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WAVES PIER 10

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Methods for Modeling and Simulation of Guided-Wave Optoelectronic Devices:

Part I: Modes and Couplings

Editor: W. P. Huang

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METHODS FOR MODELING AND SIMULATION OF GUIDED-WAVE OPTOELECTRONIC DEVICES: PART I: MODES AND COUPLINGS

PREFACE

Optoelectronics is an emerging technology that melds the capability of photonics and microelectronics and has produced a variety of novel and useful devices for telecommunications, interconnections and signal processings. There has been tremendous progress in materials, fabrication technologies, packaging and integration, as well as system applications of optoelectronic devices. For example, modern epitaxial growth and lithography techniques allow precise control and engineering of material and geometrical properties. As a result, high-performance devices such as strained-layer multiple-quantum-well lasers and modulators have been demonstrated. So far, most of the these devices are still in the research and development stage and wide-spread applications of sophisticated optoelectronic devices are yet to be realized.

One of the difficulties for engineering, manufacturing as well as research and development of optoelectronic devices for system applications is the lack of powerful computerized design tools based on modeling and simulation. Up to now, development and engineering of optoelectronic devices still follow a pattern of design, fabrication, characterization and re-design process. Such an approach, though fruitful in the early research stage in discovering and verifying new ideas, is no longer adequate for optimization and timely engineering of sophisticated optoelectronic devices to specifications. The situation becomes even more acute for the advanced optoelectronic devices in which the number of adjustable material and geometrical parameters is large. In this respect, computer-aided modeling and simulation may be used to gain useful insight beyond intuition, to assist in detailed device characterization and optimization, and to increase efficiency and reduce cost. Therefore, it is imperative to develop powerful computer-aided modeling and simulation tools that are accurate, efficient, robust and user-friendly.

The objectives of PIER 10 and 11 are to report on the state-of-the-art methods being employed and/or developed for modeling and analysis of guided-wave optoelectronic devices. The focus is on the mathematical (both analytical and numerical) techniques that are adapted and applied to the waves and fields as described by Maxwell's equations with approximations suitable for optical guided-wave devices. Physical ideas and device concepts of guided-wave optoelectronics will be discussed in conjunction with the modeling and analysis methods, but will not be the central concern. The book will serve as a reference book for the research workers and graduate students who are interested in optoelectronics. In particular, it will allow those who have been in the field of optoelectronics to familiarize the various approaches to the modeling and analysis. In addition, it will be helpful for those who wish to enter the field to understand the features and the subtleties pertinent to the electromagnetic waves in optoelectronics.

PIER Volume 10, Part I of the two-volume series entitled "methods for modeling and simulation of guided-wave optoelectronic devices", deals with techniques for calculating and analyzing guided modes and their couplings in various optical waveguide structures. Optical modes are defined as the solutions to the eigen-value problems for longitudinally-invariant waveguides. Ideal waveguide structures are used as the references and building-blocks for more complex waveguide structures (or perturbed structures), in which the fields are represented as a linear combination of the modal fields of the reference structures. Since the modal fields are not the eigen modes of the perturbed structures, these modes will therefore couple with each other. The coupling-of-modes phenomenon in an optical guided-wave structure is the basis for operation of a wide range of optoelectronic devices and a key to analysis and design of these devices.

Chapter 1 by Benson and Kendall describes a number of variational techniques to calculate both scalar and vectorial modes of buried or channel waveguide structures. It starts with a systematic description of the effective-index method (EI), which is one of the most popular approaches in optical channel waveguide mode calculation. Subsequently, the weighted-index method (WI) originally developed by Kendall and co-workers is presented in great detail. This elegant method is built on the variational principle. It is able to produce the best separable solutions to the scalar wave equations for the channel waveguides by solving successively slab problems. This algorithm is very fast and also highly accurate for waveguide structures such as rectangular and ridge waveguides. When separable solutions are no longer sufficient, one can assume other types of trial functions in the direct variational method and obtain the optimum solutions using some standard numerical optimization schemes. Once the scalar modes are obtained, the polarization effects may be considered by perturbation methods. A general formulation for the polarization corrections is presented in Chapter 1, too.

In Chapter 2, Burke presents a fast and accurate semi-analytical method – the spectral-index method (SI) in the context of optical rib and ridge waveguides. By recognizing that the lateral distribution of the refractive index below the rib is uniform, the two-dimensional wave equation in this region may be reduced to a one-dimensional one by Fourier transform. The resultant equations in the spectral-domain can be readily solved as a slab problem. On the other hand, in the rib region, the field is laterally well confined and separable trial functions may be assumed in the spatial domain. Such trial functions take particularly simple forms when the effective waveguide width is introduced, considering that the index differences between the rib and the surrounding media are usually large. Finally, the field expressions for the two regions are matched along the boundary between the rib and the slab regions, leading to a transcendental equations for the propagation constants of the guided modes supported by the structures. In comparison with the weighted-index method (WI) in Chapter 1, the trial solutions used in the spectral-index method are not separable and yield more accurate solutions for the rib/ridge waveguides as the asymmetry of the lateral field distributions within and below the rib is accounted for. One of the attractive features of the SI is that it may be readily extended to the complex domain to treat lossy and amplifying media. In addition, one can calculate leaky modes by simply extending the transcendental equation for the propagation constants to the complex domain [1]. The idea of the spectral-domain method used in the spectral-index method has also been applied to the calculation of reflections and transmissions at semiconductor laser facets [2]. Recently, the spectral-domain approach has been further developed into an exact method by rigorously treating the rib and any other regions where the lateral index distributions are not uniform [3,4]. The same approach is also used to treat the problems of junctions and laser facets [5].

Another semi-analytical method – the mode-matching method (MM) – is described by Dagli in Chapter 3. For optical rib or ridge waveguides, the mode-matching method expresses the model fields in terms of the modes of the slab waveguides in the vertical dimensions. By doing so, a set of coupled ordinary equations are derived and can be readily solved. These solutions are subsequently made to matching the boundary conditions at the rib-slab interfaces or other interfaces along the horizontal dimensions. The resultant equations are used to determine the modes supported by the structures. Both scalar and vectorial modes may be calculated, depending the types of slab waveguide modes used. While such a procedure appears to be straightforward, a difficulty in the mode-matching techniques is how to treat the field components related to the continuous radiation modes in an open waveguide structure. This difficulty may be overcome by discretizing the continuous spectra through artificially conducting planes placed far away from the guiding region. An alternative approach using an orthogonal basis function expansion is presented in Chapter 3 and a method using "transition function" is introduced in [6]. The coupled ordinary differential equations derived by the mode-matching method are similar to the conventional transmission line equations and therefore a modular equivalent circuit representation for the governing equations can be developed. Such a modular circuit formulation makes this approach very attractive in treating more complex optical waveguide structures such as lateral directional couplers. In Chapter 3, only guided modes of an optical rib waveguide are discussed. Recently, radiation modes in optical rib waveguides are derived by Rozzi and coworkers by applying a similar mode-matching technique [7].

Chapters 4 and 5 present two powerful numerical techniques, namely, the finite-difference (FD) method by Stern and the finite-element (FE) method by Rahman. These numerical methods are more versatile than the analytical or semi-analytical methods presented in the preceding chapters and may be applied to waveguides of arbitrary index profiles and geometrical shapes. In the finite-difference calculations, the governing wave equations are first discretized using a finite-difference schemes. For the scalar wave equations, such a discretization procedure is straightforward, whereas more sophisticated schemes are needed for the vector wave equations when the fields are discontinuous. A simplifying assumption often adopted in the FD method is the semi-vectorial approximation, first advanced by Stern. In the semi-vectorial formulation, the polarization effect for the dominant field component is considered, but the hybrid nature of the vector modes is ignored, an approximation acceptable for many planar optical guided-wave structures. The matrix equations derived from the discretization are subsequently solved numerically. Direct solution techniques such as the inverse power method are described in the text and more advanced iterative solvers have recently been applied [8]. Only scalar and semi-vectorial modal analyses are discussed in Chapter 4; the method can also be applied to solutions of the full-vectorial wave equations [9]. Further improvements for the FD method may include the use of nonuniform meshes [10] and more accurate discretization schemes [11]. The finite-element (FE) method, on the other hand, relies on the stationary formulas for the frequencies or the propagation constants. As shown in Chapter 5, such formulas have been derived for both scalar and vector wave equations. For the scalar or semi-vectorial modes, the variational formulations are well established and understood. As far as the full-vectorial modes are concerned, several different variational expressions have been proposed and a review and discussion of these different formulas is given in Chapter 5. In the finite-element calculations, the field distribution is represented approximately by nodal values at vertices of elements. A matrix equation for the nodal values is derived and can be solved by numerical algorithms similar to those in the finite-difference method. In comparison with the FD method, the FE method permits use of more flexible meshes to suit complex geometrical shapes, but appears to require more overhead for preprocessor. In computing the modes of open-boundary waveguide structures by either the FD or the FE method, one has to give boundary conditions at the edges of the computation region. In the FD calculations, an exponentially-decaying field distribution in the outer space is assumed with its decay constant corrected successively in the matrix iteration, whereas the infinite elements are utilized in the FE method (see Chapter 5). Recently, a radiative boundary condition is introduced in the finite-element method so that leaky modes of an optical waveguide can be calculated [12]. As the computer resources becomes more abundant and less expensive, powerful mode-solvers based the numerical techniques such as the finite-difference and the finite-element methods are expected to be more practical as design tools for device researchers and engineers.

As soon as the modal characteristics of an ideal waveguide are understood, more complex optical waveguide structures that deviate from the ideal structure may be analyzed based on the expansion of the modal fields. Two formulations, namely, the coupled-mode theory (CMT) and the transfer-matrix method (TMM), are presented in Chapter 6 by Little and Huang and in Chapter 7 by Makino, respectively. The coupled-mode theory, in this rigorous derivations, is equivalent to Maxwell's equations and hence exact. In practice, however, only limited number (often two) of guided modes are utilized and in this respect the CMT is approximate. This compromise in rigorousness is well compensated in practical applications, for the intuitive coupled-mode equations often allow for analytical solutions that are insightful and often very accurate. Three typical guided-wave structures, i.e., the uniform, the periodic and the tapered co-directional couplers, are discussed in detail in Chapter 6. It is noted that the choice of the ideal waveguide as the reference structure for the modes plays a critical role in validity and accuracy of the coupled-mode analysis. As a general guideline, one should choose the ideal waveguide as close as possible to the real structure to minimize the coupling strength between modes. Under this circumstance, the propagation constants of the guided modes have to be matched for any significant power exchange to occur. Such a coupling process is the basis for many optoelectronic guided-wave devices and can be described precisely by the CMT. The transfer matrix method (TMM) as described in Chapter 7, on the other hand, is based on a mode-matching technique, in which the boundary conditions at the index discontinuities along the waveguide axis are satisfied by the local normal modes. In this respect, the

TMM assumes the best possible trial modal solutions as the basis for the field representation. Different from the coupled-mode theory where the coupling is usually piecewise continuous, the couplings between the local normal modes in the transfer matrix formulation are discrete and occur at the index discontinuities along the waveguide axis. Continuous couping may be treated by the TMM by discretization of the change of index or geometry. Consequently, a set of transfer matrices that link the amplitudes of the local modes along the waveguide may be derived and propagation and coupling of these modes may be predicted by cascading of these transfer matrices. Such a method has been applied to contra-directional couplings in a distributed feedback structure in Chapter 7. For its application to co-directional couplings, please refer to Ref. [13]. The transfer-matrix method, due to its choice of local modes, is usually more accurate than the coupled-mode theory based on the modes of a uniform reference waveguide. These two methods are in fact similar and is closely related. For instance, one may use local modes as the trial functions in the CMT and the couplings occur only at the index discontinuities along the waveguide axis. Under this condition, the CMT and the TMM are equivalent.

As most of the two-dimensional mode solvers introduced in Chapter 1-5 calculate only the guided modes and the methods discussed in Chapters 6 and 7 treat only couplings between the guided modes, radiations are usually neglected. This approximation, though acceptable for many applications, may not be valid for modeling and analysis of structures where the radiative fields play an important role. Including the radiation effects in the modal analysis of practical optical waveguide models is possible, but often at expense of complexity and efficiency. A viable alternative approach that appears to be more straightforward is the full-wave methods that solve directly Maxwell's equations or reduced vectorial and scalar wave equations. Some of these methods are presented and discussed in PIER Volume 11 Methods for Modeling And Simulation of Guided-Wave Optoelectronic Devices: Part II: Waves and Interactions.

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