ANALYSIS AND DESIGN OF CIRCULAR FRACTAL ANTENNA USING ARTIFICIAL NEURAL NETWORKS

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Abstract—A Neural Network is a simplified mathematical model based on Biological Neural Network, which can be considered as an extension of conventional data processing technique. In this paper, an Artificial Neural Network (ANN) based simple approach is proposed as forward side for the design of a Circular Fractal Antenna (CFA) and analysis as reverse side of problem. Proposed antenna is simulated up to 2nd iteration using method of moment based IE3D software. Antenna is fabricated on Roger RT 5880 Duroid substrate (High frequency material) for validation of simulated, measured and ANN results. The main advantage of using ANN is that a properly trained neural network completely bypasses the complex iterative process for the design and analysis of this antenna. Results obtained by using artificial neural networks are in accordance with the simulated and measured results.

1. INTRODUCTION

Fractal geometry has been proved as an alternate methodology to design miniature monopole antennas such as modified Sierpinski broadband dual frequency microstrip patch antenna [1], Koach monopole [2], miniature microstrip antennas using Sierpinski bowtie [3], Sierpinski gasket [4] and dual band fractal Circular Microstrip Patch Antenna

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(CMPA) [5]. The term fractal means broken or irregular fragments to describe a family of complex shapes that possess an inherent self- similarity or self affinity in their geometrical structure. Fractals have been successfully used to model complex natural objects such as galaxies, cloud boundaries, mountain ranges and much more. Traditional approaches to the analysis and design of antenna systems use Euclidean geometry but on applying fractal geometric concepts the new research is called fractal antenna engineering [6]. Fractals are generally selfsimilar and independent of scale. It is an antenna that uses a fractal design to maximize the length or increases the perimeter (on inside sections or the outer structure) of material that can receive or transmit electromagnetic waves within a given total surface area or volume [7]. ANNs are one of the popular intelligent techniques in solving engineering and mathematical problems. An ANN consists of a few types of many, simple nonlinear functional blocks, which are called neurons. Neurons are organized into layers, which are mutually connected by highly parallel synaptic weights. The ANN exhibits a learning ability, synaptic weights can be strengthened or weakened during the learning process and in this way, information can be stored in the neural network [8,9]. In ANN model, no formula is necessary to design microstrip antenna due to its empirical nature, based on the observation of physical phenomenon. Neural networks can be used for the applications of wireless communications. In area of microwave applications, ANNs have been used to design Rectangular Microstrip Patch Antenna (RMPA) [10–13] and CMPAs [14, 15]. These can also be used to calculate different parameters such as feed position [16], resonant resistance [17], input impedance [18], radiation efficiency [19], resonant frequencies of triangular and RMPAs [20]. Similarly, ANNs have been used for calculating different parameters such as resonant frequency [21], directivity [22] and input impedance [23] of CMPAs. In case of fractal antenna design, the ANN has not been explored extensively. In this paper the concept of fractal has been applied to the geometry of CMPA in a modified way to obtain proposed CFA with multi-band frequency operations (as shown in Figure 1) and then ANN has been used for design and analysis of proposed CFA.

Figure 1 shows process for generation of different geometries of CFA up to 3rd iteration. Section 2 describes the procedure to obtain a data dictionary for training and validation of ANN model, Section 3 explains the development of ANN based model, Section 4 describes specifications for fabrication of antenna, Section 5 describes the result and Section 6 describes the conclusion of this paper.



Figure 1. Proposed circular fractal antenna. (a) Base geometry 0th iteration, (b) 1st iteration, (c) 2nd iteration, (d) 3rd iteration.

2. DESIGN, ANALYSIS AND GENERATION OF DATA DICTIONARY OF CFA

The design of CFA starts with single element using a circular conductor on a ground dielectric substrate and is termed as base geometry.

$$a_e = a \left\{ 1 + \frac{2h}{\pi a \varepsilon_r} \left[\ln \left(\frac{\pi a}{2h} \right) + 1.7726 \right] \right\}^{\frac{1}{2}}$$
(1)

$$f_r = \frac{1.8412c}{2\pi a_e \sqrt{\varepsilon_r}} \tag{2}$$

a =Radius of circular patch.

 $a_e = \text{Effective radius of circular patch.}$

c = Velocity of light in free space.

 $f_r = \text{Resonant frequency.}$

h = Height of the substrate.

 ε_r = Dielectric constant of the substrate.

Steps for design of different iterations of CFA are described below:

- Step-1: Radius of base circular geometry is calculated using Equations (1) and (2).
- Step-2: Draw a square ABCD whose sides must be equal to 1/3rd of diameter of base geometry and diagonals intersect at centre 'o' of base circular geometry as shown in Figure 1(b).
- Step-3: Cut a circle by taking A (vertex of square) as centre and having radius 1/3rd of radius of base geometry to get 1st iteration geometry.
- Step-4: Cut another three circles on the remaining vertices of square, by taking these vertices (B, C and D) as the centre of circles and having their radius equal to 1/9th of radius of base geometry circle to get 2nd iteration geometry as depicted in Figure 1(c).

- Step-5: Draw three more squares with one side of each square is equal to 1/9th of diameter of circle of base geometry and one vertex of each square coincide with centers (B, C and D) of corresponding circle.
- Step-6: With similar process cut 9 more circles at each remaining vertex of small square, having radius of each circle 1/27th of radius of base geometry circle to get 3rd iteration geometry of CFA as depicted in Figure 1(d).
- Step-7: This process can be repeated to get infinite number of iteration geometries of CFA.

For example, for operating frequency $F_1 = 4.249 \,\text{GHz}$, height of substrate h = 1.58 mm, dielectric constant $\varepsilon_r = 2.2$, and the radius of circular patch conductor is calculated by using Equations (1) and (2) and comes to 13 mm. This geometry is referred as base geometry or zero iteration as shown in Figure 1(a). A circle with radius equal to 1/3rd of radius of circular base geometry, i.e., 4.33 mm and with centre (-4.33, 4.33) is dropped from second quadrant of base geometry to get 1st iteration geometry as depicted in Figure 1(b). Three circles with radius 1/9th of radius of circular patch of base geometry, i.e., 1.44 mm and with centers (4.33, 4.33), (4.33, -4.33) and (-4.33, -4.33) are dropped from 1st iteration geometry to get 2^{nd} iteration geometry as depicted in Figure 1(c). Nine more circles each having radius 1/27th of base geometry radius, i.e., 0.48 mm and with centers (7.22, 4.33), (7.22, 7.22), (4.33, 7.22), (7.22, -4.33), (7.22, -7.22), (4.33, -7.22),(-7.22, -4.33), (-7.22, -7.22) and (-4.33, -7.22) are cut from 2nd iteration geometry to get 3rd iteration geometry of CFA. Then CFA structures from 0th to 2nd iteration geometry are simulated using IE3D software and return loss versus frequency plot are shown in Figure 2. From Figure 2(a) it is clear that the return loss is less than $-10 \,\mathrm{dB}$ only at single frequency of 4.26 GHz for 0th iteration. Thus, zero iteration geometry of CFA will work at single frequency of 4.26 GHz where as 1st and 2rd iteration geometry of CFA will work at three (4.04 GHz, 6.87 GHz and 8.5 GHz) and four frequencies (4.00 GHz, 6.9 GHz, 8.5 GHz and 9.8 GHz) respectively. Data dictionary for design of these structures is shown in Table 1. In this way a set of 75 input-output pairs for training and a set of 45 pairs (42 simulated +3measured) for validation of ANN are generated using IE3D software.

Reverse process is done for analysis of CFA. Height of substrate (h), dielectric constant (ε) , and radius of CFA (a) and number of iterations (n) are taken as input where as resonant frequencies F_1 to F_4 are taken as output.



Figure 2. Return losses versus frequency plot of fractal antenna for (a) 0th iteration, (b) 1st iteration and (c) for 2nd iteration.

Table 1. Data dictionary for design of CFA with radius of circular patch = 13 mm for 0th, 1st and 2nd iteration.

Sr. No		A	ANN Out-puts					
	F_1	F_2	F_3	F_4	h	ε	a	n
1	4.26	0	0	0	1.58	2.2	13	0
2	4.04	6.87	8.5	0	1.58	2.2	13	1
3	4.00	6.9	8.5	9.8	1.58	2.2	13	2



Figure 3. ANN model for (a) design of CFA, (b) analysis of CFA.

3. DEVELOPMENT OF ANN MODEL FOR DESIGN AND ANALYSIS OF CFA

The first step in neural model development is generation and collection of data for training and validation of neural models which are obtained in Section 2. Neural model for design and analysis of CFA is shown in Figures 3(a) and 3(b) respectively. Proposed FFBP-ANN model have six in-puts (resonant frequencies ' F_1 ' to ' F_4 ', height of substrate 'h' and dielectric constant ' ε ') and two outputs (radius 'a' and No. of iterations (n) for design where as four inputs (height of substrate (h), dielectric constant ' ε ', radius 'a' and No. of iterations 'n') and four outputs (resonant frequencies ' F_1 ' to ' F_4 ') for analysis of CFA. Both ANN models are trained with Levenberg Marquardt (LM) algorithm and structure 6-35-2 as depicted in Figure 4 is found suitable structure for design and ANN structure 4-35-4 as depicted in Figure 6 is found suitable structure for analysis of CFA. ANN structure for design of CFA has one input layer with six neurons, one hidden layer with 35 neurons and one output layer with two neurons. The first layer receives input data and its output is given as input to hidden layer with tansigmoidal non linear activation function f_1 . The output from neurons of hidden layer is transmitted to the output layer of two neurons with pure linear activation function f_2 , which finally computes the network output. Output of the proposed ANN is computed by using following equations.

$$X = f_2([OW](f_1([FW][Y] + [FB]) + [OB])$$
(3)
$$Y = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ h_i \\ \varepsilon_i \end{bmatrix}$$
(4)



Figure 4. Proposed FFBP-ANN based model for design of CFA.

$$X = \begin{bmatrix} a_i \\ n_i \end{bmatrix}$$
(5)

$$FW = \begin{bmatrix} fw_{1,1} & fw_{1,2} & fw_{1,3} & fw_{1,4} & fw_{1,5} & fw_{1,6} \\ fw_{2,1} & fw_{2,2} & fw_{2,3} & fw_{2,4} & fw_{2,5} & fw_{2,6} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ fw_{35,1} & fw_{35,2} & fw_{35,3} & fw_{35,4} & fw_{35,5} & fw_{35,6} \end{bmatrix}$$
(6)

$$FB = \begin{bmatrix} fb_1 \\ fb_2 \\ \vdots \\ fb_{35} \end{bmatrix}$$
(7)

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where $fb_1, fb_2, \ldots, fb_{35}$ are the bias values for hidden layer neurons.

$$OB = \begin{bmatrix} ob_1 \\ ob_2 \end{bmatrix}$$
(8)

where ob_1 and ob_2 are the bias values for output layer neurons.

$$OW = \begin{bmatrix} ow_{1,1} & ow_{1,2} & ow_{1,3} & \cdot & \cdot & \cdot & \cdot & ow_{1,35} \\ ow_{2,1} & ow_{2,2} & ow_{2,3} & \cdot & \cdot & \cdot & \cdot & ow_{2,35} \end{bmatrix}$$
(9)

FW and OW represent weight matrices for hidden and output layer. The MSE, i.e., performance index as given in [17].

$$MSE = \frac{1}{n} \sum_{i=1}^{n} [y_i - F_{ANN}(x_i)]^2$$
(10)

The output of the proposed ANN for analysis has been computed with similar manner as for design of CFA.

4. FABRICATION OF ANTENNA

Proposed CFAs have been fabricated on Roger RT 5880 Duroid substrate (High frequency material) for the validation of simulated, measured and ANN results. The material used for fabrication has its own specifications: dielectric constant $\varepsilon_r = 2.2 \pm 0.02$, dielectric thickness = 0.062 inch ± 0.002 (1.58 mm), top cladding 10z/sqft (35 µm) ED copper, bottom cladding 10z/sqft (35 µm) ED copper. Fabricated structures for 0th, 1st and 2nd iteration of CFA are shown in Figure 5. Experimental set up for measuring resonant frequencies of CFA is shown in Figure 7. Measured results of these antennas are used to test the ANN results and included in validation study.



Figure 5. Fabricated structures for 0th, 1st and 2nd iteration of CFA.



Figure 6. Proposed FFBP-ANN based model for analysis of CFA.

5. RESULTS AND DISCUSSION

5.1. Results of ANN Training

In order to evaluate the performance of proposed FFBP-ANN based model for the design and analysis of CFA, different 75 input-output training patterns obtained in Section 2 are used for training the proposed 6-35-2 ANN structure for design and 4-35-4 ANN structure for analysis of CFA. Results in terms of performance parameters such as number of epochs taken for training, Mean Square Error (MSE), maximum value of absolute error (difference between target and ANN output) and percentage error (%FS) for design and analysis are shown in Table 2 and Table 3 respectively. The learning characteristic of FFBP-ANN based model trained with LM training algorithm for design and analysis are shown in Figures 8(a) and 8(b) respectively. It has been observed that only 298 epochs are needed to reduce MSE level to a value 9.92e-007 for design and 386 epochs are needed to reduce MSE level to a low value 9.99e-006 for analysis of CFA. Absolute and percentage errors versus number of input-output patterns are plotted



Figure 7. Measurement of return loss of CFA using network analyzer.



Figure 8. Learning characteristics of the FFBP-ANN for (a) design, (b) analysis of CFA.

Table 2. Results of FFBP-ANN based model for the estimation ofradius and iteration No. of CFA for training data.

Training Algorithm	Number of neurons in hidden layer	Epochs	MSE	Absolute error fo	or estimation of	Percenage terror for estimation of		
Levenberg Marguardt	35	298	9 926-007	Radius 'a'	Iteration number 'n'	Radius 'a'	Iteration number 'n'	
algorithm	55	290	9.928-007	0.0072	6.9191e-004	0.3601	0.0346	

Table 3. Results of FFBP-ANN based model for the estimation of resonant frequencies of CFA for training data

Training Algorithm	Number of neurons in hidden layer	Epochs	MSE	Absol	ute error o	for estir	nation	Percentage error for estimation of			
Levenberg	25	200	0.00 000	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4
algorithm	35	386	9.90e-006	0.0104	0.0081	0.0269	0.0055	0.1779	0.0829	0.4612	0.0724



Figure 9. Absolute error for estimating the value of (a) iteration number, (b) radius. Percentage error for estimating the value of (c) iteration number, (d) radius of CFA using ANN model as a result of training study.



Figure 10. Absolute error for estimation of resonant frequency. (a) F_1 , (b) F_2 , (c) F_3 and (d) F_4 of CFA using ANN model as a result of training study.

Table 4. Maximum value of absolute and percentage error for estimating the radius and iteration No. of CFA using FFBP-ANN 6-35-2 structure for validation of data.

	Max.	absolute	Max. percentage				
	er	ror for	error for				
	estin	nation of	estimation of				
	Radius	Iteration	Radius	Iteration			
Parameter	` a `	number ' n '	a'	number ' n '			
	0.0522	0.0317	2.9819	1.5870			



Figure 11. Absolute error for estimating the value of (a) iteration number, (b) radius. Percentage error for estimating the value of (c) iteration number, (d) radius of CFA using ANN model as a result of validation study.

Table 5. Maximum value of absolute and percentage error for estimating the resonant frequencies of CFA using FFBP-ANN 4-35-4 structure for validation of data.

	Max. abso	olute erro	or for esti	mation of	Max. percentage error for estimation of						
Parameter	Freq. F ₁	Freq. F ₂	Freq. F ₃	Freq. F ₄	Freq. F_1	Freq. F ₂	Freq. F ₃	Freq. F ₄			
	0.0167	0.0360	0.1221	0.0963	0.2962	0.3750	2.1646	2.1646			

as a result of training study of ANN for design are shown in Figure 9, which shows that maximum absolute error for estimating the number of iterations and radius of CFA are 6.9191e-004 and 0.0072, respectively, whereas percentage error for estimating the number of iterations and radius of CFA are 0.0346 and 0.3601, respectively. Similarly, absolute errors versus number of input-output patterns are plotted as a result of training study of ANN for estimating the resonant frequencies as



Figure 12. Absolute error for estimating the value of resonant frequency. (a) F_1 , (b) F_2 , (c) F_3 , (d) F_4 of CFA using ANN model as a result of validation study.

shown in Figure 10, which shows that maximum errors for estimating the resonant frequencies $(F_1 \text{ to } F_4)$ of CFA are 0.0104, 0.0081, 0.0269 and 0.0055, respectively.

5.2. Results of Validation Study

A set of 45 input-output (42 patterns using IE3D software and three patterns from measured results) patterns (other than training patterns) as generated in Section 2 are used for testing proposed trained 6-35-2 ANN structure for design and 4-35-4 ANN structure for analysis of this antenna. The absolute error and percentage error (%FS) at each value of radius, number of iteration of CFA for design and resonant frequencies for analysis of this antenna as result of validation study are shown in Table 4 and Table 5 respectively. Graphical representation of absolute errors and percentage error at each value of radius and iteration number for design of CFA as result

Table 6. Comparison of ANN results with simulated and measuredresults.

Sr	ANN Inputs			IE3D Out-put				Measured Out-put				ANN Out-put				
No.	h	з	а	п	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4
1	1.58	2.2	13	0	4.26	0	0	0	4.11	0	0	0	4.215	0	0	0
2	1.58	2.2	13	1	4.04	6.87	8.5	0	4.15	6.81	8.46	0	4.038	6.865	8.469	0
3	1.58	2.2	13	2	4.00	6.9	8.5	9.8	4.15	6.75	8.45	9.85	4.017	6.872	8.449	9.803

of validation study are shown in Figure 11. Similarly, graphical representations of absolute errors at each value of resonant frequency of CFA as result of validation study are shown in Figure 12. Also, the results of ANN for estimating frequencies of CFA are compared with experimental results as shown in Table 6. Achievement of such a low value of these errors (absolute and %FS) further authenticates that the ANN model is an accurate model for the design and analysis of CFA.

6. CONCLUSIONS

The basic purpose of applying neural network in this paper is to change from the lengthy analysis and design cycles required to develop high performance system to very short product development time. The proposed technique has used FFBP-ANN with one hidden layer as an approximate model for design and analysis of circular fractal antenna. The results of ANN for estimation of design and analysis parameters are in accordance with simulated and measured results. From the results it is observed that the proposed modeling technique is very convenient to model ANN for predicting the design parameters under specified conditions.

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