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REALIZATION OF MINIATURIZED QUADRATURE HYBRID COUPLER WITH REDUCED LENGTH BRANCH ARMS USING RECURSIVELY LOADED STUBS

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Abstract—This paper presents a novel design of miniaturized microstrip quadrature coupler at 2.45 GHz. The design topology is based on reduced transmission line branch arms using recursively loaded stubs that contribute to the compact size. The proposed coupler result in a size reduction of 70.4% when compared to a conventional branch line hybrid. The designed coupler provides, at the operating frequency, a 25 dB isolation and exhibits equal power division at the output ports with quadrature phase difference. A fabricated prototype is developed with simulation and measurement in close agreement.

1. INTRODUCTION

Typical branch line coupler (BLC) at microwave frequencies [1], realized using a number of quarter-wavelength long transmission line section, is an indispensable and versatile component often used as power divider, combiner and has found applications in balanced mixer, antenna feed network and quadrature modulator [2]. This makes it an important component in wireless communication system. However, due to the length of transmission lines involved in realizing such components, the circuit size is large, particularly at lower microwave frequencies. This in turn increases the fabrication cost.

Development of wireless technology has also ushered the research towards the miniaturization of these circuit components. Therefore, numerous methods have been proposed to miniaturize these conventional designs of BLCs [3–10]. Fractal space filling shapes

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like Sierpinski curve has been used to design miniaturized branch line hybrid, rat race couplers and coupled line hybrids as reported in [3]. In [4] discontinuous microstrip lines as slow wave structures are synthesized to reduce physical size of BLC by about 60%. A 45% reduction in BLC size is proposed in [5] where only distributed components have been used without any lumped elements, bonding wires or via holes. Driven by a targeted value of size reduction for microstrip branch line coupler, design comprising of branch arms with shunt loaded high and low impedance stubs are reported in [6]. In [7] high impedance transmission line segments with distributed capacitor are used to achieve 62% miniaturization. Increased design flexibility and miniaturization by replacing branch arms of a hybrid coupler with asymmetrical-T structures is presented in [8]. With a T-model approach in one set of branch arms and implanting high low impedance based open stubs in the other pair of arms a miniaturized branch line hybrid is achieved in [9]. Branch arms are replaced by equivalent dual transmission lines where one of the lines is meandered resulting in a reduction of 68% in circuit size as reported in [10]. A technique of miniaturization of BLC by loading branch arms by complementary split ring resonators resulting in 66.14% size reduction is demonstrated in [11].

In this work, a novel design of compact BLC at 2.45 GHz is proposed that is realized using recursively loaded T-stub, which brings about a size reduction of 70.4% as compared to a conventional BLC. Rest of the paper is organized as follows. In Section 2, the analysis of recursively loaded T-stub is presented. Design process of BLC is detailed in Section 3 followed by results and discussion in Section 4 and conclusion in Section 5.

2. ANALYSIS OF RECURSIVE T-STUB LOADED TRANSMISSION LINE

A conventional branch line coupler is based on four quarter wavelength transmission lines which are composed of low impedance series arm of 35.35 Ω and high impedance shunt arm of 50 Ω. The transmission line model of such a quarter wavelength line is shown in Fig. 1(a), where the Z_0 and θ_0 is the characteristic impedance and the electrical length respectively. In order to reduce the size of each quarter wavelength branch arm, equivalent transmission lines that approximate the behavior of a quarter wave transmission line is used. An effective approach for miniaturization is to replace the quarter wavelength branch arms with recursively loaded stubs in a sequential T-model as shown in Fig. 1(b). The impedance and electrical length are as

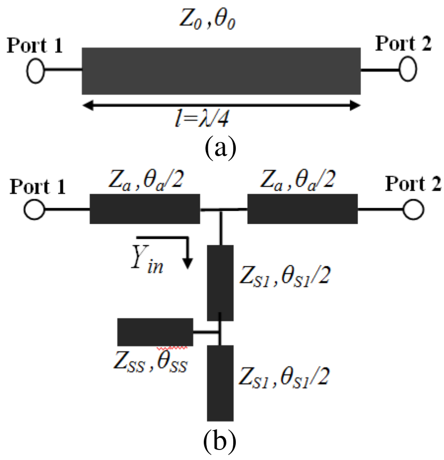

Figure 1. (a) Equivalent circuit of quarter wavelength transmission line. (b) Equivalent transmission line model of reduced transmission line using recursively loaded stubs.

indicated. Referring to Fig. 1, the net $ABCD$ matrix of the recursive loaded stub, denoted as $[ABCD]_{RS}$ is given in (1).

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{RS} = \begin{bmatrix} \cos\frac{\theta_{S1}}{2} & jZ_{S1}\sin\frac{\theta_{S1}}{2} \\ jZ_{S1}\sin\frac{\theta_{S1}}{2} & \cos\frac{\theta_{S1}}{2} \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ jY_{SS}\tan\theta_{SS} & 1 \end{bmatrix} \times \begin{bmatrix} \cos\frac{\theta_{S1}}{2} & jZ_{S1}\sin\frac{\theta_{S1}}{2} \\ jZ_{S1}\sin\frac{\theta_{S1}}{2} & \cos\frac{\theta_{S1}}{2} \end{bmatrix} \quad (1)$$

From (1), the input impedance of the reduced branch arms based on recursively loaded stubs is given below

$$Y_{in} = j\frac{\cos\frac{\theta_{S1}}{2}\left(2Y_{S1}\sin\frac{\theta_{S1}}{2} + Y_{SS}\cos\frac{\theta_{S1}}{2}\tan\theta_{SS}\right)}{\left(\cos\theta_{S1} - \frac{Z_{S1}Y_{SS}}{2}\sin\theta_{S1}\tan\theta_{SS}\right)} \quad (2)$$

To determine the impedances and electrical lengths that replicate the behavior of the quarter wavelength transmission line, the corresponding $ABCD$ matrices must be equated, such that one to one correspondence between the matrix entities must be established. In other words, $ABCD$ matrix of the quarter wavelength line, as shown in Fig. 1(a), must be same as the $ABCD$ matrix of the equivalent reduced line, as shown in Fig. 1(b) and denoted by $[ABCD]_{RL}$.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\lambda/4} = \begin{bmatrix} 0 & jZ_0 \\ jY_0 & 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{RL} \quad (3)$$

where

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{RL} = \begin{bmatrix} \cos\frac{\theta_a}{2} & jZ_a\sin\frac{\theta_a}{2} \\ jZ_a\sin\frac{\theta_a}{2} & \cos\frac{\theta_a}{2} \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ Y_{in} & 1 \end{bmatrix} \times \begin{bmatrix} \cos\frac{\theta_a}{2} & jZ_a\sin\frac{\theta_a}{2} \\ jZ_a\sin\frac{\theta_a}{2} & \cos\frac{\theta_a}{2} \end{bmatrix} \tag{4}$$

By establishing one to one correspondence in (3) results in the following solutions for Z_a, Z_{s1} and Z_{ss} as given in (5), (6) and (7), respectively.

$$Z_a = \frac{Z_0}{\left(\sin\theta_a - \tan\frac{\theta_a}{2}\cos\theta_a\right)} \tag{5}$$

$$Z_{S1} = \frac{6Z_a^2\sin^2\frac{\theta_a}{2}\cos^2\frac{\theta_{S1}}{2}}{P+Q+R} \tag{6}$$

where

$$P = 2Z_a\sin^2\frac{\theta_a}{2}\cot\theta_a\left(\sin\theta_{S1} + 2\cot\theta_{S1}\cos^2\frac{\theta_{S1}}{2}\right) \tag{6a}$$

$$Q = Z_a\sin\theta_a\left(2\cot\theta_{S1}\cos^2\frac{\theta_{S1}}{2} - \sin\theta_{S1}\right) \tag{6b}$$

$$R = Z_0\left(\sin\theta_{S1} - 2\cos^2\frac{\theta_{S1}}{2}\cot\theta_{S1}\right) \tag{6c}$$

$$Z_{SS} = \frac{Z_{S1}\cos\theta_a\sin\theta_{S1}\tan\theta_{SS} - Z_a\sin\theta_a\cos^2\frac{\theta_{S1}}{2}\tan\theta_{SS}}{2\cos\theta_{S1}\cos\theta_{SS} - Z_aY_{S1}\sin\theta_a\sin\theta_{S1}} \tag{7}$$

The design Equations (5) through (7), for reduced line BLC, are coupled transcendental equations that are solved graphically. These provide the design curves for the reduced transmission line using recursively loaded stubs. Fig. 2(a) represents the relation between normalized impedance ratio (Z_a/Z_0) and (Z_{s1}/Z_0) plotted against electrical lengths θ_a or θ_{s1} and likewise Fig. 2(b) represents the variation of normalized impedance ratio (Z_a/Z_0) and (Z_{ss}/Z_0) with change in θ_a or θ_{ss}. Design curve Fig. 2(a) indicates that for more compact size of the coupler, the transmission line length θ_a should be small and impedance ratio Z_a/Z_0 is large. Fig. 2(b) shows that the higher the characteristics impedance of Z_{s1} andZ_{ss} the longer the electrical length θ_{s1} and θ_{ss}. To miniaturize the quarter wavelength transmission line based BLC we need to choose Z_a in such a way that the other impedances can also be realized as too low values or longer electrical lengths would be impractical to design. With lower impedance and longer electrical length of the recursively loaded stubs, of the reduced branch arms, the circuit space becomes constrained. In

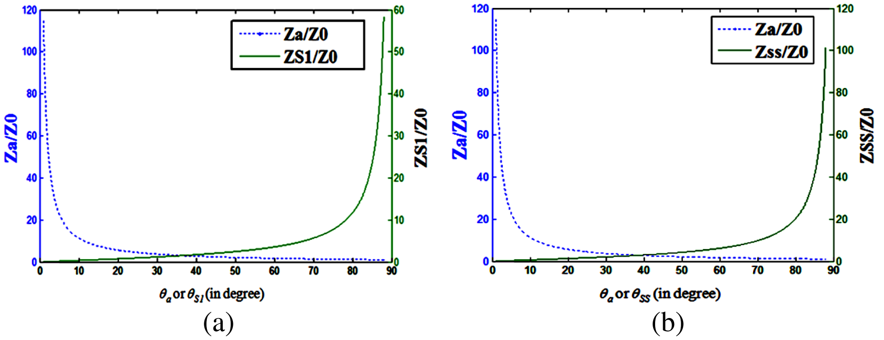

Figure 2. (a) Relations between normalized Z_a, and normalized Z_{s1} with θ_a and θ_{s1}. (b) Relations between normalized Z_a, and normalized Z_{ss} with θ_a and θ_{ss}.

extreme cases the stubs of opposite arms may not fit in to the available space. Moreover, closely spaced stub would introduce additional coupling that can impair coupler performance. These issues govern the choice of degree of miniaturization that be achieved. The value of θ_a is taken as 55° which results in an impedance ratio Z_a/Z_0 as 1.92 keeping in view design constraints. The other two impedances Z_{s1} and Z_{ss} are considered as 35 Ω each. The electrical lengths θ_{s1} and θ_{ss} is eventually obtained by (2) and is optimized to 25.7° and 15.27° respectively. The impedance ratio of shunt arm is taken as 1.92 for which value of Z_a is obtained as 95.05 Ω. For the shunt arm the values of Z_{s1} andZ_{ss} are also taken as 35 Ω each. Accordingly θ_{s1} and θ_{ss} is obtained as 18.79° and 15.38°. The length of shunt arm of miniaturized branch line coupler is 11.05 mm which accommodates the stubs of series arms. Therefore it is different from the series arm length which is 10.81 mm. Further these values are fine tuned to adjust the design frequency of 2.4 GHz. Once the series and shunt arms are synthesized as per the miniaturization technique presented in this work, the coupler layout is accomplished. This is explained in next section.

3. DESIGN OF PROPOSED MINIATURIZED BRANCH-LINE COUPLER

The reduced 3 dB BLC arms as discussed in previous Section 2 are used to design the compact BLC. The substarte is FR4 with permittivity 4.4 and height 0.8 mm with loss tangent 0.0027. The four ports are 50 Ω microstrip lines. The coupler is analyzed using electromagnetic simulation software CST Microwave Studio$^{\mathrm{TM}}$ [12]. The reduced line segments with recursively loaded stubs in sequential T model has the following two additional parameters S_1 and S_2 which denote the position of the stub of length L_3 and L_6 respectively. These are parametrically studied before going to develop a prototype. Increasing the value of S_1 increases coupling between the stub of series arm and stub of shunt arm. This is reflected in the degradation of S_{11} and S_{41}. S_2 when varied offer minute changes in the design frequency as indicated in Fig. 3. So these parameters are adjusted to fine tune the design frequency to 2.45 GHz and this is achieved for S_1 and S_2 values of 1.05 mm and 0.45 mm, respectively. The physical dimensions of the coupler as indicated in Fig. 4, are tabulated in Table 1. The overall dimension of the proposed 3 dB BLC is 11.05 mm × 10.81 mm. The proposed BLC is compared with a conventional 3 dB BLC designed with same substrate, which has dimension of 22.45 mm × 18.0 mm. The

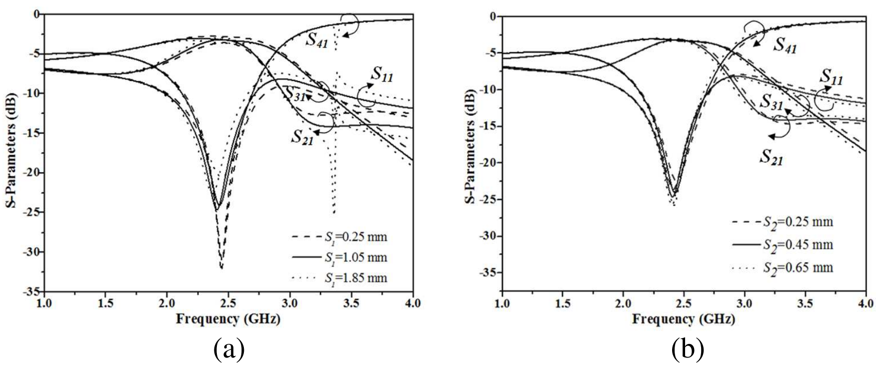

Figure 3. Effect on simulated s-parameters due to variation in (a) S_1, (b) S_2.

Table 1. Dimensions of the proposed compact BLC.

Design Parameters	**Length and Width**
L_1, W_1	10.81 mm, 1.03 mm
L_2, W_2	5.38 mm, 2.68 mm
L_3, W_3	2.90 mm, 2.68 mm
L_4, W_4	11.05 mm, 0.41 mm
L_5, W_5	3.50 mm, 2.68 mm
L_6, W_6	3.50 mm, 2.68 mm
S_1, S_2	1.05 mm, 0.45 mm

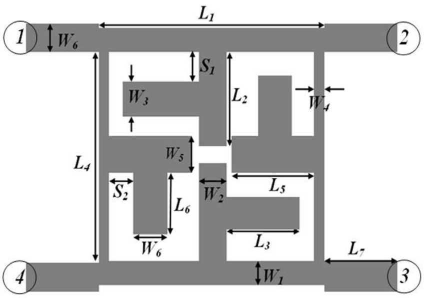

Figure 4. Layout of the proposed recursively loaded stub based branch line coupler.

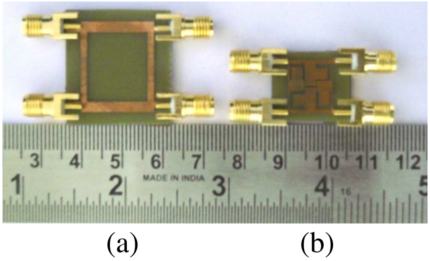

Figure 5. Photograph of fabricated prototype of (a) conventional BLC and (b) the proposed recursive stub loaded miniaturized BLC.

area of the conventional design is 404.1 mm^2 and that of the proposed compact BLC is 119.45 mm^2. This results in a 70.4% size reduction. The conventional BLC and proposed miniaturized BLC are fabricated and the photograph of the prototype is shown in Fig. 5.

4. RESULTS AND DISCUSSION

The fabricated prototype of the proposed compact 3 dB BLC is measured using Rhode and Schwarz ZVA 40 VNA over the range 1 to 4 GHz. The coupler's S-parameters obtained from electromagnetic simulation and measurement are shown in Fig. 6. They are in close agreement. Coupler's parameters are analyzed at the operating frequency 2.45 GHz. The 3 dB frequency range of S_{21} and S_{31} is 2.38 GHz to 2.51 GHz. The flatness of S_{21} and S_{31} are both within 0.25 dB in the frequency range which emphasizes the coupler's ability to divide power equally in the ports 2 and 3. The bandwidth of the proposed coupler for S_{11} better than 10 dB ranges from 2.16 GHz to 2.74 GHz. At 2.45 GHz the difference between S_{21} (dB) and S_{31} (dB) is 0.12 dB. The phase of S_{21} and S_{31} for the proposed coupler's output ports is shown in Fig. 7. It is observed that the phase difference between S_{21} and S_{31} is 89.17° at 2.45 GHz which indicates satisfactory transmission. The measured isolation between ports 1 and 4 is 24. 97 dB and due to circuit symmetry the coupler's reflection at port 1 is −24.68 dB. The electrical characteristics of the conventional and proposed miniaturized 3 dB BLC are tabulated in Table 2. A comparison between this work and other BLCs as given in reference are compared in Table 3, which is self explanatory. For fair comparison of size the overall dimensions of the BLC is computed with respect to free space wavelength corresponding to the design frequency.

Table 2. Comparison of electrical characteristics between conventional and proposed compact branch-line coupler.

Parameters	**Conventional**		**Proposed BLC**	
	@ 2.45 GHz		@ 2.45 GHz	
	Simulation	**Measured**	**Simulation**	**Measured**
S_{11} (dB)	−31.15	−29.85	−24.96	−24.68
S_{21} (dB)	−3.12	−3.21	−2.97	−3.05
S_{31} (dB)	−3.07	−3.15	−3.13	−3.17
S_{41} (dB)	−32.30	−30.5	−25.06	−24.97
Phase Difference	89.90°	89.43°	89.75°	89.17°

Table 3. Comparison of proposed work with other related work as mentioned in reference.

	Design Frequency (GHz)	Substrate Permittivity (ε_r)	Height of Substrate (mm)	Overall size (mm^2)	% Reduction in Size	$x\lambda_g \times y\lambda_g$	$x\lambda_0 \times y\lambda_0$
[6]	2.4	4.3	0.8	14.66 ×10.87	60.86%	$0.21\lambda_g \times 0.16\lambda_g$	$0.39\lambda_0 \times 0.30\lambda_0$
[7]	3.45	2.33	0.508	10.6 ×12.18	38.07%	$0.17\lambda_g \times 0.20\lambda_g$	$0.24\lambda_0 \times 0.28\lambda_0$
[9]	2.45	4.7	0.8	13.2 ×10.0	64.21%	$0.21\lambda_g \times 0.16\lambda_g$	$0.40\lambda_0 \times 0.30\lambda_0$
[10]	1.675	2.65	1.0	20 × 20	68%	$0.16\lambda_g \times 0.16\lambda_g$	$0.23\lambda_0 \times 0.23\lambda_0$
[11]	4.77	10.2	0.635	8.88 ×9.11	66.14%	$0.19\lambda_g \times 0.19\lambda_g$	$0.53\lambda_0 \times 0.53\lambda_0$
Our work	2.45	4.4	0.8	11.05 ×10.81	70.4%	$0.17\lambda_g \times 0.16\lambda_g$	$0.31\lambda_0 \times 0.30\lambda_0$

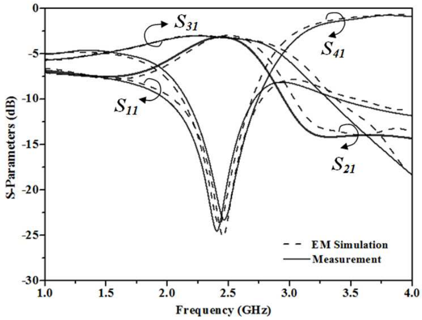

Figure 6. Simulated and measured S-parameters of the proposed BLC.

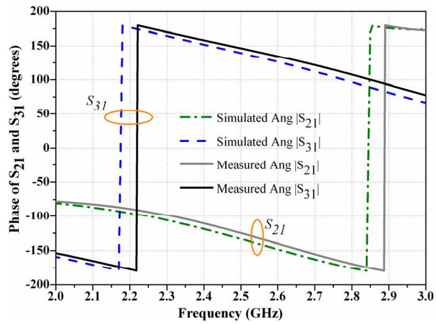

Figure 7. Phase of S_{21} and S_{31} of the proposed coupler.

5. CONCLUSION

A 70.4% miniaturized 3 dB branch line coupler at 2.45 GHz using recursively loaded stubs is presented in this paper. The proposed technique using recursively loaded stubs increases degree of design freedom. A prototype of the proposed coupler is fabricated and measured, and the simulated and measured results are in close agreement. Measured electrical characteristics at 2.45 GHz show equal power division with quadrature phase difference. The overall size is 11.05 mm × 10.81 mm.

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