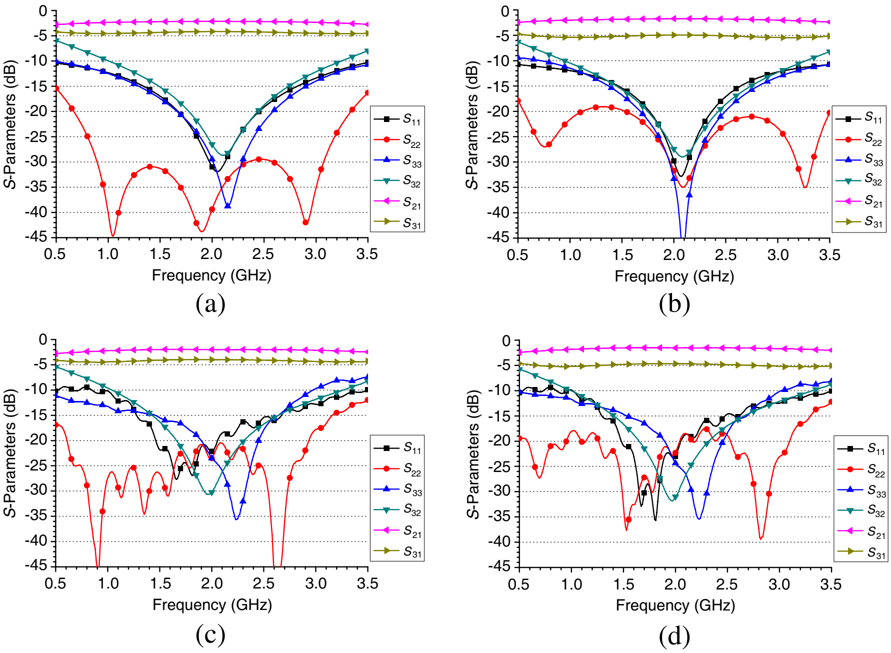


Figure 8. The simulated and measured scattering parameters: (a) the simulated results of Example A; (b) the simulated results of Example B; (c) the measured results of Example A; (d) the measured results of Example B.

are ($W_1 = 0.54$ mm, $W_2 = 1.77$ mm, $W_3 = 2.25$ mm, $W_4 = 1.29$ mm, $W_5 = 1.72$ mm, $L_1 = L_2 = L_3 = 23$ mm, $S = 1$ mm). The simulation is accomplished by Sonnet software [17]. The measured results are taken by an Agilent N5230C network analyzer. The third port is connected by a 50-Ohm load terminator when two ports of the measured power divider are measured.

Figure 8 shows the simulated and measured scattering parameters. In general, there are good agreements between the simulated and measured results. As shown in Figure 8(c), the measured transmission coefficients $|S_{21}|(|S_{31}|)$ (including additional SMA connectors) at 2 GHz for the Examples A and B are -2.01 dB (-4.00 dB) and -1.55 dB (-4.68 dB), respectively. The measured reflection coefficients ($|S_{11}|$, $|S_{22}|$, $|S_{33}|$) at all three ports in both Examples A and B are less than -22 dB at 2 GHz. In addition, the measured isolation ($|S_{32}|$) between output ports 2 and 3 at 2 GHz are better than 30 dB for Examples A and B. According to the power dividing definition, the measured power division ratios of Examples A and B are 1.99 dB and 3.13 dB, respectively. They agree well with the predicted 2-dB and 3-dB unequal performances.

5. CONCLUSION

A compact unequal power divider using asymmetrical coupled-line section has been discussed. This unequal coupled-line power divider not only has accurate unequal power dividing performance, but also occupies small circuit layout. Furthermore, this power divider can be used to match three arbitrary terminated impedances, specially two arbitrary complex impedances at output ports. According to the presented design theory and the experimental results, this unequal power divider can be easily fabricated by using microstrip circuits in PCB process.

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