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**PIER 2**

Progress in Electromagnetics Research

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**Finite Element and  
Finite Difference  
Methods in  
Electromagnetic  
Scattering**

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**PIER****Progress in Electromagnetics Research****Jin Au Kong, *Chief Editor***Massachusetts Institute of Technology, Cambridge, Massachusetts

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**PIER 1****Progress in Electromagnetics Research**

Jin Au Kong

**PIER 2****Finite Element and Finite Difference Methods in  
Electromagnetic Scattering**

Michael A. Morgan

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Editor:

**Michael A. Morgan**

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## PREFACE

This second volume of PIER considers recent advances in computational electromagnetics, with emphasis on scattering, as brought about by new formulations and algorithms which use finite element or finite difference techniques. Within the past few years, there has been a growing interest in using these "finite methods" within the electromagnetics research and development communities. A major reason for this new emphasis has been the need to solve ever more complex problems, requiring very large numbers of discrete unknowns.

Until recently, most open-region problems involving antennas and scattering have been approached using integral equations, which implicitly incorporate the far-field radiation conditions. On the other hand, numerical approximation of an integral equation usually results in global interactions between each and every discrete unknown. For the time-harmonic case, this gives fully populated system matrices, while time-domain solutions require causal updating of each unknown, as influenced by all other unknowns at each time-marching step.

In contrast, finite methods result in only local, nearest-neighbor, type interactions between discrete field values at the nodes of the spatial mesh or grid. Finite method use in solving closed-region boundary value problems in the frequency domain yields highly sparse system matrices. This can result in dramatic reductions in solution and fill times, as well as in required computer memory, when compared to dealing with a full matrix of the same rank. In the time-domain, similar efficiency is observed because explicit coupling only exists between nearest spatial nodes.

Although finite methods excel in numerical efficiency for field solutions in closed spatial regions, additional consideration is needed for the open-region case, in order to bound the spatial mesh while enforcing proper radiation conditions at infinity. This mesh termination problem is being approached by a variety of contemporary techniques, most of which are presented in detail in the eight chapters of this volume.

In Chapter 1, Morgan presents a comprehensive and fundamental overview of finite methods and their associated mesh termination techniques, both in the frequency- and time-domains. This first chapter is meant to provide basic reference and a tutorial review. A powerful

finite element algorithm is discussed by Fleming in Chapter 2 where, using the unimoment method to enforce the far-zone radiation conditions, scattering by composite (metallic and dielectric) axisymmetric structures is considered. The boundary element method is used to terminate a 2-D finite element mesh and solve several complex scattering problems in Chapter 3, as authored by Wu, Delisle, Fang and Lecours. In Chapter 4, Mittra and Ramahi derive a variety of absorbing boundary conditions and then investigate their performance in bounding finite method meshes in 2-D and axisymmetric 3-D scattering problems. McCartin, Bahrmassel and Meltz develop a finite method, called the Control Region Approximation, in Chapter 5 and apply it to 2-D scattering problems using asymptotic boundary conditions to enforce proper radiation field behavior. In Chapter 6, Morgan presents a new finite element based formulation for representing time-harmonic vector fields in 3-D inhomogeneous media using two coupled scalar potentials. Transient field problems involving corners and wedges in 2-D and 3-D are considered by Cangellaris and Mei in Chapter 7, using conforming boundary elements and leap-frog time-marching. The finite-difference time-domain method is developed by Taflov and Umashankar in Chapter 8, where further consideration is given to the Yee algorithm, radiation boundary conditions and a wide variety of applications and validations.

Finite methods (so named by Professor K. K. Mei over a decade ago) will continue to evolve as problem complexity and computational resources both increase. It is hoped that this volume will serve as a useful reference for students and practitioners of computational electromagnetics who are, or will be, involved in this evolution.

Acknowledgments are given by individual authors at the end of their respective chapters. In addition, special thanks are given to Mrs. Karen Charnley for typesetting most of the book, to Mrs. Sue Syu for detailed editorial comments, and to Professor J. A. Kong, who suggested the creation of this volume.

Michael A. Morgan

*Monterey, California*  
*May 22, 1989*



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# CONTENTS

<b>Chapter 1</b>	<b>PRINCIPLES OF FINITE METHODS IN ELECTROMAGNETIC SCATTERING</b>	<b>1</b>
1.1	Introduction	1
1.2	Finite Methods	3
1.3	Mesh Termination	29
1.4	Conclusion	62
	References	65
<b>Chapter 2</b>	<b>A FINITE ELEMENT METHOD FOR COMPOSITE SCATTERERS</b>	<b>69</b>
2.1	Introduction	69
2.2	Formulation	73
2.3	Finite Element Method	79
2.4	Numerical and Experimental Results	95
2.5	Conclusions	105
2.6	Future Developments	105
	References	110
<b>Chapter 3</b>	<b>COUPLED FINITE ELEMENT AND BOUNDARY ELEMENT METHODS IN ELECTROMAGNETIC SCATTERING</b>	<b>113</b>
3.1	Introduction	113
3.2	General Formulation	115
3.3	Implementation and Numerical Results	126
3.4	Conclusion	130
	References	131

<b>Chapter 4</b>	<b>ABSORBING BOUNDARY CONDITIONS FOR THE DIRECT SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS ARISING IN ELECTROMAGNETIC SCATTERING PROBLEMS</b>	<b>133</b>
4.1	Introduction	133
4.2	Derivation of the BGT Operators	136
4.3	Alternate Boundary Condition for 2-D Scattering	140
4.4	Performance of Boundary Operators	143
4.5	Absorbing Boundary Condition for the FEM	150
4.6	Improvement in the ABC-Based Solution	158
4.7	ABC for 3-D Scalar and Vector Fields	162
	References	172
<b>Chapter 5</b>	<b>APPLICATION OF THE CONTROL REGION APPROXIMATION TO TWO-DIMENSIONAL ELECTROMAGNETIC SCATTERING</b>	<b>175</b>
5.1	Introduction	175
5.2	Problem Formulation	177
5.3	Asymptotic Boundary Conditions	180
5.4	Discretization	186
5.5	Solution of Discrete Equations	192
5.6	Cross Section Calculation	193
5.7	Numerical Results	198
5.8	Conclusion	206
	References	208
<b>Chapter 6</b>	<b>COUPLED POTENTIALS FOR ELECTROMAGNETIC FIELDS IN INHOMOGENEOUS MEDIA</b>	<b>211</b>
6.1	Introduction	211
6.2	Coupled Potential Formulation	213
6.3	Numerical Algorithm	225
6.4	Computer Validations	234
6.5	Discussion	244
	References	245

<b>Chapter 7</b>	<b>THE METHOD OF CONFORMING BOUNDARY ELEMENTS FOR TRANSIENT ELECTROMAGNETICS</b>	<b>249</b>
7.1	Introduction	249
7.2	Initial Boundary Value Problem	251
7.3	Method of Conforming Boundary Elements	252
7.4	Radiation Boundary Condition	260
7.5	Field Singularities at Wedges and Corners	265
7.6	Numerical Results	272
7.7	Discussion	280
	References	281
<b>Chapter 8</b>	<b>THE FINITE-DIFFERENCE TIME-DOMAIN METHOD FOR NUMERICAL MODELING OF ELECTROMAGNETIC WAVE INTERACTIONS WITH ARBITRARY STRUCTURES</b>	<b>287</b>
8.1	Introduction	288
8.2	General Characteristics of FD-TD	290
8.3	Basic FD-TD Algorithm Details	293
8.4	Contour Path Interpretation	304
8.5	Radiation Boundary Conditions	312
8.6	FD-TD Modeling Validations in 2-D	329
8.7	FD-TD Modeling Validation in 3-D	340
8.8	Penetration and Coupling in 2-D and 3-D	345
8.9	Modeling Very Complex 3-D Structures	356
8.10	Microstrip and Microwave Circuits	361
8.11	Inverse Scattering Reconstructions	363
8.12	Very Large-Scale Software	366
8.13	Conclusion	368
	References	369
	<b>INDEX</b>	<b>375</b>