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ENHANCEMENT OF PHOTONIC BAND GAP IN A DISORDERED QUARTER-WAVE DIELECTRIC PHOTONIC CRYSTAL

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Abstract—The enhancement of the photonic band gap in visible region for a disordered one-dimensional dielectric-dielectric photonic crystal (DDPC) is theoretically investigated. The DDPC is made of alternating two high/low-index quarter-wave dielectric layers stacked periodically. A disordered DDPC is modeled by randomly changing the real thicknesses, or, the optical lengths, of the two dielectrics. In a single disorder case, where the disorder only appears in one of the two constituents, it is found the photonic band gap can be preferably enhanced for the disordered high-index layer. In the double disorder stack, in which both the constituent layers are disordered, the photonic band gap can, however, be significantly enlarged. In addition, numerical results illustrate that a flat band gap can be obtained by the use of disorder in the optical length.

1. INTRODUCTION

A planar periodic stack structure made of high/low-index dielectric bilayers is called a dielectric mirror (DM) or a distributed Bragg reflector (DBR). Now it is generally referred to as a one-dimensional dielectric-dielectric photonic crystal (1D DDPC) and has attracted much attention because it can be easily fabricated by modern experimental techniques. One of the most useful applications for a 1D DDPC is that it can act as an excellent optical reflector [1], playing an important part in the modern solid-state laser systems. The principle of an optical reflector comes from the existence of the high-reflectance bands or photonic band gaps (PBG) in some certain frequency (or wavelength) ranges in a PC. Other possible useful applications of 1DPC

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include the time delay device [2], high quality filter [3, 4], nonlinear diode [5], temperature and optical sensors [6, 7], and polarizer [8], as well.

A simple 1D DDPC can be commonly formed by using the so-called quarter-wave stack, $(HL)^N$, where N is the number of periods and H, L are the quarter-wave layers with their optical thicknesses equal to $n_H d_H = n_L d_L = \lambda_0/4$, where λ_0 is the design wavelength usually designed to fall around the center of PBG and n_i, d_i, $(i = H, L)$ are their refractive indices and real thicknesses, respectively. In modern photonic applications a PC with wider PBG is often needed and useful for some purposes. There are several methods for widening the PBG. According to the theory of 1DPC, the band gap can be enlarged by the increase in the refractive index contrast n_H/n_L [9]. Also as mentioned in Ref. [9], "Broadband reflectors can also be made by using aperiodic layered medium in which the local period is an increasing (or decreasing) function of position. Such structures are called chirped periodic layered medium." For the band gap enlargement based on the chirped PCs, we mention Refs. [10–13]. Using the unequal optical lengths in the constituent bilayer, PBG is also expected to be enhanced [14]. The PBG can be further significantly extended in a heterostructured PC that is formed by cascading two or more different PCs [15–18].

In addition to the above-mentioned methods of enlargement of PBG, there is another alternative, that is, a disordered 1DPC [19–23]. In Ref. [23], a disordered PC is studied in a 1D metal-dielectric photonic crystal (MDPC), in which the system contains a metal film, Ag, sandwiched by two dielectric layers. With the additional metal film, it becomes 1D ternary MDPC and consequently the band gap can be extended pronouncedly. In this paper, we design a disordered 1D binary DDPC that is operated in the visible region. We shall show that the band gap extension can be preferably seen when the disorder is incorporated in the high-index layer. Moreover, a salient extension in the band gap can be seen as the both layers are disordered in the system. The analysis will be made through the reflectance calculated by making use of the transfer matrix method (TMM) [9].

2. THEORY

The 1D DDPC is modeled as an ideal dielectric mirror, $Air/(HL)^N/S$, as depicted in Fig. 1, in which the high/low-index quarter-wavelength layers H and L are stacked periodically on the substrate S (assumed to be semi-infinite). The real thicknesses for H and L are respectively denoted as $d_H = \lambda_0/4n_H$ and $d_L = \lambda_0/4n_L$, where λ_0 is the design

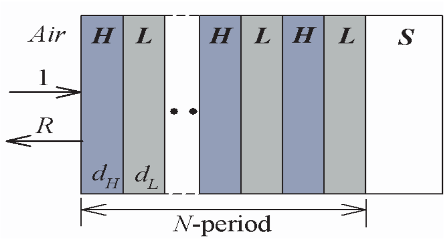

Figure 1. An ideal dielectric mirror made of high/low-index quarter-wavelength layers H and L, i.e., $n_H d_H = n_L d_L = \lambda_0/4$, where λ_0 is the design wavelength.

wavelength. In addition, the number of periods is N.

The ideal dielectric mirror then no longer exists when the disorder is incorporated in the structure, leading to a so-called disorder dielectric mirror or disorder DDPC. The disorder is obtained by a certain statistical distribution of layer thickness, that is, the degree of disorder D is defined by the deviation from the ideal real thickness [20]:

$$D = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[(d_{H,i} - d_H)^2 + (d_{L,i} - d_L)^2\right]}}{d_H + d_L}, \tag{1}$$

where d_H and d_L are the real thicknesses for an ideal dielectric mirror, and $d_{H,i}$ and $d_{L,i}$ are the real thicknesses in the presence of the disorder in the ith period, where i runs over from 1 to N. In general, $d_{H,I}$, $d_{L,i}$ and d_H, d_L are related by the variational parameters Δx_H and Δx_L in the following relationship,

$$d_{H,i} = d_H + m_i \Delta x_H, \quad d_{L,i} = d_L + m_i \Delta x_L, \tag{2}$$

where the discrete numbers m_i are randomly chosen from a Gaussian-like distribution around d_H or d_L, the real thickness of the non-disordered ideal quarter-wave stack, which is the case of $m_i = 0$. For $m_i > 0$, the disordered real thickness is larger than the ideal real thickness, whereas it will be smaller than the ideal real thickness at $m_i < 0$. A disordered thickness distribution is achieved as follows: First, we arbitrarily select a fixed value in D. Second, with a chosen λ_0, we have the non-disordered real thicknesses, $d_H = \lambda_0/4n_H$ and $d_L = \lambda_0/4n_L$. Third, a set of discrete numbers in m_i is given. For the single disorder case such as disorder in high-index layer, we have $\Delta x_H \neq 0$ and $\Delta x_L = 0$. With the above conditions and substituting Eq. (2) into Eq. (1) enables us to determine Δx_H, which, according to Eq. (2), in turn gives the real thickness for the high-index layer in each period. Similarly, we can obtain Δx_L for the single disorder in low-index layer. As for the double disorder, we take $\Delta x_H = \Delta x_L = \Delta x$. We can also easily determine Δx according to Eqs. (1) and (2). It should be mentioned that the sets of m_i can, of course, be arbitrarily assigned for both high- and low-index layers. However, for the convenience of comparison, in the later calculation, we shall take the same set of m_i for the two constituent layers.

In addition to the definition of order of disorder in Eq. (1) to make a disordered 1D DDPC, another method based on the deviation of optical length is also available. In this case, the order of disorder is defined as [21]:

$$D = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[n_H^2\left(d_{H,i}-d_H\right)^2+n_L^2\left(d_{L,i}-d_L\right)^2\right]}}{n_H d_H + n_L d_L}. \tag{3}$$

Here the disordered and ideal thicknesses are also related by Eq. (2). We shall calculate the PBG for the disordered 1D DDPC by using these two schemes simultaneously later.

Having obtained the disordered thicknesses for the entire system, the normal-incidence reflectance R can be calculated by making use of transfer matrix method (TMM). According to TMM, the total transfer matrix is expressed as [9]

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \mathbf{D}_A^{-1} \prod_{i=1}^{N}\left[\mathbf{D}_H\mathbf{P}_{H,i}\mathbf{D}_H^{-1}\mathbf{D}_L\mathbf{P}_{L,i}\mathbf{D}_L^{-1}\right]\mathbf{D}_S, \tag{4}$$

where the dynamical matrix in Eq. (4) for the medium q is dependent only on its refractive index given by

$$\mathbf{D}_q = \begin{pmatrix} 1 & 1 \\ n_q & -n_q \end{pmatrix}, \quad q = A(Air),\ H,\ L, \text{ and } S, \tag{5}$$

and the translational matrix in H is dependent on the real disordered thickness expressed as

$$\mathbf{P}_{H,i} = \begin{pmatrix} \exp\left(j\phi_{H,i}\right) & 0 \\ 0 & \exp\left(-j\phi_{H,i}\right) \end{pmatrix}, \tag{6}$$

where the phase is $\phi_{H,i} = 2\pi n_H d_{H,i}/\lambda$ with λ being the wavelength of the incident wave. Here the time part, $\exp(j\omega t)$, have been used for all fields. Similarly, $\mathbf{P}_{L,i}$ can be obtained with $\phi_{L,i} = 2\pi n_L d_{L,i}/\lambda$. Then the reflection coefficient r is calculated through the matrix elements in Eq. (4), namely

$$r = \frac{M_{11}}{M_{21}}, \tag{7}$$

which in turn gives rise to the reflectance, i.e., $R = |r|^2$.

3. NUMERICAL RESULTS AND DISCUSSION

Let us now present the numerical results for a typical example. The high/low-index layers H and L are taken to be zinc sulfide (ZnS, $n_H = 2.3$) and magnesium fluoride (MgF_2, $n_L = 1.38$), respectively. The substrate S is glass with $n_S = 1.52$. All the above refractive indices are available in Ref. [14]. The design wavelength is $\lambda_0 = 500\,\text{nm}$ and the number of periods is $N = 11$. The discrete numbers, m_i, $i = 1, 2, \ldots, N$ in Eq. (2) are taken to be -40, -26, -15, -13, -4, 0, 4, 13, 15, 26, and 40.

In Fig. 2, we plot the wavelength-dependent reflectance for a single disorder dielectric mirror at three different degrees of disorder $D = 0.05$ (curve B), 0.1 (curve C), and 0.15 (curve D). Here the single disorder means that the disorder is made only in the high-index layers of ZnS, but the low-index MgF_2-layer remains unchanged. The left Figure 2(a) is calculated based on the disorder in the real thickness as in Eq. (1), while in the right Figure 2(b) the disorder is made based on the optical length in Eq. (3). In addition, the curve A is for an ideal quarter-wave dielectric mirror whose band edges can be analytically determined based on the theory of dielectric mirror, namely [14]

$$\lambda_L = \frac{\pi\left(n_H d_H + n_L d_L\right)}{a\cos\left(-\rho\right)}, \quad \lambda_R = \frac{\pi\left(n_H d_H + n_L d_L\right)}{a\cos\left(\rho\right)}, \tag{8}$$

where the Fresnel coefficient is $\rho = (n_H - n_L)/(n_H + n_L)$. With the given material parameters, the calculated left and right band edges are $\lambda_L = 430.7\,\text{nm}$ and $\lambda_R = 595.8\,\text{nm}$, in good agreement with the locations shown in Fig. 2.

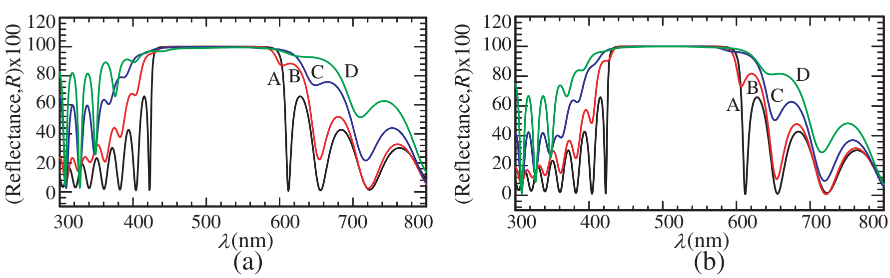

Figure 2. Reflectance response as a function of the wavelength for the ideal (A) and disorder dielectric mirrors (B, C, and D). The single disorder is designed on ZnS layers with $D = 0.05$ (B), 0.1 (C), and 0.15 (D) and $N = 11$. Here (a) is for the disorder made by Eq. (1) and (b) is by Eq. (3).

With the introduction of disorder, it can be seen that the side bands are totally raised up. The high-reflectance range (HRR) is obviously enhanced as D is larger than 0.1 (curves C and D). For a small disorder $D = 0.01$ (curve B), HRR is not enlarged, but even shrinks a little. That is, in order to extend HRR, the degree of disorder D cannot be chosen too small, and it should be better as D is larger than 0.1. In addition, it is found the HRR is more flat based on the disorder in optical length.

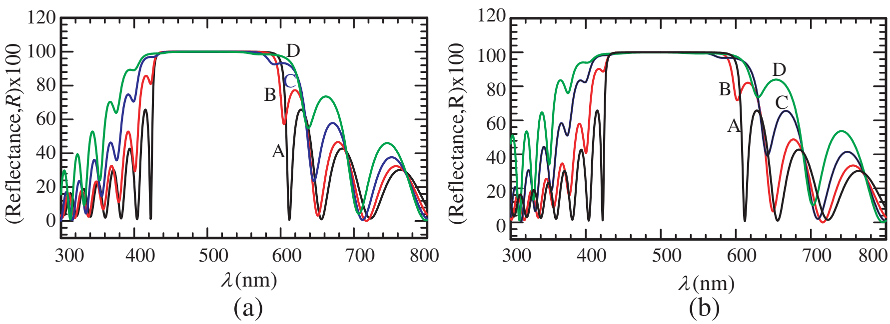

Figure 3. Reflectance response as a function of the wavelength for the ideal (A) and disorder dielectric mirrors (B, C, and D). The single disorder is designed on MgF_2 layers with $D = 0.05$ (B), 0.1 (C), and 0.15 (D) and $N = 11$. Here (a) is for the disorder made by Eq. (1) and (b) is by Eq. (3).

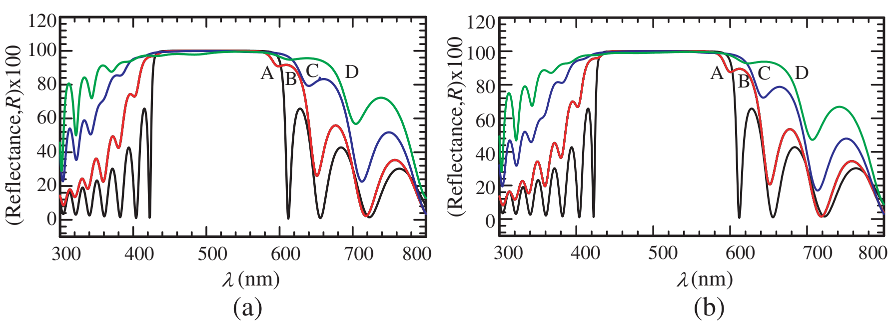

Figure 4. Reflectance response as a function of the wavelength for the ideal (A) and double disorder dielectric mirrors (B, C, and D). The double disorder is designed on both ZnS and MgF_2 layers with equal $D = 0.05$ (B), 0.1 (C), and 0.15 (D) and $N = 11$. Here (a) is for the disorder made by Eq. (1) and (b) is by Eq. (3).

If now the single disorder is incorporated in the thickness of the low-index layer (MgF_2), but the thickness of high-index layer is kept unchanged. In this case, the wavelength-dependent reflectance at three different degrees of disorder $D = 0.05$, 0.1, and 0.15 is plotted in Fig. 3, where (a) is for the disorder in the real thickness and (b) is for the disorder in the optical length, respectively. It is seen that the side bands are lifted up as the disorder is introduced, showing the same feature as in Fig. 2. However, HRR is not changed so salient as in Fig. 2, indicating that the effect of disorder on HRR due to the low-index layer is weak. For $D = 0.1$ and 0.15, the HRR near right band edge is more flat in the disorder scheme based on the optical length.

In Fig. 4, we plot the reflectance response for the case of double-disorder DDPC, in which the disorder in real thickness is shown in (a) and the disorder in optical length is in (b). Here both the high/low-index layers are disordered with an equal degree of disorder $D = 0.01$ (curve B), 0.1 (curve C), and 0.15 (curve D), respectively. It can be seen from the figure that side bands are greatly raised up and consequently HRR is considerably enhanced. For $D = 0.15$, the flat top HRR is seen. However, there are some slow and small variations in the original flat top band at a larger D-value in (a). The existence of such slow and small variations appearing in HRR can be ascribed to the limited random numbers, m_i in Eq. (2), (in fact, $N = 11$) in making a disorder DDPC. If more numbers of m_i are taken, say $N = 19$, it is believed that a more flat top HRR will be recovered [20]. This slow varying HRR is not seen when we use the disorder from the optical length, as illustrated in (b), where a wide HRR is shown.

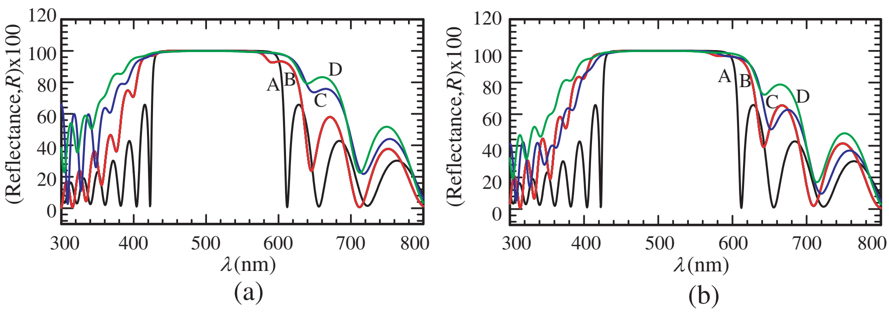

Figure 5. Reflectance response as a function of the wavelength at $N = 11$. Curve A: The ideal quarter-wavelength stack; Curve B: $D = 0.1$, single disorder for MgF_2; Curve C: $D = 0.1$, single disorder for ZnS; Curve D: $D = 0.1$ double disorder. Here (a) is for the disorder made by Eq. (1) and (b) is by Eq. (3).

The above results are summarized and illustrated in Fig. 5, in which we have taken single disorder in Fig. 2 (with $D = 0.1$) and 3 (with $D = 0.1$), and double disorder in Fig. 4 ($D = 0.1$) for disorder in the real thickness (a) and in optical length (b), respectively. The effect of disorder on the HRR is seen in this figure. In the single disorder design, introducing the disorder in the high-index layer can be used to apparently enhance band gap. As for the double disorder case, the band gap can be further enlarged considerably.

4. CONCLUSION

By designing the disorder in the real thickness or the optical length of the constituent layer of a 1D DDPC, the high-reflectance photonic band gap can be enhanced. For the single disorder design we have seen that the disorder in high-index layer is better than in low-index layer in order to widen the HRR. In the case of double disorder a much wider band gap can be obtained. It also shows that a better and more flat HRR is seen when we design the disorder in the optical length.

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