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PLANE WAVE SCATTERING BY A SPHERICAL DIELECTRIC PARTICLE IN MOTION: A RELATIVISTIC EXTENSION OF THE MIE THEORY

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Abstract—Light scattering from small spherical particles has applications in a vast number of disciplines including astrophysics, meteorology optics and particle sizing. Mie theory provides an exact analytical characterization of plane wave scattering from spherical dielectric objects. There exist many variants of the Mie theory where fundamental assumptions of the theory has been relaxed to make generalizations. Notable such extensions are generalized Mie theory where plane waves are replaced by optical beams, scattering from lossy particles, scattering from layered particles or shells and scattering of partially coherent (non-classical) light. However, no work has yet been reported in the literature on modifications required to account for scattering when the particle or the source is in motion relative to each other. This is an important problem where many applications can be found in disciplines involving moving particle size characterization. In this paper we propose a novel approach, using special relativity, to address this problem by extending the standard Mie theory for scattering by a particle in motion with a constant speed, which may be very low, moderate or comparable to the speed of light. The proposed

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technique involves transforming the scattering problem to a reference frame co-moving with the particle, then applying the Mie theory in that frame and transforming the scattered field back to the reference frame of the observer.

1. INTRODUCTION

Applications of light scattering from small spherical particles can be found in a vast number of disciplines such as astrophysics, meteorological optics, particle sizing and many other areas. The problem of scattering of electromagnetic waves by moving particles is encountered in many different disciplines of science. For example, scattering by celestial particles such as the solar corona and the solar wind, and scattering of radio waves by rocket-exhaust plasmas which is an important problem in space technology [1]. Aerosols play a critical role in a range of scientific disciplines [2]. Scattering of light by these aerosol droplets provides information about their size, composition, morphology and temperature [2]. Accurate determination of the location and flow velocity of moving particles in highly scattering media has applications in a diverse range of disciplines. Another discipline which considers the scattering of electromagnetic waves by moving particles is the investigation of dynamic processes in sprays and dusts [3]. In this field particle sizes are measured using optical techniques to achieve high resolution in space and time while keeping the behaviour of the dispersion undisturbed [3].

In addition to the aforementioned potential applications of this work in different disciplines, we plan to use the results of this work to improve the accuracy of Laser Doppler flowmeters, which are routinely used in flow rate analysis of blood vessels. It is interesting to note that at present Laser Doppler flowmeters fail to give accurate results at high flow rates due to nonlinear dependency of flow velocity with Doppler shift. Especially, this becomes a problem when flow rates are measured through scattering media. This problem can be addressed by modifying the photon transport equation to account for scattering of light by moving targets. This approach requires a phase function [4] to be derived for electromagnetic scattering by a moving particle. The derivation of such a phase function based on the results of Mie theory motivated us to extend the standard Mie theory to account for motion of the particle, without any additional assumptions. Therefore, we decided to look at the Mie scattering problem in its most general terms using a special relativity framework and then come up with approximations of that exact result at a later stage to describe the corrections for much lower speeds associated with flow rates in kidneys etc. Thus, this paper describe the full-blown analysis in its complete form which may be approximated, disregarding higher order terms of the speed of the particle (v) to speed of light (c) ratio (i.e., higher order v/c terms), to address the specific Doppler flowmetry issue and photon transport equation modifications. Nevertheless, the present work does not contain any approximations and provides a complete analysis of the problem in the frame work of special relativity. Hence, the results of this work can be applied in a vast number of disciplines as previously outlined, with or without approximations that disregard higher order (v/c) terms, as desired.

Mie theory provides a general solution to Maxwell's equations and it can be applied rigorously to solve the problem of electromagnetic wave scattering by a particle whose size is comparable to the incident wavelength [2]. Mie theory is widely used in practice to characterize scattering by spherical particles because it provides an exact solution to the Maxwell's equations when an electromagnetic wave interact with a stationary spherical particle [5]. The original Mie theory formulation assumes that the medium surrounding the scatterer is non-absorbing, the scatterer is spherical, homogeneous and isotropic and the size of the scatterer is comparable to the incident wavelength [2, 6, 7]. However, many variants of this theory has been reported in the literature. There are techniques developed for modifying the Mie theory for spheres immersed in an absorbing medium [6]. The Generalized Lorenz-Mie theory [7] is used to deal with scattering of waves by spheres whose sizes are not small enough with respect to the beam diameter of the wave. (Note: the Generalized Mie theory involves an optical beam instead of a plane wave). The Mie theory had been modified to study the scattering of waves by non-spherical particles [8, 9] as well.

All of these variants of the Mie theory were developed to study electromagnetic wave scattering by stationary particles. However, as stated previously, the problem of wave scattering by moving particles is an important problem in many disciplines. There are some techniques and theories proposed in the literature to model the scattering of electromagnetic waves by particles in motion. For example, Censor [10] proposed a technique to solve the problem of scattering of a plane wave by a moving sphere using Lorentz force formulas. In this technique, the boundary conditions were derived from Lorentz force formulas, which agree only to the first order v/c terms. Therefore, his technique is valid only for moderate speeds relative to the speed of light [10]. The implementation of the technique proposed by Censor is much more complicated compared to implementing Mie problem in the absence of motion [10]. His technique involves investigation of the behaviour of the plane waves for a stationary sphere and exploiting Sommerfeld-type integrals [11] for the vector spherical waves [12]. The calculation of the scattering coefficients is a tedious process involving expressing the fields in a series of vector spherical harmonics [10]. Shiozawa [1] investigated the problem of scattering of a plane wave by a small particle moving uniformly in vacuum. He obtained the scattered field and the scattering cross section based on the covariance of Maxwell's equations and the principle of phase invariance. In his technique, he used the assumption that the particles are much smaller than the wavelength of the incident wave and described the scattering using electric and magnetic dipole moments induced on it [1].

Zutter [13] investigated the problem of time harmonic plane wave scattering by objects in translational motion. In his work, the scattered fields were expressed in terms of the precursor position of the scatterer. He used Lorentz transformations to transform the problem of scattering by a moving object to that of a stationary object in a reference frame co-moving with the particle. To evaluate the scattered field he then used a dipole-moment approach [13]. Chu et al. [14] analyzed the scattering of two crossed coherent plane waves by a moving spherical particle based upon an exact solution to Maxwell's equations for the scattered wave fields that can be integrated over a signal collection aperture that is centered along either the forward or backward scattering direction. Konig et al. [3] proposed a light-scattering technique for measuring the diameter of transparent droplets. In their technique, light scattering was approximated by ray optics.

To the best our knowledge, the Mie theory in its original form has not been applied to the problem of light scattering by a moving spherical particle previously. However, having a formulation which uses the Mie theory is very much desired because of its popularity and the vast amount of numerical software techniques available. In this paper, we propose a technique to determine the scattered field of a plane wave due to scattering from a particle in motion, while closely following the steps of the standard Mie theory. Our approach is to first transform the problem to a reference frame co-moving with the particle using Lorentz transformations and then apply the standard Mie theory in this frame. Once the scattered field is calculated in the reference frame co-moving with the particle, it is then transformed back to a reference frame which is stationary relative to the observer.

Since this extension of the Mie theory is carried out taking relativistic effects into account, it can be used in any application irrespective of the speed of the scatterer compared to that of light. For example, the results of this work can be applied in astrophysics where the particles move at speeds comparable to the speed of light.

For other applications where the speed of the scatterer is much smaller compared to that of light the results can be approximated by neglecting higher order terms of the ratio (v/c). In order to present the proposed technique without an increased mathematical complexity that might mask the main theme of the work, we consider a lossless medium and a lossless particle throughout this paper.

This paper is organized as follows. In Section 2, a novel technique is proposed to solve the problem of plane wave scattering by particles involving a relative motion between the particle and the source which emits the plane wave. In Section 3, we show that the proposed generalized solution reduces to the standard Mie theory when the particle is stationary, clearly demonstrating the accuracy and the versatility of the proposed theory. In addition, we show that the proposed theory is compatible with the relativistic Doppler formula under scattering free conditions. This result also shows that our formulation is well behaved under limiting cases and thus has a wide applicability in varying conditions. Section 4 provides simulation results followed by some concluding remarks in Section 5.

2. SCATTERING OF A PLANE WAVE INVOLVING MOVING PARTICLES AND SOURCES

In this section, we extend the Mie theory to a spherical particle stationary with respect to a moving inertial reference frame relative to the electromagnetic source. This section is further divided into four subsections. In Section 2.1, we present the proposed theory considering a stationary source and a moving particle relative to the observer. In Section 2.2, we show how this result can be used for an application involving a moving source and a stationary particle and in Section 2.3, we show how the proposed theory can be applied to a system with a moving source and a moving particle. In order to reduce the mathematical complexity, in all these three cases we deal with parallel motion between the plane wave and the particle. In Section 2.4, we provide a discussion on how to apply the proposed technique to systems involving non-parallel motion.

2.1. Stationary Source and Moving Particle

Consider an x-polarized plane wave $\mathbf{E}_i$ hitting a small particle moving (relative to an observer in an inertial reference frame [15, 16]) with velocity $\mathbf{v}$ along the z-axis, as shown in Fig. 1.

Consider two inertial frames S and S' in standard configuration [17] with a relative velocity $\mathbf{v}$ along the z-axis , as shown in

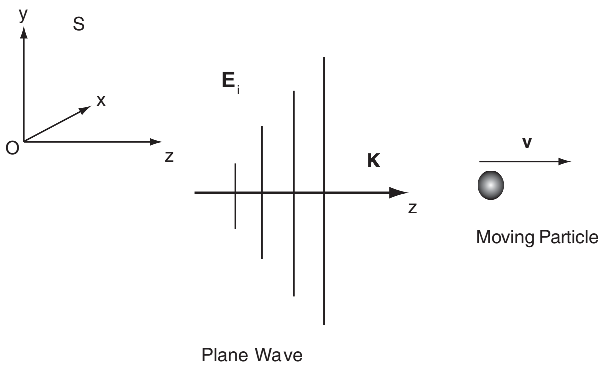


Figure 1. Plane wave incident on a moving particle.

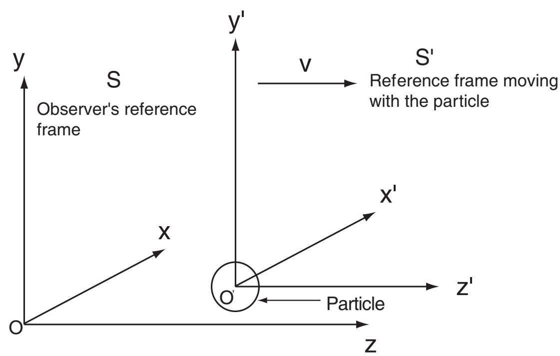


Figure 2. Two inertial frames in standard configuration.

Fig. 2. Frame S is stationary relative to the observer and frame S' is stationary relative to the particle.

The plane wave relative to frame S can be written as

$$\mathbf{E}_i\left(x, y, z, t\right) = E_0 e^{jkz} e^{-j\omega t} \mathbf{u}_x. \tag{1}$$

In order to apply the Mie theory to determine the scattered field, we first transform the incident plane wave given above to frame S'. Using the Lorentz transformations [17, 18] (1) can be written in the coordinates of inertial frame S' as,

$$\begin{aligned} \mathbf{E}_i'\left(x', y', z', t'\right) &= E_0 e^{j\gamma\left(k-\omega v/c^2\right)z'} e^{-j\gamma(\omega-kv)t'} \mathbf{u}_{x'}, \\ &= E_0 e^{jk'z'} e^{-j\omega' t'} \mathbf{u}_{x'}, \end{aligned} \tag{2}$$

where v denotes the magnitude of $\mathbf{v}$, $k' = \gamma\left(k - \omega v/c^2\right)$ and $\omega' = \gamma\left(\omega - kv\right)$. γ is the Lorentz factor where $\gamma =$

$1/(\sqrt{1-v^2/c^2})$. $\mathbf{E}'_i(x', y', z', t')$ is time harmonic and can be expressed as $\mathbf{E}'_i(x', y', z', t') = \mathbf{E}'_i(x', y', z')\, e^{-j\gamma(\omega-kv)t'}$.

Writing $\mathbf{E}'_i(x', y', z')$ in spherical coordinates results in

$$\mathbf{E}'_i\left(x', y', z'\right) = E_0 e^{jk'r'\cos\theta'}\mathbf{u}_{x'}, \tag{3}$$

where $\mathbf{u}_{x'} = S_{\theta'}C_{\phi'}\mathbf{u}_{r'} + C_{\theta'}C_{\phi'}\mathbf{u}_{\theta'} - S_{\phi'}\mathbf{u}_{\phi'}$ and $S_{\theta'} = \sin\theta'$, $C_{\theta'} = \cos\theta'$, $S_{\phi'} = \sin\phi'$ and $C_{\phi'} = \cos\phi'$. We use this notation throughout this paper. We can expand $\mathbf{E}'_i$ in spherical harmonics as [19],

$$\mathbf{E}'_i = E_0\sum_{n=1}^{\infty} j^n\frac{2n+1}{n(n+1)}\left(\mathbf{M}^{(1)}_{o1n} - j\mathbf{N}^{(1)}_{e1n}\right), \tag{4}$$

where $\mathbf{M}_{o1n} = C_{\phi'}\pi_n z_n(\rho)\hat{\mathbf{u}}_{\theta'} - S_{\phi'}\tau_n z_n(\rho)\hat{\mathbf{u}}_{\phi'}$, $\mathbf{N}_{e1n} = C_{\phi'}n(n+1)S_{\theta'}\pi_n z_n(\rho)/\rho\hat{\mathbf{u}}_{r'} + C_{\phi'}\tau_n D[\rho z_n(\rho)]/\rho\hat{\mathbf{u}}_{\theta'} - S_{\phi'}\pi_n D[\rho z_n(\rho)]/\rho\hat{\mathbf{u}}_{\phi'}$, $\mathbf{N}_{e1n} = \nabla'\times\mathbf{M}_{e1n}/k'$, $\mathbf{N}_{o1n} = \nabla'\times\mathbf{M}_{o1n}/k'$. Thus, $\mathbf{M}_{e1n} = -S_{\phi'}\pi_n z_n(\rho)\hat{\mathbf{u}}_{\theta'} - C_{\phi'}\tau_n z_n(\rho)\hat{\mathbf{u}}_{\phi'}$, $\mathbf{N}_{o1n} = S_{\phi'}n(n+1)S_{\theta'}\pi_n z_n(\rho)/\rho\hat{\mathbf{u}}'_r + S_{\phi'}\tau_n D[\rho z_n(\rho)]/\rho\hat{\mathbf{u}}_{\theta'} + C_{\phi'}\pi_n D[\rho z_n(\rho)]/\rho\hat{\mathbf{u}}_{\phi'}$. Superscripts appended to the functions $\mathbf{M}$ and $\mathbf{N}$ denote the kind of the spherical Bessel function z_n: (1) denotes $j_n(k_1 r)$ (i.e., spherical Bessel functions of the first kind [20]) and (3) denotes $h^{(1)}_n(kr)$ (i.e., spherical Bessel functions of the third kind, or spherical Hankel functions [20]). The angle-dependent functions $\pi_n = P^1_n(C_{\theta'})/S_{\theta'}$ and $\tau_n = dP^1_n(C_{\theta'})/d\theta'$ where P^1_n are the associate Legendre polynomials of order 1 and degree n. These functions can be computed by upward recurrence using $\pi_n = (2n-1/n-1)C_{\theta'}\pi_{n-1} - (n/n-1)\pi_{n-2}$, $\tau_n = nC_{\theta'}\pi_n - (n+1)\pi_{n-1}$, beginning with $\pi_0 = 0$ and $\pi_1 = 1$. $\rho = k'r'$ and $\nabla' \equiv \partial/\partial x'\mathbf{u}_{x'} + \partial/\partial y'\mathbf{u}_{y'} + \partial/\partial z'\mathbf{u}_{z'}$ denotes the gradient operator with respect to inertial frame S'. k'_1 is the wave number of the wave inside the scatterer (moving particle) and k' is the wave number of the wave outside the scatterer. D represents the first derivative with respect to ρ. (A complete derivation of the expansion of the electric field using spherical harmonics can be found in [19]).

From (2) we have $\partial/\partial t' \equiv -j\omega'$. Thus, from Maxwell's equations

$$\begin{aligned}\nabla'\times\mathbf{E}' &= j\omega'\mathbf{B}',\\ \mathbf{B}' &= -j\frac{1}{\omega'}\nabla'\times\mathbf{E}'.\end{aligned} \tag{5}$$

In frame S', Minkowski constitutive relations hold. These are given by the following two equations [21].

$$\mathbf{D}' + \frac{\mathbf{v}\times\mathbf{H}'}{c^2} - \epsilon\mathbf{E}' - \epsilon\mathbf{v}\times\mathbf{B}' = \mathbf{0}, \tag{6}$$

$$\mathbf{B}' - \frac{\mathbf{v}\times\mathbf{E}'}{c^2} - \mu_0\mathbf{H}' + \mu_0\mathbf{v}\times\mathbf{D}' = \mathbf{0}. \tag{7}$$

Eliminating $\mathbf{D}$ from the above two equations and using the identity $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$ results in

$$\frac{\mu_0}{c^2}\mathbf{v}\left(\mathbf{v} \cdot \mathbf{H}'\right) - \frac{\mu_0}{c^2}v^2\mathbf{H}' - \epsilon\mu_0\mathbf{v} \times \mathbf{E}' - \epsilon\mu_0\mathbf{v}\left(\mathbf{v} \cdot \mathbf{B}'\right) + \epsilon\mu_0 v^2\mathbf{B}' - \mathbf{B}'$$
$$+\frac{1}{c^2}\mathbf{v} \times \mathbf{E}' + \mu_0\mathbf{H}' = \mathbf{0}. \tag{8}$$

Since the velocity of the particle is perpendicular to $\mathbf{B}$, $\mathbf{H}$, $\mathbf{D}$ and $\mathbf{E}$ (in addition, the magnetic field is perpendicular to the electric field due to the properties of the plane wave), $\mathbf{v} \cdot \mathbf{H}' = 0$ and $\mathbf{v} \cdot \mathbf{B}' = 0$. Then, (8) becomes

$$-\frac{\mu_0}{c^2}v^2\mathbf{H}' - \epsilon\mu_0\mathbf{v} \times \mathbf{E}' + \epsilon\mu_0 v^2\mathbf{B}' - \mathbf{B}' + \frac{1}{c^2}\mathbf{v} \times \mathbf{E}' + \mu_0\mathbf{H}' = \mathbf{0}. \tag{9}$$

Using (5) in (9) and assuming free-space propagation we can express the incident magnetic field using spherical harmonics as (see Appendix A)

$$\begin{aligned}\mathbf{H}'_i &= \frac{1}{\mu_0}\tilde{P}_0\sum_{n=1}^{\infty} E_n\left(\mathbf{v} \times \mathbf{M}^{(1)}_{o1n} - j\mathbf{v} \times \mathbf{N}^{(1)}_{e1n}\right)\\ &\quad -\frac{k'c^2}{\mu_0\omega'}Q_0\sum_{n=1}^{\infty} E_n\left(\mathbf{M}^{(1)}_{e1n} + j\mathbf{N}^{(1)}_{o1n}\right),\end{aligned} \tag{10}$$

where $\tilde{P}_0 = \left(c^2\epsilon_0\mu_0 - 1\right)/\left(c^2 - v^2\right)$, $Q_0 = \left(1 - \epsilon_0\mu_0 v^2\right)/\left(c^2 - v^2\right)$ and $E_n = j^n\left(2n+1\right)/\left(n(n+1)\right)E_0$.

Let the expansion of the electric field, $\mathbf{E}'_1$, inside the sphere relative to frame S' be

$$\mathbf{E}'_1 = \sum_{n=1}^{\infty} E_n\left(c_n\mathbf{M}^{(1)}_{o1n} - jd_n\mathbf{N}^{(1)}_{e1n}\right). \tag{11}$$

Then the magnetic field, $\mathbf{H}'_1$, inside the sphere is

$$\begin{aligned}\mathbf{H}'_1 &= 1/\mu_0\tilde{P}_1\mathbf{v} \times \mathbf{E}'_1 - j\frac{c^2}{\mu_0\omega'}Q_1\nabla' \times \mathbf{E}'_1,\\ &= 1/\mu_0\tilde{P}_1\sum_{n=1}^{\infty} E_n\left(c_n\mathbf{v} \times \mathbf{M}^{(1)}_{o1n} - jd_n\mathbf{v} \times \mathbf{N}^{(1)}_{e1n}\right)\\ &\quad -\frac{k'_1c^2}{\mu_0\omega'}Q_1\sum_{n=1}^{\infty} E_n\left(d_n\mathbf{M}^{(1)}_{e1n} + jc_n\mathbf{N}^{(1)}_{o1n}\right),\end{aligned} \tag{12}$$

where $\tilde{P}_1 = (c^2\epsilon_1\mu_0 - 1)/(c^2 - v^2)$, $Q_1 = (1 - \epsilon_1\mu_0 v^2)/(c^2 - v^2)$. Let the expansion of the scattered electric field with respect to frame S'

be [19]

$$\mathbf{E}'_s = \sum_{n=1}^{\infty} E_n \left(j a_n \mathbf{N}^{(3)}_{e1n} - b_n \mathbf{M}^{(3)}_{o1n} \right). \tag{13}$$

Then the scattered magnetic field will be

$$\begin{aligned} \mathbf{H}'_s &= \frac{1}{\mu_0} \tilde{P}_0 \mathbf{v} \times \mathbf{E}'_s - j \frac{c^2}{\mu_0 \omega'} Q_0 \nabla' \times \mathbf{E}'_s, \\ &= \frac{1}{\mu_0} \tilde{P}_0 \sum_{n=1}^{\infty} E_n \left(j a_n \mathbf{v} \times \mathbf{N}^{(3)}_{e1n} - b_n \mathbf{v} \times \mathbf{M}^{(3)}_{o1n} \right) \\ &\quad + \frac{k' c^2}{\mu_0 \omega'} Q_0 \sum_{n=1}^{\infty} E_n \left(j b_n \mathbf{N}^{(3)}_{o1n} + a_n \mathbf{M}^{(3)}_{e1n} \right). \end{aligned} \tag{14}$$

On the scatterer-medium boundary (i.e., on the surface of the scatterer) we have the following boundary conditions [19]

$$\left(\mathbf{E}'_i + \mathbf{E}'_s - \mathbf{E}'_1 \right) \times \mathbf{u}_{r'} = \left(\mathbf{H}'_i + \mathbf{H}'_s - \mathbf{H}'_1 \right) \times \mathbf{u}_{r'} = \mathbf{0}. \tag{15}$$

That is, the tangential components of $\mathbf{E}'$ and $\mathbf{H}'$ are continuous across a boundary separating two media of different properties [19]. The above boundary conditions can be written in component form as (for each n)

$$\mathbf{F}'_{i\beta} + \mathbf{F}'_{s\beta} = \mathbf{F}'_{1\beta}, \tag{16}$$

where $\mathbf{F} = \mathbf{E}, \mathbf{H}$ and $\beta = \theta, \phi$. On the boundary $r' = a$ where a is the radius of the spherical scatterer. Let χ represents ρ on the boundary where $\chi = k'a$, $\chi_1 = k'_1 a$ and $m = k'_1/k'$.

Equations (4), (11) and (13) can be written in component form and used in the boundary condition given by (16) to obtain two equations relating the four scattering coefficients, a_n, b_n, c_n and d_n (see Appendix B). Similarly, (10), (12) and (14) can be written in component form and used in the boundary condition given by (16) to obtain two equations relating the four scattering coefficients (see Appendix B). By solving these four equations we obtain the four scattering coefficients that are required to calculate the Mie solution.

$$a_n = \frac{\Psi}{\Omega}, \tag{17}$$

$$b_n = \frac{Y}{Z}, \tag{18}$$

$$c_n = \frac{1}{M_a Z} \left(Z J_a - H_a Y \right), \tag{19}$$

$$d_n = \frac{m}{B_a \Omega} \left(A_a \Omega - I_a \Psi \right), \tag{20}$$

where

$$\begin{aligned}
\Omega &= \left(jP_0\Gamma I_a - jP_0\Lambda H_a - k'Q_0\Delta H_a\chi - jP_1\Gamma I_a + jP_1\Lambda\frac{M_aI_a}{B_a}\right.\\
&\quad\left.+k_1'Q_1\Delta m\chi\frac{M_aI_a}{B_a}\right),\\
\Psi &= \left(jP_0\Gamma A_a - jP_0\Lambda J_a - k'Q_0\Delta J_a\chi - jP_1\Gamma A_a + jP_1\Lambda\frac{M_aA_a}{B_a}\right.\\
&\quad\left.+k_1'Q_1\Delta\frac{m\chi M_aA_a}{B_a}\right),\\
Y &= \left(P_0\Gamma J_a\chi - jP_0XJ_a - jk'Q_0\Delta A_a + jP_0XH_a\frac{\Psi}{\Omega} - P_1\Gamma J_a\chi\right.\\
&\quad\left.+jP_1\frac{XM_aA_a}{B_a} - jP_1\frac{XM_aI_a}{B_a}\frac{\Psi}{\Omega} + jk_1'Q_1\Delta\frac{B_aJ_a}{mM_a}\right),\\
Z &= \left(P_0\Gamma H_a\chi - jk'Q_0\Delta I_a - P_1\Gamma H_a\chi + jk_1'Q_1\Delta\frac{B_aH_a}{mM_a}\right),
\end{aligned}$$

$P_0 = v\times\tilde{P}_0$, $P_1 = v\times\tilde{P}_1$, $\Pi = \pi_n(C_{\theta'})$, $J_a = j_n(\chi)$, $M_a = j_n(\chi_1) = j_n(m\chi)$, $A_a = D[\chi j_n(\chi)] = (d/d\chi)\,[\chi j_n(\chi)]$, $B_a = D[\chi_1 j_n(\chi_1)] = D[m\chi j_n(m\chi)] = (d/d\chi)\,[m\chi j_n(m\chi)]$, $T = \tau_n(C_{\theta'})$, $H_a = h_n^{(1)}(\chi)$, $I_a = D[\chi h_n^{(1)}(\chi)]$, $\Gamma = C_{\theta'}(\Pi\times\Pi - T\times T)$, $\Delta = c^2/\omega'(\Pi\times\Pi - T\times T)$, $\Lambda = n(n+1)S_{\theta'}^2\Pi\times T$ and $X = n(n+1)S_{\theta'}^2\Pi\times\Pi$.

Thus, we have calculated the scattering coefficients a_n, b_n, c_n and d_n. Hence, the scattered field and the field inside the scatterer are known with respect to frame S'. We then need to transform these fields to reference frame S, which is stationary relative to the observer. We first write the field components with respect to the Cartesian coordinate system and then use the standard field transformations from frame S' to frame S.

The Cartesian field components can be constructed from the spherical field components using the following transformations

$$F_x' = F_r'S_{\theta'}C_{\phi'} + F_\theta C_{\theta'}C_{\phi'} - F_\phi'S_{\phi'}, \tag{21}$$

$$F_y' = F_r'S_{\theta'}S_{\phi'} + F_\theta'C_{\theta'}S_{\phi'} + F_\phi'C_{\phi'}, \tag{22}$$

$$F_z' = F_r'C_{\theta'} - F_\theta'S_{\theta'}, \tag{23}$$

where $F = E, H$.

Using the expression for the scattered electric field (i.e., (13)) in (21) to (23), we obtain the Cartesian components of the electric field as

$$
\begin{aligned}
E'_{s,x} &= \sum_{n=1}^{\infty} E_n \Big[j a_n \left(N^{(3)}_{e1n,r} S_{\theta'} C_{\phi'} + N^{(3)}_{e1n,\theta} C_{\theta'} C_{\phi'} - N^{(3)}_{e1n,\phi} S_{\phi'} \right) \\
&\quad - b_n \left(M^{(3)}_{o1n,\theta} C_{\theta'} C_{\phi'} - M^{(3)}_{o1n,\phi} S_{\phi'} \right) \Big], \qquad (24) \\
E'_{s,y} &= \sum_{n=1}^{\infty} E_n \Big[j a_n \left(N^{(3)}_{e1n,r} S_{\theta'} S_{\phi'} + N^{(3)}_{e1n,\theta} C_{\theta'} S_{\phi'} + N^{(3)}_{e1n,\phi} C_{\phi'} \right) \\
&\quad - b_n \left(M^{(3)}_{o1n,\theta} C_{\theta'} S_{\phi'} + M^{(3)}_{o1n,\phi} C_{\phi'} \right) \Big], \qquad (25) \\
E'_{s,z} &= \sum_{n=1}^{\infty} E_n \Big[j a_n \left(N^{(3)}_{e1n,r} C_{\theta'} - N^{(3)}_{e1n,\theta} S_{\phi'} \right) + b_n M^{(3)}_{o1n,\theta} S_{\theta'} \Big]. \qquad (26)
\end{aligned}
$$

Similarly, using the expression for the scattered magnetic field (i.e., (14)) in (21) to (23), we obtain the Cartesian components of the magnetic field as,

$$
\begin{aligned}
H'_{s,x} &= \frac{P_0}{\mu_0} \sum_{n=1}^{\infty} E_n \Big[j a_n \left(-N^{(3)}_{e1n,\phi} C_{\phi'} - N^{(3)}_{e1n,\theta} C_{\theta'} S_{\phi'} - N^{(3)}_{e1n,r} S_{\theta'} S_{\phi'} \right) \\
&\quad + b_n \left(M^{(3)}_{o1n,\phi} C_{\phi'} + M^{(3)}_{o1n,\theta} C_{\theta'} S_{\phi'} \right) \Big] \\
&\quad + \frac{k' c^2}{\mu_0 \omega'} Q_0 \sum_{n=1}^{\infty} E_n \Big[j b_n \left(N^{(3)}_{o1n,r} S_{\theta'} C_{\phi'} + N^{(3)}_{o1n,\theta} C_{\theta'} C_{\phi'} - N^{(3)}_{o1n,\phi} S_{\phi'} \right) \\
&\quad a_n \left(M^{(3)}_{e1n,\theta} C_{\theta'} C_{\phi'} - M^{(3)}_{e1n,\phi} S_{\phi'} \right) \Big], \qquad (27) \\
H'_{s,y} &= \frac{P_0}{\mu_0} \sum_{n=1}^{\infty} E_n \Big[j a_n \left(-N^{(3)}_{e1n,\phi} S_{\phi'} + N^{(3)}_{e1n,\theta} C_{\theta'} C_{\phi'} + N^{(3)}_{e1n,r} S_{\theta'} C_{\phi'} \right) \\
&\quad + b_n \left(M^{(3)}_{o1n,\phi} S_{\phi'} - M^{(3)}_{o1n,\theta} C_{\theta'} C_{\phi'} \right) \Big] \\
&\quad + \frac{k' c^2}{\mu_0 \omega'} Q_0 \sum_{n=1}^{\infty} E_n \Big[j b_n \left(N^{(3)}_{o1n,r} S_{\phi'} S_{\theta'} + N^{(3)}_{o1n,\theta} C_{\theta'} S_{\phi'} + N^{(3)}_{o1n,\phi} C_{\phi'} \right) \\
&\quad + a_n \left(M^{(3)}_{e1n,\theta} C_{\theta'} S_{\phi'} + M^{(3)}_{e1n,\phi} C_{\phi'} \right) \Big], \qquad (28)
\end{aligned}
$$

$$H'_{s,z} = \frac{P_0}{\mu_0} \sum_{n=1}^{\infty} E_n \Big[-ja_n \left(N^{(3)}_{e1n,\phi} S_{\theta'} C_{\theta'} + N^{(3)}_{e1n,\theta} C_{\theta'} S_{\theta'} + N^{(3)}_{e1n,r} S^2_{\theta'} \right)$$
$$+ b_n \left(M^{(3)}_{o1n,\phi} S_{\theta'} C_{\theta'} + M^{(3)}_{o1n,\theta} C_{\theta'} S_{\theta'} \right) \Big]$$
$$+ \frac{k'c^2}{\mu_0 \omega'} Q_0 \sum_{n=1}^{\infty} E_n \Big[jb_n \left(N^{(3)}_{o1n,r} C_{\theta'} - N^{(3)}_{o1n,\phi} S_{\theta'} \right) - a_n M^{(3)}_{e1n,\phi} S_{\theta'} \Big], \quad (29)$$

where $P_0 = v \times \tilde{P}_0$.

These components of the scattered electromagnetic field with respect to frame S' should then be transformed back to the inertial reference frame S of the observer. Using the Minkowski constitutive relations [21] in the standard field transformation equations [22] we obtain the following field transformations between the two inertial frames (see Appendix C)

$$E_x = \gamma \left(E'_x + \frac{v}{c} \mu_0 L_1 L_2 H'_y + \frac{v^2}{c} L_1 L_3 E'_x \right), \quad (30)$$

$$E_y = \gamma \left(E'_y - \frac{v}{c} \mu_0 L_1 L_2 H'_x + \frac{v^2}{c} L_1 L_3 E'_y \right), \quad (31)$$

$$E_z = E'_z, \quad (32)$$

$$H_x = \gamma \left(L_1 L_2 H'_x - \frac{v}{\mu_0} L_1 L_3 E'_y - \frac{v}{\mu_0 c} E'_y \right), \quad (33)$$

$$H_y = \gamma \left(L_1 L_2 H'_y + \frac{v}{\mu_0} L_1 L_3 E'_x + \frac{v}{\mu_0 c} E'_x \right), \quad (34)$$

$$H_z = L_1 L_2 H'_z, \quad (35)$$

where $L_1 = 1/(1 - \epsilon_0 \mu_0 v^2)$, $L_2 = 1 - (v^2/c^2)$ and $L_3 = 1/c^2 - \epsilon_0 \mu_0$.

Hence, using (24) to (26) and (27) to (29) in (30) to (35), we can calculate all the components of electric and magnetic fields with respect to inertial frame S.

2.2. Moving Source and Stationary Particle

Consider a source moving with a velocity $\mathbf{v}_s$ along the z-axis, relative to an observer. The source is emitting x-polarized plane waves $\mathbf{E}_i$ which hit a small stationary particle (relative to the observer), as shown in Fig. 3. Consider two inertial frames S and S' in standard configuration with a relative velocity $\mathbf{v}_s$. Frame S is stationary relative to the source and frame S' is stationary relative to the particle and the observer, as shown in Fig. 3. The plane wave with respect to frame S can be written

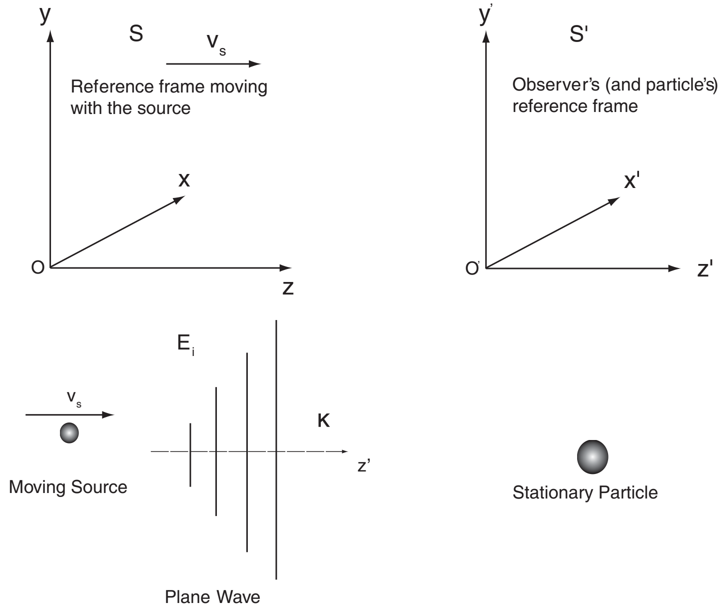


Figure 3. Two inertial frames in standard configuration.

as

$$\mathbf{E}_i(x,y,z,t) = E_0 e^{jkz} e^{-j\omega t}\mathbf{u}_x. \tag{36}$$

Equation (36) can be transformed to frame S' using the Lorentz transformations [17, 18] as

$$\begin{aligned}\mathbf{E}_i'\left(x',y',z',t'\right) &= E_0 e^{j\left(\gamma k + v_s\omega\gamma/c^2\right)z'} e^{-j\gamma(v_s k+\omega)t'}\mathbf{u}_{x'},\\ &= E_0 e^{jk'z'} e^{-j\omega' t'}\mathbf{u}_{x'},\end{aligned}$$

where v_s denotes the magnitude of $\mathbf{v}_s$, $k' = \gamma\left(k + \omega v_s/c^2\right)$, $\omega' = \gamma\left(v_s k + \omega\right)$ and $\gamma = 1/\sqrt{1 - v_s^2/c^2}$.

Now consider frame S'' which is stationary with respect to frame S' but whose origin is situated at the centre of the scatterer (so that we can have the same boundary conditions as in the standard Mie theory). The incident wave can be written with respect to frame S'' as

$$\begin{aligned}\mathbf{E}_i''\left(x'',y'',z'',t''\right) &= E_0 e^{jk'(z''-c_z)} e^{-j\omega' t''}\mathbf{u}_{x''},\\ &= E_0 e^{-jk'c_z} e^{jk'z''} e^{-j\omega' t''}\mathbf{u}_{x''}, = E_0'' e^{jk'z''} e^{-j\omega' t''}\mathbf{u}_{x''}.\end{aligned}$$

$\mathbf{E}_i''$ can be expanded in spherical harmonics as follows [19].

$$\mathbf{E}_i'' = E_0'' \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} \left(\mathbf{M}_{o1n}^{(1)} - j\mathbf{N}_{e1n}^{(1)}\right). \tag{37}$$

From Maxwell's equations

$$\mathbf{B}'' = -j\frac{1}{\omega'}\nabla'' \times \mathbf{E}''. \tag{38}$$

In frame S'' the following relation holds:

$$\mathbf{B}'' = \mu_0 \mathbf{H}''. \tag{39}$$

Using (39) and (37) in (38), for the incident magnetic field, we get

$$\mathbf{H}_i'' = -\frac{k'}{\mu_0\omega'}\sum_{n=1}^{\infty} E_n''\left(\mathbf{M}_{e1n}^{(1)} + j\mathbf{N}_{o1n}^{(1)}\right). \tag{40}$$

From the format of (37) and (40) it is evident that we can obtain the scattered field by replacing k by k', ω by ω' and E_n by E_n'' in the results of the standard Mie theory (which is derived for a stationary source and a stationary particle). Thus, the scattered field is

$$\begin{aligned}
\mathbf{E}_s'' &= \sum_{n=1}^{\infty} E_n''\left(ja_n\mathbf{N}_{e1n}^{(3)} - b_n\mathbf{M}_{o1n}^{(3)}\right), \\
\mathbf{H}_s'' &= \frac{k'}{\omega'\mu_0}\sum_{n=1}^{\infty} E_n''\left(jb_n\mathbf{N}_{o1n}^{(3)} + a_n\mathbf{M}_{e1n}^{(3)}\right),
\end{aligned}$$

where

$$\begin{aligned}
a_n &= \frac{m^2 j_n(m\chi)D[\chi j_n(\chi)] - j_n(\chi)D[m\chi j_n(m\chi)]}{m^2 j_n(m\chi)D[\chi h_n^{(1)}(\chi)] - h_n^{(1)}(\chi)D[m\chi j_n(m\chi)]}, \\
b_n &= \frac{j_n(m\chi)D[\chi j_n(\chi)] - j_n(\chi)D[m\chi j_n(m\chi)]}{j_n(m\chi)D[\chi h_n^{(1)}(\chi)] - h_n^{(1)}(\chi)D[m\chi j_n(m\chi)]},
\end{aligned}$$

and $\chi = k'a$, $m = k_1'/k'$.

2.3. Source and Particle Both in Motion

Consider a source moving with a velocity $\mathbf{v}_s$ along the z-axis relative to an observer. This source is emitting x-polarized plane waves $\mathbf{E}_i$ which hit a small particle moving (relative to an observer) with velocity $\mathbf{v}$ along z-axis, as shown in Fig. 4. Consider three inertial frames S, S' and S'' in standard configuration. Frame S'' is stationary relative to the observer, frame S' is stationary relative to the particle and frame S is stationary relative to the source, as shown in Fig. 4. The incident plane wave with respect to frame S can be written as

$$\mathbf{E}_i(x, y, z, t) = E_0 e^{jkz} e^{-j\omega t}\mathbf{u}_x. \tag{41}$$

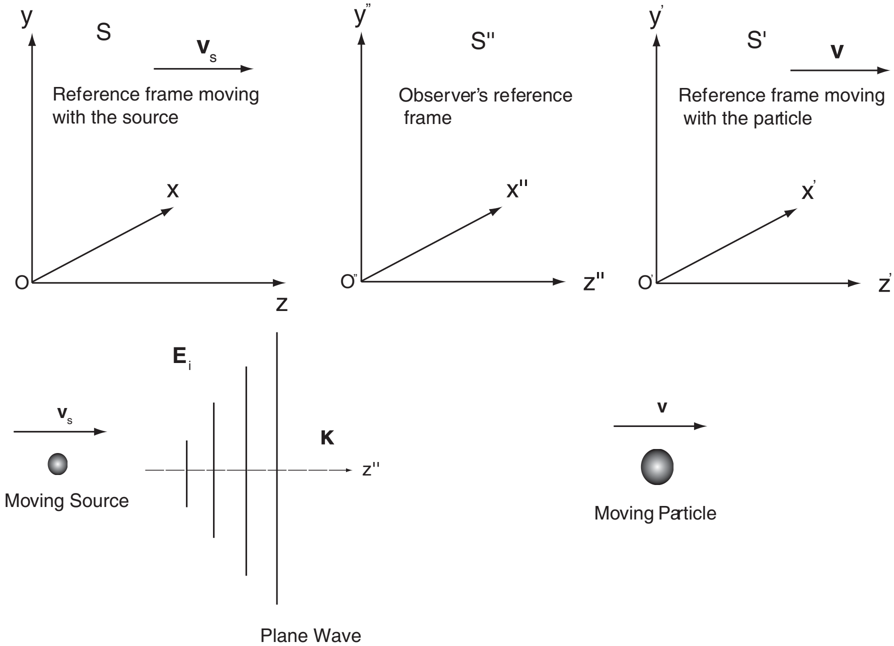


Figure 4. Two inertial frames in standard configuration.

Equation (41) can be transformed to frame S'' as

$$\begin{aligned}\mathbf{E}_i''(x'',y'',z'',t'') &= E_0 e^{j\gamma_1\left(k+v_s\omega/c^2\right)z''} e^{-j\gamma_1(\omega+v_sk)t''}\mathbf{u}_{x''},\\ &= E_0 e^{jk''z''}e^{-j\omega''t''}\mathbf{u}_{x''},\end{aligned}\tag{42}$$

where $k'' = \gamma_1\left(k+v_s\omega/c^2\right)$, $\omega'' = \gamma_1(\omega+v_sk)$ and $\gamma_1 = 1/\sqrt{1-v_s^2/c^2}$. Equation (42) can then be transformed to frame S' as

$$\begin{aligned}\mathbf{E}_i'(x',y',z',t') &= E_0 e^{j\gamma_2\left(k''-v\omega''/c^2\right)z'} e^{-j\gamma_2(\omega''-vk'')t'}\mathbf{u}_{x'},\\ &= E_0 e^{jk'z'}e^{-j\omega't'}\mathbf{u}_{x'},\end{aligned}$$

where $k' = \gamma_2\left(k''-v\omega''/c^2\right) = \gamma_1\gamma_2\left(\left(1-v_sv/c^2\right)k+\left(v_s/c^2-v/c^2\right)\omega\right)$, $\omega' = \gamma_2(\omega''-vk'') = \gamma_1\gamma_2\left(\left(1-v_sv/c^2\right)\omega+(v_s-v)k\right)$ and $\gamma_2 = 1/\sqrt{1-v^2/c^2}\cdot\mathbf{E}_i'$ can be expanded using spherical harmonics as

$$\mathbf{E}_i' = E_0\sum_{n=1}^{\infty} j^n\frac{2n+1}{n(n+1)}\left(\mathbf{M}_{o1n}^{(1)}-j\mathbf{N}_{e1n}^{(1)}\right).\tag{43}$$

From Maxwell's equations

$$\mathbf{B}' = -j\frac{1}{\omega'}\nabla'\times\mathbf{E}'.\tag{44}$$

In frame S' Minkowski constitutive relations hold. Using (44), (6) and (7), we get

$$\mathbf{H}' = \frac{1}{\mu_0}\tilde{P}\mathbf{v} \times \mathbf{E}' - j\frac{c^2}{\mu_0\omega'}Q\nabla' \times \mathbf{E}', \tag{45}$$

where $\tilde{P} = \left(c^2\epsilon\mu_0 - 1\right) / \left(c^2 - v^2\right)$ and $Q = \left(1 - \epsilon\mu_0 v^2\right) / \left(c^2 - v^2\right)$.

Using (43) in (45) we get

$$\begin{aligned} \mathbf{H}'_i \;=&\; \frac{1}{\mu_0}\tilde{P}_0\sum_{n=1}^{\infty} E_n\left(\mathbf{v} \times \mathbf{M}^{(1)}_{o1n} - j\mathbf{v} \times \mathbf{N}^{(1)}_{e1n}\right) \\ &- \frac{k'c^2}{\mu_0\omega'}Q_0\sum_{n=1}^{\infty} E_n\left(\mathbf{M}^{(1)}_{e1n} + j\mathbf{N}^{(1)}_{o1n}\right). \end{aligned} \tag{46}$$

Equations (43) and (46) are exactly similar to (4) and (10). Therefore, following a similar argument (and using the same boundary conditions) as in Section 2.1, we can determine the scattered field relative to frame S'. In addition, the Cartesian components of electric and magnetic fields relative to frame S' can be obtained using (24) to (26) and (27) to (29). Since the observer is stationary relative to frame S'', we should convert these fields to frame S''. Using the Minkowski constitutive relations [21] in the standard field transformations [22], we obtain the scattered field as seen by the observer using the following transformations

$$E''_x = \gamma_2\left(E'_x + \frac{v}{c}\mu_0 L_1 L_2 H'_y + \frac{v^2}{c}L_1 L_3 E'_x\right), \tag{47}$$

$$E''_y = \gamma_2\left(E'_y - \frac{v}{c}\mu_0 L_1 L_2 H'_x + \frac{v^2}{c}L_1 L_3 E'_y\right), \tag{48}$$

$$E''_z = E'_z, \tag{49}$$

$$H''_x = \gamma_2\left(L_1 L_2 H'_x - \frac{v}{\mu_0}L_1 L_3 E'_y - \frac{v}{\mu_0 c}E'_y\right), \tag{50}$$

$$H''_y = \gamma_2\left(L_1 L_2 H'_y + \frac{v}{\mu_0}L_1 L_3 E'_x + \frac{v}{\mu_0 c}E'_x\right), \tag{51}$$

$$H''_z = L_1 L_2 H'_z. \tag{52}$$

where $L_1 = 1/(1 - \epsilon\mu_0 v^2)$, $L_2 = 1 - (v^2/c^2)$ and $L_3 = 1/c^2 - \epsilon\mu_0$.

2.4. Non-parallel Motion between the Wave and the Particle

In all the three cases discussed in the previous three sections, we have considered the relative motion of the plane wave and the particle in the same direction. This configuration made it possible for us to closely follow the standard Mie theory derivation in reference frame S'. However, the proposed method can be easily adopted to non-parallel motion between the wave and the particle as described in this section.

Consider the case discussed in Section 2.1 where the source is stationary and the particle is in motion relative to the observer. For the non-parallel case we prefer to choose the coordinate axes of reference frame S such that the z-axis makes an angle α with the direction of propagation of the wave, where $\cos\alpha = (\mathbf{k}\cdot\mathbf{v})/(kv)$ as shown by Fig. 5. The z'-axis of frame S' is chosen to be along the direction of motion of the particle. This choice of coordinate axes makes it possible for us to use the standard simple Lorentz transformations and field transformations due to the fact that the two reference frames are in standard configuration.

Equation of the plane wave relative to frame S is

$$\mathbf{E}_i(x,y,z,t) = E_0 e^{jk(\sin\alpha y+\cos\alpha z)} e^{-j\omega t}\mathbf{u}_x. \tag{53}$$

Relative to frame S' (53) can be written as

$$\begin{aligned}\mathbf{E}'_i(x',y',z',t') &= E_0 e^{j\left(k\sin\alpha y'+\gamma\left(k\cos\alpha-\omega v/c^2\right)z'\right)} e^{-j\gamma(\omega-kv\cos\alpha)t'}\mathbf{u}_{x'}, \\ &= E_0 e^{j\left(k'_y y'+k'_z z'\right)} e^{-j\omega' t'}\mathbf{u}_{x'}.\end{aligned} \tag{54}$$

In spherical coordinates (54) can be written as

$$\mathbf{E}'_i\left(x',y',z'\right) = E_0 e^{j\left(k'_y r' S_{\theta'} S_{\phi'}+k'_z r' C_{\theta'}\right)}\mathbf{u}_{x'}. \tag{55}$$

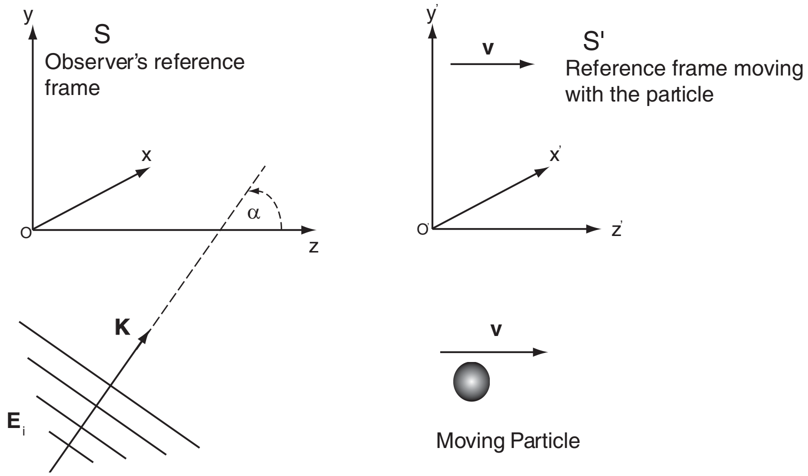


Figure 5. Non-parallel relative motion between the wave and the particle.

The next step is to expand the plane wave in spherical harmonics.

$$\mathbf{E}_i' = \sum_{n=1}^{\infty} \left(B_{o1n} \mathbf{M}_{o1n}^{(1)} + A_{e1n} \mathbf{N}_{e1n}^{(1)} \right). \tag{56}$$

The form of (55) is different to that of (3) of Section 2.1. The form of the transformed equation of the plane wave in Section 2.1 is similar to that of the standard Mie theory and therefore the spherical harmonic expansion coefficients B_{o1n} and A_{e1n} were equal to those of the standard Mie theory. (i.e., $B_{o1n} = j^n E_0(2n+1)/(n(n+1))$ and $A_{e1n} = -jE_0 j^n(2n+1)/(n(n+1))$). However, for the non-parallel case these coefficients will not be the same and the integrals involved in the determination of these coefficients are much more complicated than those involved in the parallel motion case. Nevertheless, once these coefficients are evaluated, the scattered field can be obtained using the method discussed in Section 2.1. That is, once B_{o1n} and A_{e1n} are evaluated, the same steps and field transformations can be used to determine the scattered field relative to the observer.

3. VALIDATION OF THE SCATTERING THEORY RESULTS

In this section, we carryout two tests to validate the results of the proposed theory. First, we show that the expressions derived for a plane wave scattering by a moving particle reduces to those of the standard Mie theory when the relative velocity is set to zero. Second, we show that when the refractive index of the particle is equal to that of the medium, no scattering takes place, as expected, and the field experienced by an observer moving with the particle is the same as that predicted by the relativistic Doppler formula [23].

3.1. Reduction to Standard Mie Theory When the Relative Velocity is Zero

Consider setting the relative velocity $\mathbf{v}$ to zero in the problem discussed in Section 2.1 so that the particle is stationary relative to the observer. Transformations given by (30) to (35) then reduce to

$$E_x = \gamma E_x', \tag{57}$$
$$E_y = \gamma E_y', \tag{58}$$
$$E_z = E_z', \tag{59}$$
$$H_x = \gamma L_1 L_2 H_x', \tag{60}$$
$$H_y = \gamma L_1 L_2 H_y', \tag{61}$$

$$H_z = L_1 L_2 H_z'. \tag{62}$$

In addition, when $v = 0$, $\gamma = 1$, $L_1 = 1$ and $L_2 = 1$. Therefore, (57) to (62) further reduce to

$$F_n = F_n', \tag{63}$$

where $F = E, H$ and $n = x, y, z$. Thus, when $v = 0$

$$\mathbf{E} = \mathbf{E}', \tag{64}$$

$$\mathbf{H} = \mathbf{H}'. \tag{65}$$

It is also evident that in this special case the two inertial reference frames S and S' are the same and thus the spherical harmonics $\mathbf{M}$ and $\mathbf{N}$ would remain unchanged. Using (13) and (14) in (64) and (65) with $v = 0$, for the scattered field with respect to frame S when the particle is stationary, we get

$$\mathbf{E}_s = \sum_{n=1}^{\infty} E_n \left(j a_n \mathbf{N}_{e1n}^{(3)} - b_n \mathbf{M}_{o1n}^{(3)} \right), \tag{66}$$

$$\mathbf{H}_s = \frac{k' c^2}{\mu_0 \omega'} Q \sum_{n=1}^{\infty} E_n \left(j b_n \mathbf{N}_{o1n}^{(3)} + a_n \mathbf{M}_{e1n}^{(3)} \right). \tag{67}$$

$$Q = \left(1 - \epsilon \mu_0 v^2\right) / \left(c^2 - v^2\right). \tag{68}$$

With respect to frame S, $\epsilon \mu_0 = 1/c^2$. Hence,

$$Q = \left(1 - v^2/c^2\right) / \left(c^2 - v^2\right) = 1/c^2.$$

When $v = 0$, $k' = k$ and $\omega' = \omega$. Equation (67) thus reduces to

$$\mathbf{H}_s = \frac{k}{\mu_0 \omega} \sum_{n=1}^{\infty} E_n \left(j b_n \mathbf{N}_{o1n}^{(3)} + a_n \mathbf{M}_{e1n}^{(3)} \right). \tag{69}$$

Using a similar argument for the field inside the scatterer, we get

$$\mathbf{E}_1 = \sum_{n=1}^{\infty} E_n \left(c_n \mathbf{M}_{o1n}^{(1)} - j d_n \mathbf{N}_{e1n}^{(1)} \right), \tag{70}$$

$$\mathbf{H}_1 = -\frac{k_1}{\mu_0 \omega} \sum_{n=1}^{\infty} E_n \left(d_n \mathbf{M}_{e1n}^{(1)} + j c_n \mathbf{N}_{o1n}^{(1)} \right). \tag{71}$$

Thus, it is evident from (66), (69), (70) and (71) that the expressions for the scattered electric and magnetic fields have reduced to the standard expressions (of Mie theory), given that the expressions for coefficients a_n, b_n, c_n and d_n reduce to the standard expressions when v is set to zero.

3.1.1. Reduction of Scattering Coefficients When the Particle is Stationary

When $v = 0$, $P_0 = 0$ and $P_1 = 0$. In addition, $k' = k$ and $k'_1 = k_1$. Hence (17) reduces to

$$\begin{aligned} a_n &= \frac{m^2 M_a A_a - J_a B_a}{m^2 M_a I_a - H_a B_a} \\ &= \frac{m^2 j_n(m\chi) D[\chi j_n(\chi)] - j_n(\chi) D[m\chi j_n(m\chi)]}{m^2 j_n(m\chi) D[\chi h_n^{(1)}(\chi)] - h_n^{(1)}(\chi) D[m\chi j_n(m\chi)]}. \end{aligned} \tag{72}$$

Equation (18) reduces to

$$b_n = \frac{M_a A_a - J_a B_a}{M_a I_a - H_a B_a} = \frac{j_n(m\chi) D[\chi j_n(\chi)] - j_n(\chi) D[m\chi j_n(m\chi)]}{j_n(m\chi) D[\chi h_n^{(1)}(\chi)] - h_n^{(1)}(\chi) D[m\chi j_n(m\chi)]}. \tag{73}$$

Equation (19) reduces to

$$c_n = \frac{J_a I_a - H_a A_a}{M_a I_a - H_a B_a} = \frac{j_n(\chi) D[\chi h_n^{(1)}(\chi)] - h_n^{(1)} D[\chi j_n(\chi)]}{j_n(m\chi) D[\chi h_n^{(1)}(\chi)] - h_n^{(1)}(\chi) D[m\chi j_n(m\chi)]}. \tag{74}$$

Equation (20) reduces to

$$\begin{aligned} d_n &= \frac{m J_a I_a - m H_a A_a}{m^2 M_a I_a - H_a B_a} \\ &= \frac{m j_n(\chi) D[\chi h_n^{(1)}(\chi)] - m h_n^{(1)}(\chi) D[\chi j_n(\chi)]}{m^2 j_n(m\chi) D[\chi h_n^{(1)}(\chi)] - h_n^{(1)}(\chi) D[m\chi j_n(m\chi)]}. \end{aligned} \tag{75}$$

Equations (66), (69), (70) and (71) together with (72) to (75) are the expressions for the scattered field of a plane wave scattered by a small stationary spherical particle [19]. Thus, the modified theory for the moving particle has been reduced to the standard Mie theory when the velocity of the particle is set to zero.

In the problem of moving source and stationary particle (relative to the observer) discussed in Section 2.2, when the relative velocity $\mathbf{v}_s$ is set to zero, $\gamma = 1$, $k' = k$ and $\omega' = \omega$. Hence, the problem reduces to the standard Mie scattering problem.

In the problem where both the source and the particle are in motion, as discussed in Section 2.3, when $\mathbf{v}_s$ and $\mathbf{v}$ is set to zero, $\gamma_1 = 1$, $\gamma_2 = 1$, $k' = k$ and $\omega' = \omega$. In addition, in the transformations of the field given by (47) to (52), $L_1 = 1$ and $L_2 = 1$. Then, those field transformations reduce to the same form as in (63). Therefore, using the same argument as for the case of the stationary source and the moving particle, we have the expressions reduced to those of the standard Mie theory when $\mathbf{v}_s = \mathbf{v} = 0$.

3.2. Comparison with Doppler Shift Formula

In this section, we show that the proposed formulation is comparable with the Doppler shift formula. We consider a particle with the same refractive index as that of the medium so that no scattering takes place and the whole system behaves as if there is a non-scattering object present. We show that, in this scenario, no scattering takes place and the field inside the particle is equal to the incident field. In addition, we show that an observer moving with the particle experiences a frequency shift of the plane wave equal to that predicted by the Doppler shift formula, as expected. For this comparison, we consider the case discussed in Section 2.1, where the source is stationary while the particle is in motion.

When the refractive index of the particle is equal to that of the medium, $\epsilon_1 = \epsilon_0$. Then in (17) to (20), $k_1' = k'$, $m = 1$, $Q_1 = Q_0$ and $P_1 = P_0$. In addition, since $\rho_1 = \rho$, $\chi_1 = \chi$, $B_a = A_a$ and $M_a = J_a$ (see Appendix B). Then,

$$\begin{aligned}
\Psi &= \left(jP_0\Gamma A_a - jP_0\Lambda J_a - k'Q_0\Delta J_a\chi - jP_0\Gamma A_a + jP_0\Lambda\frac{J_aA_a}{A_a} \right. \\
&\quad \left. +k'Q_0\Delta\frac{\chi J_aA_a}{A_a} \right) = 0. \\
\Omega &= \left(jP_0\Gamma I_a - jP_0\Lambda H_a - k'Q_0\Delta H_a\chi - jP_0\Gamma I_a + jP_0\Lambda\frac{J_aI_a}{A_a} \right. \\
&\quad \left. +k'Q_0\Delta\chi\frac{J_aI_a}{A_a} \right) = \left(jP_0\Lambda + k'Q_0\Delta\chi \right) \left(\frac{J_aI_a}{A_a} - H_a \right) \neq 0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
a_n &= \Psi/\Omega = 0. \\
Y &= \left(P_0\Gamma J_a\chi - jP_0XJ_a - jk'Q_0\Delta A_a + jP_0XH_a\frac{\Psi}{\Omega} - P_0\Gamma J_a\chi \right. \\
&\quad \left. +jP_0\frac{XJ_aA_a}{A_a} - jP_0\frac{XJ_aI_a}{A_a}\frac{\Psi}{\Omega} + jk'Q_0\Delta\frac{A_aJ_a}{J_a} \right) = 0. \\
Z &= \left(P_0\Gamma H_a\chi - jk'Q_0\Delta I_a - P_0\Gamma H_a\chi + jk'Q_0\Delta\frac{A_aH_a}{J_a} \right) \\
&= jk'Q_0\Delta\left(\frac{A_aH_a}{J_a} - I_a \right) \neq 0.
\end{aligned}$$

Therefore,

$$b_n = Y/Z = 0.$$

Then,

$$c_n = 1/J_a\left(J_a - H_a b_n\right) = 1,$$
$$d_n = 1/A_a\left(A_a - I_a a_n\right) = 1.$$

Using a_n and b_n in (13) and (14), for the scattered field with respect to frame S', we get

$$\mathbf{E}'_s = \mathbf{0},$$
$$\mathbf{H}'_s = \mathbf{0}.$$

Using c_n and d_n in (11) and (12) (and combining the time dependency term), for the field inside the particle with respect to frame S', we get

$$\begin{aligned}
\mathbf{E}'_1\left(x', y', z', t'\right) &= \sum_{n=1}^{\infty} E_n\left(\mathbf{M}^{(1)}_{o1n} - j\mathbf{N}^{(1)}_{e1n}\right)e^{-j\omega' t'}, \\
&= \mathbf{E}'_i\left(x', y', z'\right)e^{-j\omega' t'} \\
\mathbf{H}'_1\left(x', y', z', t'\right) &= \left[\frac{1}{\mu_0}\tilde{P}_0\sum_{n=1}^{\infty} E_n\left(\mathbf{v}\times\mathbf{M}^{(1)}_{o1n} - j\mathbf{v}\times\mathbf{N}^{(1)}_{e1n}\right)\right. \\
&\quad \left. -\frac{k'c^2}{\mu_0\omega'}Q_0\sum_{n=1}^{\infty} E_n\left(\mathbf{M}^{(1)}_{e1n} + j\mathbf{N}^{(1)}_{o1n}\right)\right]e^{-j\omega' t'}, \\
&= \mathbf{H}'_i\left(x', y', z'\right)e^{-j\omega' t'}.
\end{aligned}$$

From (2)

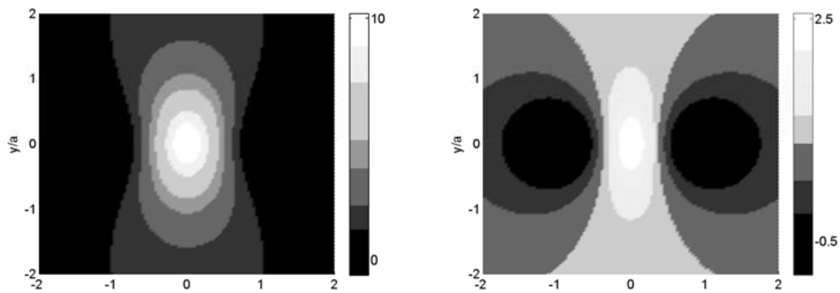
$$\begin{aligned}
\omega' &= \gamma(\omega - kv) = \gamma\left(\omega - \omega v/v_p\right), \\
&= \frac{c}{\sqrt{c^2 - v^2}}\left(1 - v/v_p\right)\omega,
\end{aligned}$$

where v is the speed of the observer, ω is the angular frequency of the plane wave relative to the medium and v_p is the phase velocity of the plane wave relative to the medium.

Thus, when the refractive index of the particle is equal to that of the medium, there is no scattered field, while the field inside the particle, as seen by an observer moving with the particle, is equal to the incident field in magnitude, but the frequency is shifted by the amount predicted by the special relativistic Doppler formula [23]. This shows that the derived formula is well-behaved even under the limiting conditions, making it widely usable for both strong and weak scattering scenarios.

4. NUMERICAL RESULTS AND DISCUSSION

In this section we present some numerical results that were obtained by simulating the proposed derivation.



(a) (b)

Figure 6. Distribution of the z-component of the Poynting vector within the xy-plane when the particle is stationary. (a) At $z = -a$ (plane for incoming radiation), (b) at $z = a$ (plane for outgoing radiation).

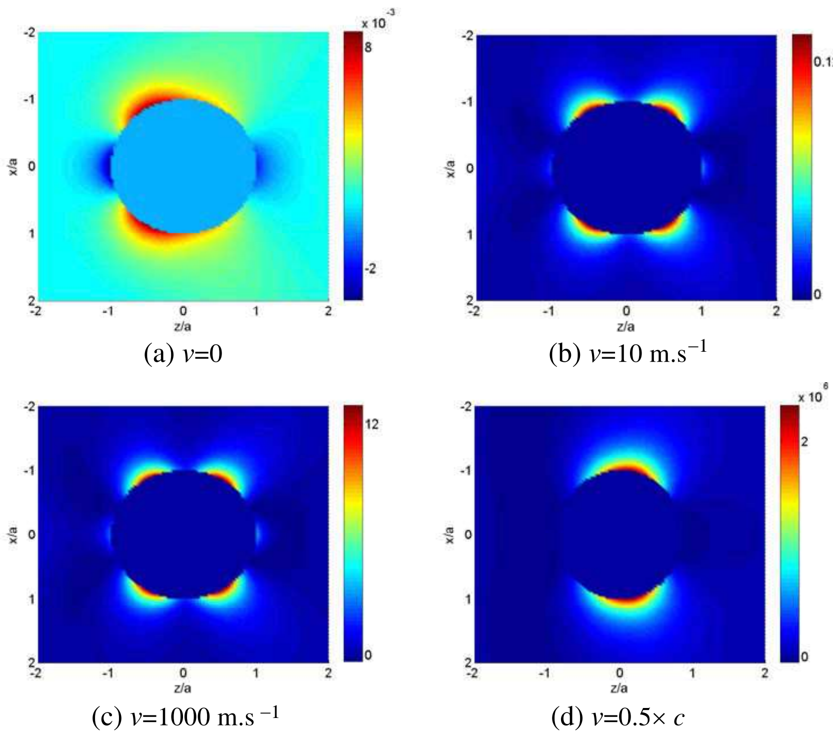


Figure 7. Distribution of the z-component of the Poynting vector within the xz-plane for different velocities.

Figure 6 shows the distribution of the z-component of the Poynting vector within the xy-plane at $z = -a$ and $z = a$ planes, when v is set to zero (i.e., the particle is stationary). For this figure we have set the simulation parameters to those used for Fig. 4 of [24]. In [24], Wang et al. obtained Fig. 4 using the classical Mie theory. Hence, Fig. 6 illustrates that the theory proposed in this paper produces the results of the classical Mie theory when the velocity of the particle is set to zero.

Figure 7 shows the distribution of the z-component of the Poynting vector within the xz-plane, around the particle, for different velocities. For this simulation we have set $E_0 = 1\,\mathrm{V/m}$, $a/\lambda = 3/4\pi$ and a non-magnetic particle with relative permittivity 12 was considered. Figs. 7(a), 7(b), 7(c) and 7(d) were obtained for $v = 0$, $v = 10\,\mathrm{m\cdot s^{-1}}$, $v = 1000\,\mathrm{m\cdot s^{-1}}$ and $v = 0.5$ times the speed of light, respectively. Hence, Fig. 7 illustrates the versatility of the proposed theory, which is accurate from very low to very high velocities. A comparison of Figs. 7(a) and 7(b) shows that the standard Mie theory is not a good approximation even at very low speeds, such as $10\,\mathrm{m\cdot s^{-1}}$.

5. CONCLUSION

In this paper, we addressed the problem of electromagnetic scattering by particles involving relative motion between the source and the particle. The proposed technique involves first transforming the problem to a reference frame co-moving with the particle using the Lorentz transformations. Then the Mie theory is applied relative to this frame. We have closely followed the steps of the standard derivation and obtained the scattered field relative to the reference frame co-moving with the particle. Field transformations were then used to transform this field to the reference frame of the observer. When the velocity of the particle and the source were set to zero relative to the observer, the results reduced to those of the standard Mie theory. Using simulations we showed that the standard Mie theory is not a good approximation even at very low speeds such as $10\,\mathrm{m\cdot s^{-1}}$. By assuming that the refractive index of the particle is equal to that of the medium, we showed that the results of the derivation are compatible with the relativistic Doppler formula.

APPENDIX A. EXPANSION OF THE MAGNETIC FIELD USING SPHERICAL HARMONICS

Here, we show how to express the magnetic field in terms of spherical harmonics using the expression for the electric field.

Using (5) in (9) and simplifying, we get

$$\begin{aligned}\mathbf{H}' &= \frac{1}{\mu_0}\left(\frac{c^2\epsilon\mu_0-1}{c^2-v^2}\right)\mathbf{v}\times\mathbf{E}' - j\frac{c^2}{\mu_0\omega'}\left(\frac{1-\epsilon\mu_0 v^2}{c^2-v^2}\right)\nabla'\times\mathbf{E}', \\ &= \frac{1}{\mu_0}\tilde{P}\mathbf{v}\times\mathbf{E}' - j\frac{c^2}{\mu_0\omega'}Q\nabla'\times\mathbf{E}', \end{aligned} \tag{A1}$$

where $\tilde{P} = \left(c^2\epsilon\mu_0 - 1\right)/\left(c^2 - v^2\right)$ and $Q = \left(1-\epsilon\mu_0 v^2\right)/\left(c^2-v^2\right)$. We then replace the terms $\mathbf{v}\times\mathbf{E}'$ and $\nabla'\times\mathbf{E}'$ using spherical harmonics.

$$\mathbf{v}\times\mathbf{E}_i' = \sum_{n=1}^{\infty} E_n\left[\mathbf{v}\times\mathbf{M}_{o1n}^{(1)} - j\mathbf{v}\times\mathbf{N}_{e1n}^{(1)}\right], \tag{A2}$$

$$\nabla'\times\mathbf{E}_i' = -jk'\sum_{n=1}^{\infty} E_n\left(\mathbf{M}_{e1n}^{(1)} + j\mathbf{N}_{o1n}^{(1)}\right), \tag{A3}$$

where $E_n = j^n\left(2n+1\right)/\left(n(n+1)\right)E_0$.

The velocity $\mathbf{v}$ can be decomposed into components as $\mathbf{v} = v\mathbf{u}_{z'} = vC_{\theta'}\mathbf{u}_{r'} - vS_{\theta'}\mathbf{u}_{\theta'}$. Using (A2) and (A3) in (A1) the incident magnetic field can be written using spherical harmonics as

$$\begin{aligned}\mathbf{H}_i' = &\frac{1}{\mu_0}\tilde{P}_0\sum_{n=1}^{\infty}E_n\left(\mathbf{v}\times\mathbf{M}_{o1n}^{(1)} - j\mathbf{v}\times\mathbf{N}_{e1n}^{(1)}\right) \\ &-\frac{k'c^2}{\mu_0\omega'}Q_0\sum_{n=1}^{\infty}E_n\left(\mathbf{M}_{e1n}^{(1)} + j\mathbf{N}_{o1n}^{(1)}\right).\end{aligned}$$

APPENDIX B. SOLUTION FOR THE SCATTERING COEFFICIENTS

Here, we present a detailed solution procedure for obtaining the scattering coefficients from the boundary conditions, as mentioned in Section 2.1.

Let $\Pi = \pi_n(C_{\theta'})$, $J = j_n(\rho)$, $M = j_n(\rho_1) = j_n(m\rho)$, $A = D[\rho j_n(\rho)] = \frac{d}{d\rho}[\rho j_n(\rho)]$, $B = D[\rho_1 j_n(\rho_1)] = D[m\rho j_n(m\rho)] = \frac{d}{d\rho}[m\rho j_n(m\rho)]$, $T = \tau_n(C_{\theta'})$, $H = h_n^{(1)}(\rho)$, $I = D[\rho h_n^{(1)}(\rho)]$, and let us use the subscript a assigned to the above notation to denote the evaluation of those functions on the boundary, where $r' = a$ and hence $\rho = \chi = k'a$ and $\rho_1 = \chi_1 = k_1'a$. (e.g., $J_a = j_n(\chi) = j_n(k'a)$ where as $J = j_n(\rho) = j_n(k'r')$).

Using $\mathbf{F} = \mathbf{E}$ and $\beta = \theta$ in the boundary condition given by (16) we get,

$$\Pi J_a - jT\frac{A_a}{\chi} + ja_nT\frac{I_a}{\chi} - b_n\Pi H_a - c_n\Pi M_a + jd_nT\frac{B_a}{m\chi} = 0. \tag{B1}$$

Using $\mathbf{F} = \mathbf{E}$ and $\beta = \phi$ in the boundary condition given by (16), we get,

$$-TJ_a + j\Pi\frac{A_a}{\chi} - ja_n\Pi\frac{I_a}{\chi} + b_nTH_a + c_nTM_a - jd_n\Pi\frac{B_a}{m\chi} = 0. \quad \text{(B2)}$$

Using $\mathbf{F} = \mathbf{H}$ and $\beta = \theta$ in the boundary condition given by (16), we get,

$$\begin{aligned}
&P_0C_{\theta'}TJ_a - jP_0C_{\theta'}\Pi\frac{A_a}{\chi} + \frac{k'c^2}{\omega'}Q_0\Pi J_a - j\frac{k'c^2}{\omega'}Q_0T\frac{A_a}{\chi}\\
&+jP_0a_nC_{\theta'}\Pi\frac{I_a}{\chi} - P_0b_nC_{\theta'}TH_a + j\frac{k'c^2}{\omega'}Q_0b_nT\frac{I_a}{\chi} - \frac{k'c^2}{\omega'}Q_0a_n\Pi H_a\\
&-P_1c_nC_{\theta'}TM_a + jP_1d_nC_{\theta'}\Pi\frac{B_a}{m\chi} - \frac{k_1'c^2}{\omega'}Q_1d_n\Pi M_a\\
&+j\frac{k_1'c^2}{\omega'}Q_1c_nT\frac{B_a}{m\chi} = 0. \qquad \text{(B3)}
\end{aligned}$$

Using $\mathbf{F} = \mathbf{H}$ and $\beta = \phi$ in the boundary condition given by (16), we get,

$$\begin{aligned}
&P_0C_{\theta'}\Pi J_a - jP_0C_{\theta'}T\frac{A_a}{\chi} - jP_0n(n+1)S^2_{\theta'}\Pi\frac{J_a}{\chi} + \frac{k'c^2}{\omega'}Q_0TJ_a\\
&-j\frac{k'c^2}{\omega'}Q_0\Pi\frac{A_a}{\chi} + jP_0a_nC_{\theta'}T\frac{I_a}{\chi} + jP_0a_nn(n+1)S^2_{\theta'}\Pi\frac{H_a}{\chi}\\
&-P_0b_nC_{\theta'}\Pi H_a + j\frac{k'c^2}{\omega'}Q_0b_n\Pi\frac{I_a}{\chi} - \frac{k'c^2}{\omega'}a_nQ_0TH_a - P_1c_nC_{\theta'}\Pi M_a\\
&+jP_1d_nC_{\theta'}T\frac{B_a}{m\chi} + jP_1d_nn(n+1)S^2_{\theta'}\Pi\frac{M_a}{m\chi} - \frac{k_1'c^2}{\omega'}Q_1d_nTM_a\\
&+j\frac{k_1'c^2}{\omega'}Q_1c_n\Pi\frac{B_a}{m\chi} = 0 \qquad \text{(B4)}
\end{aligned}$$

Multiplying (B1) by T and (B2) by Π and adding the two resulting equations we get

$$mI_aa_n + B_ad_n = mA_a. \quad \text{(B5)}$$

Multiplying (B1) by Π, multiplying (B2) by T and adding the resulting two equations, we get

$$M_ac_n + H_ab_n = J_a. \quad \text{(B6)}$$

Multiplying (B3) by T, multiplying (B4) by Π and taking the difference of the resulting two equations, we get

$$\begin{aligned}
&-P_0 C_{\theta'} J_a(\Pi\Pi - TT) + jP_0 n(n+1) S_{\theta'}^2 \Pi\Pi \frac{J_a}{\chi} \\
&+j\frac{k'c^2}{\omega'} Q_0 \frac{A_a}{\chi}(\Pi\Pi - TT) - jP_0 a_n n(n+1) S_{\theta'}^2 \Pi\Pi \frac{H_a}{\chi} \\
&+P_0 b_n C_{\theta'} H_a(\Pi\Pi - TT) - j\frac{k'c^2}{\omega'} Q_0 b_n \frac{I_a}{\chi}(\Pi\Pi - TT) \\
&+P_1 c_n C_{\theta'} M_a(\Pi\Pi - TT) - jP_1 d_n n(n+1) S_{\theta'}^2 \Pi\Pi \frac{M_a}{m\chi} \\
&-j\frac{k_1'c^2}{\omega'} c_n Q_1 \frac{B_a}{m\chi}(\Pi\Pi - TT) = 0. \qquad \text{(B7)}
\end{aligned}$$

where $\Pi\Pi = \Pi \times \Pi$, $TT = T \times T$. Multiplying (B3) by Π, multiplying (B4) by T and taking the difference of the resulting two equations, we get

$$\begin{aligned}
&-jP_0 C_{\theta'} \frac{A_a}{\chi}(\Pi\Pi - TT) + jP_0 n(n+1) S_{\theta'}^2 \Pi T \frac{J_a}{\chi} \\
&+\frac{k'c^2}{\omega'} Q_0 J_a(\Pi\Pi - TT) + jP_0 a_n C_{\theta'} \frac{I_a}{\chi}(\Pi\Pi - TT) \\
&-jP_0 a_n n(n+1) S_{\theta'}^2 T\Pi \frac{H_a}{\chi} - \frac{k'c^2}{\omega'} Q_0 a_n H_a(\Pi\Pi - TT) \\
&+jP_1 d_n C_{\theta'} \frac{B_a}{m\chi}(\Pi\Pi - TT) - jP_1 d_n n(n+1) S_{\theta'}^2 T\Pi \frac{M_a}{m\chi} \\
&-\frac{k_1'c^2}{\omega'} Q_1 d_n M_a(\Pi\Pi - TT) = 0. \qquad \text{(B8)}
\end{aligned}$$

Let $\Gamma = C_{\theta'}(\Pi\Pi - TT)$, $\Delta = \frac{c^2}{\omega'}(\Pi\Pi - TT)$, $\Lambda = n(n+1)S_{\theta'}^2 \Pi T$, $X = n(n+1)S_{\theta'}^2 \Pi\Pi$. Using (B5) in (B8) (i.e., substituting for d_n in terms of a_n), we get

$$a_n = \frac{\Psi}{\Omega}, \qquad \text{(B9)}$$

where

$$\begin{aligned}
\Omega = \Bigg(& jP_0 \Gamma I_a - jP_0 \Lambda H_a - k' Q_0 \Delta H_a \chi - jP_1 \Gamma I_a + jP_1 \Lambda \frac{M_a I_a}{B_a} \\
&+ k_1' Q_1 \Delta m\chi \frac{M_a I_a}{B_a} \Bigg),
\end{aligned}$$

$$\Psi = \left(jP_0\Gamma A_a - jP_0\Lambda J_a - k'Q_0\Delta J_a\chi - jP_1\Gamma A_a + jP_1\Lambda\frac{M_aA_a}{B_a} + k_1'Q_1\Delta\frac{m\chi M_aA_a}{B_a}\right).$$

Using (B9) in (B5) we get

$$d_n = \frac{m}{B_a}\left(A_a - \frac{I_a\Psi}{\Omega}\right). \tag{B10}$$

Using (B6), (B9) and (B10) in (B7), we get

$$b_n = \frac{Y}{Z},$$

where

$$Y = \left(P_0\Gamma J_a\chi - jP_0XJ_a - jk'Q_0\Delta A_a + jP_0XH_a\frac{\Psi}{\Omega} - P_1\Gamma J_a\chi + jP_1\frac{XM_aA_a}{B_a} - jP_1\frac{XM_aI_a}{B_a}\frac{\Psi}{\Omega} + jk_1'Q_1\Delta\frac{B_aJ_a}{mM_a}\right),$$

$$Z = \left(P_0\Gamma H_a\chi - jk'Q_0\Delta I_a - P_1\Gamma H_a\chi + jk_1'Q_1\Delta\frac{B_aH_a}{mM_a}\right).$$

Then

$$c_n = \frac{1}{M_a}\left(J_a - H_a\frac{Y}{Z}\right).$$

APPENDIX C. FIELD TRANSFORMATIONS BETWEEN FRAMES S AND S'

Here we show how the Cartesian field components of inertial frame S' are transformed to those of inertial frame S.

The electromagnetic field transformations between the two inertial reference frames S and S', which are in standard configuration, are given by [22]

$$\begin{aligned}
E_x &= \gamma\left(E_x' + \frac{v}{c}B_y'\right),\\
E_y &= \gamma\left(E_y' - \frac{v}{c}B_x'\right),\\
E_z &= E_z',\\
B_x &= \gamma\left(B_x' - \frac{v}{c}E_y'\right),\\
B_y &= \gamma\left(B_y' + \frac{v}{c}E_x'\right),\\
B_z &= B_z'.
\end{aligned}$$

From Minkowski constitutive relations (i.e., from (6) and (7)) we can determine $\mathbf{B}'$ from $\mathbf{E}'$ and $\mathbf{H}'$ as follows.

$$\mathbf{B}' = \frac{1}{1-\epsilon\mu_0 v^2}\left(-\frac{\mu_0 v^2}{c^2}\mathbf{H}' - \epsilon\mu_0 \mathbf{v}\times\mathbf{E}' + \frac{1}{c^2}\mathbf{v}\times\mathbf{E}' + \mu_0\mathbf{H}'\right). \quad \text{(C1)}$$

$$\mathbf{v}\times\mathbf{E}' = -vE'_y\mathbf{u}_{x'} + vE'_x\mathbf{u}_{y'}. \quad \text{(C2)}$$

Using (C2) in (C1), for the x, y and z-components of magnetic flux density with respect to frame S' we have,

$$\begin{aligned}
\mathbf{B}'_x &= \frac{1}{1-\epsilon\mu_0 v^2}\left(\mu_0\left(1-\frac{v^2}{c^2}\right)H'_x - v\left(\frac{1}{c^2}-\epsilon\mu_0\right)E'_y\right), \\
\mathbf{B}'_y &= \frac{1}{1-\epsilon\mu_0 v^2}\left(\mu_0\left(1-\frac{v^2}{c^2}\right)H'_y + v\left(\frac{1}{c^2}-\epsilon\mu_0\right)E'_x\right), \\
\mathbf{B}'_z &= \frac{1}{1-\epsilon\mu_0 v^2}\mu_0\left(1-\frac{v^2}{c^2}\right)H'_z.
\end{aligned}$$

Therefore, the electromagnetic field transformations can be written as

$$\begin{aligned}
E_x &= \gamma\left(E'_x + \frac{v}{c}\mu_0 L_1 L_2 H'_y + \frac{v^2}{c}L_1 L_3 E'_x\right), \\
E_y &= \gamma\left(E'_y - \frac{v}{c}\mu_0 L_1 L_2 H'_x + \frac{v^2}{c}L_1 L_3 E'_y\right), \\
E_z &= E'_z, \\
H_x &= \gamma\left(L_1 L_2 H'_x - \frac{v}{\mu_0}L_1 L_3 E'_y - \frac{v}{\mu_0 c}E'_y\right), \\
H_y &= \gamma\left(L_1 L_2 H'_y + \frac{v}{\mu_0}L_1 L_3 E'_x + \frac{v}{\mu_0 c}E'_x\right), \\
H_z &= L_1 L_2 H'_z,
\end{aligned}$$

where $L_1 = 1/(1-\epsilon\mu_0 v^2)$, $L_2 = 1-(v^2/c^2)$ and $L_3 = 1/c^2 - \epsilon\mu_0$.

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