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HIGH FREQUENCY SCATTERING BY A SECOND-ORDER GENERALIZED IMPEDANCE DISCONTINUITY ON A CYLINDRICALLY CURVED SURFACE

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Abstract—The aim of the present paper is to obtain explicit asymptotic expressions for the "*transfer (diffraction) coefficients*" related to the diffraction of high frequency cylindrical waves from the discontinuity occurred in the material properties as well as in the thicknesses of a coated cylindrically curved metallic sheet characterized by the second order GIBCs. Relying on the locality of the high frequency diffraction phenomenon, the angular interval $\varphi \in (-\pi, \pi)$ is extended to the abstract infinite space $\varphi \in (-\infty, \infty)$ wherein the diffracting structure is replaced by a two-part cylindrically curved second order impedance sheet $\rho = a$ extending from $\varphi = -\infty$ to $\varphi = \infty$. The resulting boundary value problem is formulated as a Hilbert equation which is solved asymptotically in the high frequency limit. Some graphical results showing the effects of various parameters on the transfer coefficients are presented.

1. INTRODUCTION

The diffraction of electromagnetic waves by a two-part surface is an important topic in diffraction theory because it constitutes a canonical problem for analyzing the scattering caused by an abrupt change in the material properties of a surface and has been subjected to intensive past investigations. These studies are mostly restricted to the cases where the two-part surfaces are planar junctions between two thin material half-planes or two thin material coatings applied on a metallic plane, characterized by first order resistive, conductive or impedance

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(Leontovich) boundary conditions [1–5] extended the analysis to the case where the discontinuity occurs on a cylindrically curved first order impedance surface.

A better simulation of thin material sheets or dielectric coatings on metallic surfaces are provided by the generalized impedance boundary conditions (GIBCs) which involve field derivatives higher than the first [6–10]. The application of GIBCs provides more accurate models for coated metallic surface than the classical first order impedance (Leontovich) boundary condition but the solutions of the corresponding diffraction problems are neither unique nor reciprocal even after the edge condition is imposed. In such a case an additional constraint referred to as "*junction condition*" or "*contact condition*" has to be taken into account. This junction condition is derived and used extensively by [11–16].

The aim of the present paper is to obtain explicit asymptotic expressions for the "*transfer (diffraction) coefficients*" related to the diffraction of high frequency cylindrical waves generated by a magnetic line source from the discontinuity occurred in the material properties as well as in the thicknesses of a coated cylindrically curved metallic sheet characterized by the second order GIBCs.

Relying on the locality of the high frequency diffraction phenomena, the angular interval $\varphi \in (-\pi, \pi)$ is extended to the abstract infinite space $\varphi \in (-\infty, \infty)$ [17] wherein the diffracting structure is replaced by a two-part cylindrically curved second order impedance sheet $\rho = a$ extending from $\varphi = -\infty$ to $\varphi = \infty$. The resulting boundary value problem is then formulated as a Hilbert equation which is solved asymptotically in the high frequency limit upon replacing the Bessel and Hankel functions involved with their Debye approximations. The procedure of solving Hilbert and Wiener-Hopf equations are described in detail in [18]. Some graphical results showing the effects of various parameters on the transfer coefficients are also presented.

2. FORMULATION OF THE PROBLEM

Let the magnetic line source K, with strength I and time dependence $e^{-i\omega t}$, be located at $\rho = b$, $\varphi = \varphi_K$, $z \in (-\infty, \infty)$ while the cylindrical surface is defined by $S : \{\rho = a < b,\ \varphi \in (-\infty, \infty),\ z \in (-\infty, \infty)\}$. We assume that the extended metallic cylinders $\{\rho = a,\ \varphi \in (0, \infty), z \in (-\infty, \infty)\}$ and $\{\rho = a,\ \varphi \in (-\infty, 0),\ z \in (-\infty, \infty)\}$ are coated with thin material layers whose relative constitutive parameters and thicknesses are $(\varepsilon_{r1},\ \mu_{r1},\ t_1)$ and $(\varepsilon_{r2},\ \mu_{r2},\ t_2)$, respectively (see Fig. 1). For $kt_j\sqrt{\varepsilon_{rj}\mu_{rj}} \ll 1$ $(j = 1, 2)$ they can be characterized by the

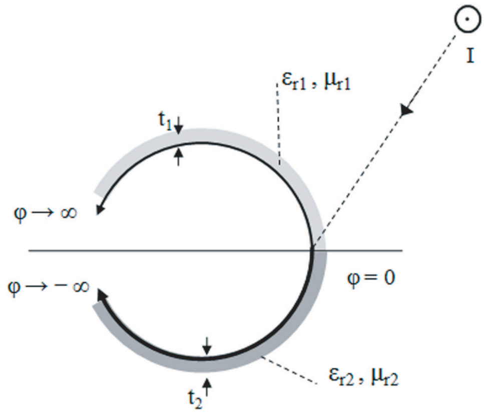

Figure 1. Geometry of the problem.

following second order GIBCs:

$$\left[\frac{\alpha_j}{(ka)^2}\frac{\partial^2}{\partial\varphi^2} \pm \frac{i\beta_j}{k}\frac{\partial}{\partial\rho} - \gamma_j\right] H_z\left(a \pm 0, \varphi\right) = 0, \tag{1a}$$

$$\alpha_j = kt_j\left(1 - \frac{1}{\varepsilon_{rj}}\right), \tag{1b}$$

$$\beta_j = -i\left\{t\left(1 - \frac{1}{\varepsilon_{rj}}\right)\left(\frac{1}{a} - \frac{1}{a+t_j}\right) - 1\right\} \tag{1c}$$

and

$$\gamma_j = kt_j\left(\mu_{rj} - 1\right). \tag{1d}$$

The Fourier transform technique is used within this paper, which is appropriate to be defined as

$$\hat{u}\left(\nu\right) = \int_{-\infty}^{\infty} u\left(\varphi\right) e^{i\nu\varphi} d\varphi \tag{2}$$

Here $\hat{u}\left(\nu\right)$ is the Fourier transform of the function $u\left(\varphi\right)$. For analysis purposes, it is convenient to express the total magnetic field $H_T\left(\rho, \varphi\right) = u_T\left(\rho, \varphi\right)$ which satisfies the Helmholtz equation in the infinitely extended angular space $\varphi \in (-\infty, \infty)$

$$\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + k^2\right] u_T\left(\rho, \varphi\right) = i\frac{Ik}{Z_o\rho}\delta\left(\rho - \rho_K\right)\delta\left(\varphi - \varphi_K\right) \tag{3}$$

with

$$u_T\left(\rho, \varphi\right) = u_{sc}\left(\rho, \varphi\right) + u_{inc}\left(\rho, \varphi\right), \tag{4}$$

where $u_{sc}(\rho,\varphi)$ and $u_{inc}(\rho,\varphi)$ are the scattered and the incident fields, respectively. The incident field is generated by the line source and it is the particular solution of the Helmholtz equation given by (3). By applying Fourier transform to this equation and solving the particular solution by Green's function method, the Fourier transform of the incident field yields

$$\hat{u}_{inc}(\rho,\nu)=\frac{\pi}{2}\frac{Ik}{Z_o}e^{i\nu\varphi_K}\begin{cases}J_{|\nu|}(k\rho)\,H_\nu^{(1)}(y)\,, & \rho<\rho_K\\ H_\nu^{(1)}(k\rho)\,J_{|\nu|}(y)\,, & \rho>\rho_K\end{cases}\tag{5}$$

where we put $y=k\rho_K$.

2.1. Derivation of the Simultaneous Hilbert Equations

Since $\hat{u}_{inc}(\rho,\nu)$ is the particular solution of the equation given by (5), the Fourier transform of the scattered field $\hat{u}_{sc}(\rho,\nu)$ yields the homogeneous solution which is

$$\hat{u}_{sc}(\rho,\nu)=A(\nu)\,H_\nu^{(1)}(k\rho)\tag{6}$$

where $A(\nu)$ is the spectral coefficients to be solved and $\rho>a$. On the other hand, the Fourier transform of the total field is the sum of the Fourier transforms of the incident and scattered fields and can be denoted by $\hat{u}_T(\rho,\nu)$. The total field satisfies the conditions

$$\left[\frac{\alpha_1}{x^2}\frac{\partial^2}{\partial\varphi^2}+\frac{i\beta_1}{k}\frac{\partial}{\partial\rho}-\gamma_1\right]u_T(a,\varphi)=0,\quad \varphi>0,\tag{7a}$$

$$\left[\frac{\alpha_2}{x^2}\frac{\partial^2}{\partial\varphi^2}+\frac{i\beta_2}{k}\frac{\partial}{\partial\rho}-\gamma_2\right]u_T(a,\varphi)=0,\quad \varphi<0,\tag{7b}$$

$$u_T(a,\varphi)=\begin{cases}0, & \varphi<0\\ -\mathcal{J}_e(\varphi)\,, & \varphi>0\end{cases}\tag{7c}$$

$$\frac{\partial}{\partial\rho}u_T(a,\varphi)=\begin{cases}0, & \varphi<0\\ -\dfrac{ik}{Z_o}\mathcal{J}_m(\varphi)\,, & \varphi>0\end{cases}\tag{7d}$$

with

$$\mathcal{J}_e(\varphi)=\mathcal{O}\left(\sqrt{a\varphi}\right),\quad \varphi\to 0\tag{7e}$$

and

$$\mathcal{J}_m(\varphi)=\mathcal{O}\left(\frac{1}{\sqrt{a\varphi}}\right),\quad \varphi\to 0.\tag{7f}$$

In the above equations, Z_o is the intrinsic impedance of the surrounding medium, and $\mathcal{J}_e(\varphi)$ and $\mathcal{J}_m(\varphi)$ denote the densities of the induced electric and magnetic surface currents, respectively. Applying Fourier transforms to the Equations (7a) and (7b).

$$\Phi^{-}(\nu) = \left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]H_\nu^{(1)}(x)-i\beta_1 H_\nu^{'(1)}(x)\right\}A(\nu) + \frac{\pi}{2}\frac{Ik}{Z_o}e^{i\nu\varphi_K}H_\nu^{(1)}(y)\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]J_{|\nu|}(x)-i\beta_1 J'_{|\nu|}(x)\right\} \tag{8a}$$

and

$$\Phi^{+}(\nu) = \left\{\left[\alpha_2\left(\frac{\nu}{x}\right)^2+\gamma_2\right]H_\nu^{(1)}(x)-i\beta_2 H_\nu^{\prime(1)}(x)\right\}A(\nu) + \frac{\pi}{2}\frac{Ik}{Z_o}e^{i\nu\varphi_K}H_\nu^{(1)}(y)\left\{\left[\alpha_2\left(\frac{\nu}{x}\right)^2+\gamma_2\right]J_{|\nu|}(x)-i\beta_2 J'_{|\nu|}(x)\right\} \tag{8b}$$

are obtained where

$$\Phi^{-}(\nu) = -\int_{-\infty}^{0}\left[\frac{\alpha_1}{x^2}\frac{\partial^2}{\partial\varphi^2}+\frac{i\beta_1}{k}\frac{\partial}{\partial\rho}-\gamma_1\right]u_T(a,\varphi)\,e^{i\nu\varphi}d\varphi \tag{9a}$$

and

$$\Phi^{+}(\nu) = -\int_{0}^{\infty}\left[\frac{\alpha_2}{x^2}\frac{\partial^2}{\partial\varphi^2}+\frac{i\beta_2}{k}\frac{\partial}{\partial\rho}-\gamma_2\right]u_T(a,\varphi)\,e^{i\nu\varphi}d\varphi. \tag{9b}$$

In the Equations (8a) and (8b), the prime over the Bessel and Hankel functions denote the first order derivatives with respect to the argument. With

$$H_\nu^{\prime(1)}(x)\,J_{|\nu|}(x) - H_\nu^{(1)}(x)\,J'_{|\nu|}(x) = \frac{2i}{\pi x}, \tag{10}$$

the elimination of the spectral coefficient $A(\nu)$ from the Equations (8a) and (8b) yields

$$\Phi^{+}(\nu) - \frac{\left\{\left[\alpha_2\left(\frac{\nu}{x}\right)^2+\gamma_2\right]H_\nu^{(1)}(x)-i\beta_2 H_\nu^{\prime(1)}(x)\right\}}{\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]H_\nu^{(1)}(x)-i\beta_1 H_\nu^{\prime(1)}(x)\right\}}\Phi^{-}(\nu) = \frac{Ik}{xZ_o}e^{i\nu\varphi_K}H_\nu^{(1)}(y)\frac{\left\{\beta_1\left[\alpha_2\left(\frac{\nu}{x}\right)^2+\gamma_2\right]-\beta_2\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]\right\}}{\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]H_\nu^{(1)}(x)-i\beta_1 H_\nu^{\prime(1)}(x)\right\}} \tag{11}$$

which is nothing but the Hilbert equation needed to be solved, valid for $\Im m\nu = 0$. It can be shown that the functions $\Phi^{+}(\nu)$ and $\Phi^{-}(\nu)$ can be approximated by the functions which satisfy the equations obtained from (11) by replacing the Bessel and Hankel functions with their Debye approximations given by [19] felsen, namely

$$H_\nu^{(1)}(z) \sim \sqrt{\frac{2}{\pi z \sin\theta_z}} e^{-i\pi/4} e^{i\nu[\tan\theta_z - \theta_z]} \tag{12a}$$

$$J_\nu(z) \sim \sqrt{\frac{1}{2\pi z \sin\theta_z}} e^{i\pi/4} e^{-i\nu[\tan\theta_z - \theta_z]}. \tag{12b}$$

This gives

$$\Phi^+(\nu) - \frac{\chi_1(\nu)}{\chi_2(\nu)}\Phi^-(\nu) = \frac{Ik}{xZ_o} e^{i\nu\varphi_K} H_\nu^{(1)}(y) \times \frac{\left\{\beta_1\left[\alpha_2\left(\frac{\nu}{x}\right)^2+\gamma_2\right]-\beta_2\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]\right\}}{\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]H_\nu^{(1)}(x)-i\beta_1 H_\nu^{\prime(1)}(x)\right\}}. \tag{13a}$$

where

$$\chi_j(\nu) = \left\{\left[\alpha_j\left(\frac{\nu}{x}\right)^2+\gamma_j\right]+\beta_j\sqrt{1-\left(\frac{\nu}{x}\right)^2}\right\}^{-1}, \qquad j=1,2. \tag{13b}$$

When the factorization procedure is applied, (13a) becomes

$$\frac{\chi_2^+(\nu)}{\chi_1^+(\nu)}\Phi^+(\nu) - \frac{\chi_1^-(\nu)}{\chi_2^-(\nu)}\Phi^-(\nu) = \frac{Ik}{xZ_o} e^{i\nu\varphi_K} H_\nu^{(1)}(y)\,\frac{\chi_2^+(\nu)}{\chi_1^+(\nu)} \times \frac{\left\{\beta_1\left[\alpha_2\left(\frac{\nu}{x}\right)^2+\gamma_2\right]-\beta_2\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]\right\}}{\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]H_\nu^{(1)}(x)-i\beta_1 H_\nu^{\prime(1)}(x)\right\}}. \tag{13c}$$

$\chi_j^+(\nu)$ and $\chi_j^-(\nu)$ $(j=1,2)$ appearing in the above equation, are the split functions, regular and free of zeros in the half-planes $\Im m\nu > x$ and $\Im m\nu < x$, respectively, resulting from the factorization of $\chi_j(\nu)$ in (13b) as

$$\chi_j(\nu) = \chi_j^+(\nu)\,\chi_j^-(\nu) \tag{15a}$$

$$\chi_j^+(\nu) = \chi_j^-(-\nu). \tag{15b}$$

Note that $\chi^\pm(\nu)$ can be expressed as

$$\chi_j^\pm(\nu) = \frac{ix\kappa^\pm\left(\frac{1}{\eta_j^{(1)}},\nu\right)\kappa^\pm\left(\frac{1}{\eta_j^{(2)}},\nu\right)}{\sqrt{\alpha_j}\sqrt{\eta_j^{(1)}\eta_j^{(2)}}\,(x\pm\nu)} \tag{16}$$

with

$$K\left(\frac{1}{\eta},\nu\right)=\left\{\frac{1}{\eta}+\frac{1}{\sqrt{1-\left(\frac{\nu}{x}\right)^2}}\right\}^{-1} \tag{17}$$

and

$$\eta_j^{(1)} = -\frac{1}{2\alpha_j}\left[\beta_j+\sqrt{\beta_j^2+4\alpha_j\left(\alpha_j+\gamma_j\right)}\,\right] \tag{18a}$$

$$\eta_j^{(2)} = -\frac{1}{2\alpha_j}\left[\beta_j-\sqrt{\beta_j^2+4\alpha_j\left(\alpha_j+\gamma_j\right)}\,\right] \tag{18b}$$

As it was shown in [12], $K^{\pm}(1/\eta,\nu)$ can be expressed explicitly in terms of the well-known Maliuzhinetz function. When the second term of the Equation (14) is decomposed, the Hilbert equation become

$$\frac{\chi_2^+(\nu)}{\chi_1^+(\nu)}\Phi^+(\nu)-\frac{\chi_1^-(\nu)}{\chi_2^-(\nu)}\Phi^-(\nu)=Q^+(\nu)+Q^-(\nu) \tag{19}$$

where $Q^+(\nu)$ and $Q^-(\nu)$ stand for the integrals

$$\begin{aligned}Q^{\pm}(\nu) &= \pm\frac{1}{2\pi i}\frac{Ik}{xZ_o}\int_{-\infty}^{\infty}e^{i\tau\varphi_K}H_\tau^{(1)}(y)\,\frac{\chi_2^+(\tau)}{\chi_1^+(\tau)}\\ &\quad\times\frac{\left\{\beta_1\left[\alpha_2\left(\frac{\tau}{x}\right)^2+\gamma_2\right]-\beta_2\left[\alpha_1\left(\frac{\tau}{x}\right)^2+\gamma_1\right]\right\}}{\left\{\left[\alpha_1\left(\frac{\tau}{x}\right)^2+\gamma_1\right]H_\tau^{(1)}(x)-i\beta_1H_\tau'^{(1)}(x)\right\}}\frac{d\tau}{(\tau-\nu)}.\end{aligned} \tag{20}$$

Replacing the Hankel functions by their Debye approximations allows us to calculate the integral $Q^-(\nu)$ by the well-known saddle-point method. The saddle-point occurs at $\tau = x\sin\psi_K$ when $\psi_K < \pi/2$ and the asymptotic evaluation gives

$$Q^-(\nu)=-\mathcal{Q}\frac{\left(w^2-s^2\right)}{(\nu-w)} \tag{21a}$$

with $w = x\sin\psi_K$,

$$s^2=\frac{(\beta_2\gamma_1-\beta_1\gamma_2)}{(\beta_1\alpha_2-\beta_2\alpha_1)}x^2 \tag{21b}$$

and

$$\mathcal{Q}=\frac{Ik}{x^2Z_o}\frac{e^{i\pi/4}}{\sqrt{2\pi}}\frac{e^{ikR_K}}{\sqrt{kR_K}}\frac{\chi_2^+(w)}{\chi_1^+(w)}\frac{(\beta_1\alpha_2-\beta_2\alpha_1)\cos\psi_K}{\left[\left(\alpha_1\sin^2\psi_K+\gamma_1\right)+\beta_1\cos\psi_K\right]}. \tag{21c}$$

The meanings of ψ_K and R_K are shown in Fig. 3.

If $\psi_K > \pi/2$, it can be shown that no saddle-point exists for the integral related to $Q_1^-(\nu)$. In this case this integral can be evaluated by using the residue technique. By virtue of the Jordan's Lemma and Cauchy's Theorem, it is equal to $(2\pi i)$ times the residues associated with the poles occurring at the zeros of $f(\nu)$ lying in the upper half-plane, namely; at ν_1, ν_2, ν_3, ... Since the contribution of the first pole is dominant over the others, one can write

$$Q^-(\nu) = -\tilde{\mathcal{Q}} \frac{\left(\nu_1^2 - s^2\right)}{(\nu - \nu_1)} \tag{22a}$$

with

$$\tilde{\mathcal{Q}} = -\frac{Ik}{x^3 Z_o} e^{i\nu_1\varphi_K} H_{\nu_1}^{(1)}(y) \frac{\chi_2^+(\nu_1)}{\chi_1^+(\nu_1)} \frac{(\beta_1\alpha_2 - \beta_2\alpha_1)}{f'(\nu_1)} \tag{22b}$$

and

$$f(\nu) = \left[\alpha_1 \left(\frac{\nu}{x}\right)^2 + \gamma_1\right] H_\nu^{(1)}(x) - i\beta_1 H_\nu'^{(1)}(x). \tag{22c}$$

The solution of the Hilbert equation is then found to be

$$\Phi^-(\nu) = \frac{\chi_2^-(\nu)}{\chi_1^-(\nu)} \begin{cases} \mathcal{Q}\left[\dfrac{\left(w^2 - s^2\right)}{(\nu - w)} + \mathcal{C}\right], & \psi_K < \pi/2 \\ \tilde{\mathcal{Q}}\left[\dfrac{\left(\nu_1^2 - s^2\right)}{(\nu - \nu_1)} + \tilde{\mathcal{C}}\right], & \psi_K > \pi/2 \end{cases}. \tag{23}$$

2.2. Determination of Unknown Constants $\mathcal{C}$ and $\tilde{\mathcal{C}}$

Since the standard edge conditions are not sufficient to obtain a unique solution, additional constraints must be considered to determine the constants $\mathcal{C}$ and $\tilde{\mathcal{C}}$. For the edged structures with second-order GIBC's, the required constraint is obtained by the help of the approach given in [11] and [20] [1975] as

$$\lim_{\varphi\to 0^+} \mathcal{J}_\rho(\varphi) = \lim_{\varphi\to 0^-} \mathcal{J}_\rho(\varphi) \tag{24a}$$

or

$$\frac{\alpha_1}{\beta_1} \lim_{\varphi\to 0^+} \frac{\partial}{\partial\varphi} I(\varphi) = \frac{\alpha_2}{\beta_2} \lim_{\varphi\to 0^-} \frac{\partial}{\partial\varphi} I(\varphi) \tag{24b}$$

with

$$I(\varphi) = u_T(a, \varphi). \tag{24c}$$

Taking into account the expressions (4) and (6) for $u_T(\rho, \varphi)$, one gets

$$I(\varphi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\hat{u}_{inc}(a,\nu) + A(\nu) H_\nu^{(1)}(x)\right] e^{-i\nu\varphi} d\nu. \tag{25}$$

Equation (8a) together with the Debye approximations of the Bessel and Hankel functions yield

$$\begin{aligned} A(\nu) &= \chi_1(\nu)\Phi^-(\nu) - \frac{\pi}{2}\frac{Ik}{Z_o} e^{i\nu\varphi_K} H_\nu^{(1)}(x) H_\nu^{(1)}(y) \\ &\quad \times \frac{\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2 + \gamma_1\right] J_{|\nu|}(x) - i\beta_1 J'_{|\nu|}(x)\right\}}{\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2 + \gamma_1\right] H_\nu^{(1)}(x) - i\beta_1 H_\nu'^{(1)}(x)\right\}}. \end{aligned} \tag{26}$$

For $|\psi_K| < \pi/2$ we may use Debye approximations for the second term at the right-hand side of the above equation which gives

$$A(\nu) = \chi_1(\nu)\Phi^-(\nu) - \hat{u}_{inc}(a,\nu) \frac{\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2 + \gamma_1\right] - \beta_1\sqrt{1-\left(\frac{\nu}{x}\right)^2}\right\}}{\left\{\left[\alpha_1\left(\frac{\nu}{x}\right)^2 + \gamma_1\right] + \beta_1\sqrt{1-\left(\frac{\nu}{x}\right)^2}\right\}}. \tag{27}$$

Substituting the above relation in (25), one can obtain

$$I(\varphi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\chi_1(\nu)\Phi^-(\nu) + 2\hat{u}_{inc}(a,\nu)\chi_1(\nu)\beta_1\sqrt{1-\left(\frac{\nu}{x}\right)^2}\,\right] e^{-i\nu\varphi} d\nu. \tag{28}$$

The above relation can be rewritten as

$$I(\varphi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[V(\nu) + W(\nu)\right] e^{-i\nu\varphi} d\nu \tag{29}$$

with

$$V(\nu) = \frac{\pi I k e^{i\nu\varphi_K}\beta_1}{Z_o} J_{|\nu|}(x) H_\nu^{(1)}(y) \chi_1(\nu) \sqrt{1 - \left(\frac{\nu}{x}\right)^2} \tag{30a}$$

and

$$W(\nu) = \chi_1^+(\nu)\chi_2^-(\nu)\mathcal{Q}\left[\frac{(w^2 - s^2)}{(\nu - w)} + \mathcal{C}\right]. \tag{30b}$$

Here, $V(\nu)$ can be decomposed as $V(\nu) = V^-(\nu) + V^+(\nu)$ where

$$V^+(\nu) = \frac{Ik\beta_1}{2iZ_o}\int_{-\infty}^{\infty} e^{i\tau\varphi_K} J_{|\tau|}(x) H_\tau^{(1)}(y)\chi_1(\tau)\sqrt{1-\left(\frac{\tau}{x}\right)^2}\frac{d\tau}{(\tau-\nu)}. \tag{31}$$

This integral can be evaluated via the saddle-point technique where the saddle-point occurs at $\tau = x \sin\psi_K$. This gives

$$V^+(\nu) = \frac{x^2}{(\beta_1\alpha_2 - \beta_2\alpha_1)} \mathcal{Q}\beta_1 \frac{\chi_1^+(w)}{\chi_2^+(w)} \frac{1}{(\nu - w)} \tag{32}$$

On the other hand, $W(\nu)$ needs to be decomposed as well. The term $\chi_1^+(\nu)\chi_2^-(\nu)$ can be arranged as

$$\chi_1^+(\nu)\chi_2^-(\nu) = \frac{x^2}{(\beta_1\alpha_2 - \beta_2\alpha_1)} \frac{[S^+(\nu) - S^-(\nu)]}{(\nu^2 - s^2)} \tag{33a}$$

with

$$S^+(\nu) = \beta_1 \frac{\chi_1^+(\nu)}{\chi_2^+(\nu)} + a_o + a_1\nu \tag{33b}$$

and

$$S^-(\nu) = \beta_2 \frac{\chi_2^-(\nu)}{\chi_1^-(\nu)} + a_o + a_1\nu. \tag{33c}$$

Here, the constants a_o and a_1 are introduced to eliminate the poles of $S^-(\nu)$ and $S^+(\nu)$ at $\nu = -s$ and $\nu = s$, respectively. Thus a_o and a_1 read

$$a_o = \frac{1}{2}\left[\beta_1 \frac{\chi_1^+(s)}{\chi_2^+(s)} + \beta_2 \frac{\chi_2^+(s)}{\chi_1^+(s)}\right] \tag{34a}$$

and

$$a_1 = \frac{1}{2s}\left[\beta_1 \frac{\chi_1^+(s)}{\chi_2^+(s)} - \beta_2 \frac{\chi_2^+(s)}{\chi_1^+(s)}\right]. \tag{34b}$$

Finally, $W(\nu)$ is decomposed as

$$W^+(\nu) = \mathcal{Q}\frac{x^2}{(\beta_1\alpha_2 - \beta_2\alpha_1)}\left\{-\frac{S^+(w)}{(\nu - w)} + \frac{S^+(\nu)}{(\nu^2 - s^2)}\left[\frac{(w^2 - s^2)}{(\nu - w)} + \mathcal{C}\right]\right\} \tag{35a}$$

and

$$W^-(\nu) = -\mathcal{Q}\frac{x^2}{(\beta_1\alpha_2 - \beta_2\alpha_1)}\left\{\frac{S^+(w)}{(\nu - w)} + \frac{S^-(\nu)}{(\nu^2 - s^2)}\left[\frac{(w^2 - s^2)}{(\nu - w)} + \mathcal{C}\right]\right\} \tag{35b}$$

The integral in (29) can be rearranged as

$$I(\varphi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[V^+(\nu) + W^+(\nu)\right] e^{-i\nu\varphi} d\nu + \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[V^-(\nu) + W^-(\nu)\right] e^{-i\nu\varphi} d\nu. \tag{36}$$

where

$$V^{+}(\nu)+W^{+}(\nu) = \mathcal{Q}\frac{x^2}{(\beta_1\alpha_2-\beta_2\alpha_1)} \times\left\{\frac{S^{+}(\nu)}{(\nu^2-s^2)}\left[\frac{(w^2-s^2)}{(\nu-w)}+\mathcal{C}\right]-\frac{(a_o+a_1w)}{(\nu-w)}\right\} \tag{37a}$$

and

$$V^{-}(\nu)+W^{-}(\nu) = \frac{\pi Ik}{Z_o}e^{i\nu\varphi_K}J_{|\nu|}(x)H_{\nu}^{(1)}(y)\chi_1(\nu)\beta_1\sqrt{1-\left(\frac{\nu}{x}\right)^2} +\mathcal{Q}\frac{x^2}{(\beta_1\alpha_2-\beta_2\alpha_1)}\left\{\frac{(a_o+a_1w)}{(\nu-w)}-\frac{S^{-}(\nu)}{(\nu^2-s^2)}\left[\frac{(w^2-s^2)}{(\nu-w)}+\mathcal{C}\right]\right\} \tag{37b}$$

For $\varphi \to 0^{+}$, the second integral at the right-hand side of (36) vanishes as $V^{-}(\nu)$ and $W^{-}(\nu)$ are both regular in the lower half-plane. Considering now the asymptotic expansions of $V^{+}(\nu)$ and $W^{+}(\nu)$, using

$$\frac{1}{(\nu-w)} \approx \frac{1}{\nu}\left[1+\frac{w}{\nu}+O\left(\nu^{-2}\right)\right] \tag{38a}$$

and

$$\chi_j^{+}(\nu) \sim \frac{x}{\sqrt{\alpha_j}}\frac{1}{\nu} \tag{38b}$$

one obtains

$$V^{+}(\nu)+W^{+}(\nu)=\mathcal{Q}\frac{x^2}{(\beta_1\alpha_2-\beta_2\alpha_1)} \times\left\{\frac{(a_1\mathcal{C}-a_o-a_1w)}{\nu}+\frac{\left(p_1\mathcal{C}-a_1s^2-a_ow\right)}{\nu^2}+O\left(\nu^{-3}\right)\right\} \tag{39a}$$

with

$$p_1=\left(\beta_1\frac{\sqrt{\alpha_2}}{\sqrt{\alpha_1}}+a_o\right) \tag{39b}$$

which gives

$$u^T\left(a,0^{+}\right) \approx -2\pi i\mathcal{Q}\frac{x^2}{(\beta_1\alpha_2-\beta_2\alpha_1)} \times\left\{(a_1\mathcal{C}-a_o-a_1w)+i\varphi\left(p_1\mathcal{C}-a_1s^2-a_ow\right)+O\left(\varphi^2\right)\right\}. \tag{39c}$$

By applying a similar approach for $\varphi \to 0^{-}$, the second integral at the right-hand side of (36) vanishes as $V^{+}(\nu)$ and $W^{+}(\nu)$ are both regular in the lower half-plane. Considering now the asymptotic expansions of $V^-(\nu)$ and $W^-(\nu)$, using (38a) and (38b), one gets

$$V^-(\nu)+W^-(\nu)=\mathcal{Q}\frac{x^2}{(\beta_1\alpha_2-\beta_2\alpha_1)} \times\left\{\frac{(a_o+a_1w-a_1\mathcal{C})}{\nu}+\frac{\left(a_ow+a_1s^2-p_2\mathcal{C}\right)}{\nu^2}+O\left(\nu^{-3}\right)\right\} \quad (40a)$$

with

$$p_2=\left(\beta_2\frac{\sqrt{\alpha_1}}{\sqrt{\alpha_2}}+a_o\right) \quad (40b)$$

which gives

$$u^T\left(a,0^-\right)\approx 2\pi i\mathcal{Q}\frac{x^2}{(\beta_1\alpha_2-\beta_2\alpha_1)} \times\left\{(a_o+a_1w-a_1\mathcal{C})+i\varphi\left(a_ow+a_1s^2-p_2\mathcal{C}\right)+O\left(\varphi^2\right)\right\}. \quad (40c)$$

Substituting (39c) and (40c) in (24b) yields

$$\mathcal{C}=\frac{a_1}{a_o}s^2+w. \quad (41)$$

For $\psi_K>\pi/2$ a similar procedure is applied giving

$$\tilde{\mathcal{C}}=\frac{a_1}{a_o}s^2+\nu_1. \quad (42)$$

3. ANALYSIS OF THE FIELDS

The explicit expression of the field at any point (ρ,φ) outside the reflector and the source can be found by asymptotically evaluating the inverse Fourier transform of $\hat{u}(\rho,\nu)$. According to the positions of the line source and the observation point, the expressions of the field components have different physical interpretations. These cases will be considered separately. Taking into account (6) and (8a) the scattered field in the region can be obtained by evaluating the integrals

$$u_{sc}^{(1)}(\rho,\varphi)=-\frac{Ik}{4Z_o}\int_{-\infty}^{\infty}e^{i\nu\varphi_K}H_\nu^{(1)}(y)\,H_\nu^{(1)}(k\rho)\,\frac{g(\nu)}{f(\nu)}e^{-i\nu\varphi}d\nu \quad (43a)$$

with

$$g(\nu)=\left[\alpha_1\left(\frac{\nu}{x}\right)^2+\gamma_1\right]J_{|\nu|}(x)-i\beta_1J'_{|\nu|}(x) \quad (43b)$$

and

$$u_{sc}^{(2)}(\rho,\varphi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Phi^{-}(\nu)\,H_{\nu}^{(1)}(k\rho)}{f(\nu)}e^{-i\nu\varphi}d\nu. \tag{43c}$$

The scattered field is then

$$u_{sc}(\rho,\varphi) = u_{sc}^{(1)}(\rho,\varphi) + u_{sc}^{(2)}(\rho,\varphi). \tag{44}$$

3.1. The Case in Which the Edge is Illuminated by a Direct Ray

3.1.1. Reflected Field

When $\psi_K < \pi/2$, the edge is illuminated directly by the line source. Additionally, if the observation point lies in the region where $\psi < \pi/2$, the integral given by (43a) can be evaluated asymptotically by the saddle-point technique. In this condition, the saddle-point occurs at $\nu_s = x\sin\vartheta$. Hence, the explicit expression of $u^{(1)}(\rho,\varphi)$ reads

$$u^{(1)}(\rho,\varphi) = u_r = u_i(R_r)\,T_{rr}\sqrt{\frac{lR_r}{\left(R_r+\tilde{R}_r\right)l+2R_r\tilde{R}_r}}e^{ik\tilde{R}_r} \tag{45}$$

which is nothing but the reflected field. Here $u_i(R_r)$ is the incident field evaluated at the point M_r.

$$u_i(R_j) = \left(\frac{Ik}{2Z_o}\right)\frac{e^{-i\pi/4}}{\sqrt{2\pi}}\frac{e^{ikR_j}}{\sqrt{kR_j}} \tag{46}$$

and the reflection coefficient T_{rr} is defined by

$$T_{rr} = -\frac{\left(\alpha_1\sin^2\vartheta+\gamma_1-\beta_1\cos\vartheta\right)}{\left(\alpha_1\sin^2\vartheta+\gamma_1+\beta_1\cos\vartheta\right)}. \tag{47}$$

The meanings of the parameters l, R_r, $\tilde{R}_r$ and ϑ are shown in Fig. 2.

3.1.2. Edge Diffracted Field

The integral (43c) can also be evaluated asymptotically by the saddle-point method when the observation point is in the same region as above. For this integral, the saddle-point occurs at $\nu_s = -x\sin\psi$ with ψ being the observation angle. This gives

$$u^{(2)}(\rho,\varphi) = u_e(\rho,\varphi) = u_i(R_K)\,T_{ee}\frac{e^{ikR_1}}{\sqrt{kR_1}} \tag{48}$$

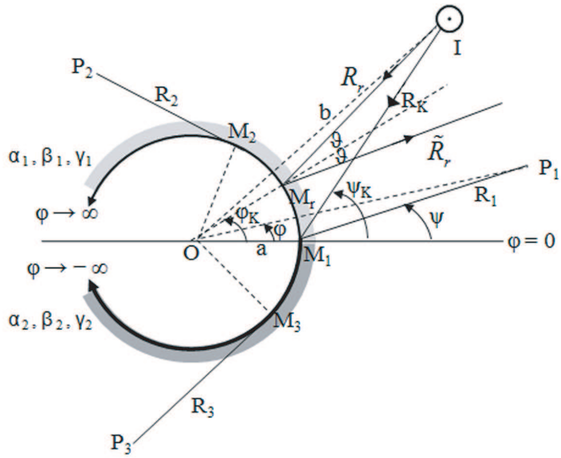

Figure 2. Geometrical parameters pertaining to the reflected field.

with the edge diffraction coefficient T_{ee} is defined by

$$T_{ee}(\psi_K,\psi)=\sqrt{\frac{2}{\pi}}e^{i\pi/4}\frac{\cos\psi}{\left(\alpha_1\sin^2\psi+\gamma_1+\beta_1\cos\psi\right)}$$
$$\times\frac{\cos\psi_K}{\left(\alpha_1\sin^2\psi_K+\gamma_1+\beta_1\cos\psi_K\right)}$$
$$\times\frac{\chi_2^+(x\sin\psi)\,\chi_2^+(x\sin\psi_K)}{\chi_1^+(x\sin\psi)\,\chi_1^+(x\sin\psi_K)}\frac{1}{(\sin\psi+\sin\psi_K)}$$
$$\times\left\{(\beta_2\gamma_1-\beta_1\gamma_2)\left[1+\frac{a_1}{a_o}(\sin\psi+\sin\psi_K)\right]+(\beta_1\alpha_2-\beta_2\alpha_1)\sin\psi\sin\psi_K\right\}. \tag{49}$$

The meanings of R_1 and ψ are shown in Fig. 3.

3.1.3. Creeping Modes

When the observation point lies in the region $\psi > \pi/2$ the integral (43c) can no longer be evaluated by the saddle-point method. In order to get an expression valid in this region, this integral can be evaluated by the residue method. The singularities of the integrand of (43c) lying below the real axis consist of the simple poles at the zeros of $f(\nu)$, namely at $-\nu_1$, $-\nu_2$, ... By using the Jordan's Lemma, we can show that the integral (43c) is equal to $(-2\pi i)$ times the sum of the residues at these poles. The dominant contribution comes from the

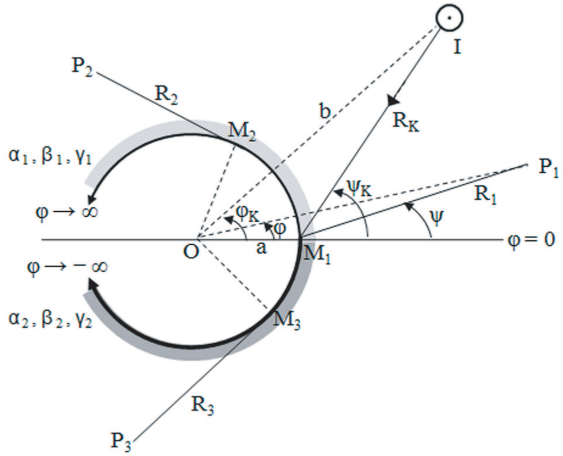

Figure 3. Illumination by a direct ray.

first pole and is equal to

$$u_{sc}^{(2)}(\rho,\varphi) = ie^{i\nu_1\varphi}\frac{\Phi^-(-\nu_1)\,H_{\nu_1}^{(1)}(k\rho)}{f'(\nu_1)} \tag{50}$$

with $\Phi^-(\nu)$ is given in (23). By replacing $H_{\nu_1}^{(1)}(k\rho)$ by its uniform asymptotic expression valid for $k\rho \to \infty$ we arrive at a result which can be arranged as

$$u^{(2)}(\rho,\varphi) \approx u_i(R_K)\,T_{ec}^{(1)}e^{i\nu_1\widehat{M_1M_2}/a}T_{cs}^{(1)}\frac{e^{ikR_2}}{\sqrt{kR_2}}. \tag{51}$$

The meanings of M_2 and R_2 are shown in Fig. 3 and the factor $T_{cs}^{(1)}$ is the surface diffraction coefficient related to the coating as

$$T_{cs}^{(1)} = \frac{2^{5/4}}{\pi^{1/4}}e^{i\pi/8}\sqrt{\frac{\beta_1}{xH_{\nu_1}^{(1)}(x)\,f'(\nu_1)}}. \tag{52}$$

$T_{ec}^{(1)}$ appearing in (51) is the transfer coefficient showing the modifications to be considered when the incident field is transformed into a creeping mode at the edge M_1. $T_{ec}^{(1)}$ is defined by

$$\begin{aligned}T_{ec}^{(1)} &= \left(\frac{2}{\pi}\right)^{1/4}e^{i5\pi/8}\frac{\chi_2^+(\nu_1)}{\chi_1^+(\nu_1)}\frac{\chi_2^+(w)}{\chi_1^+(w)}\sqrt{\frac{H_{\nu_1}^{(1)}(x)}{\beta_1 f'(\nu_1)}}\\ &\times\frac{\cos\psi_K}{(\alpha_1\sin^2\psi_K+\gamma_1+\beta_1\cos\psi_K)}\frac{\sqrt{x}}{(\nu_1+x\sin\psi_K)}\\ &\times\left[(\beta_2\gamma_1-\beta_1\gamma_2)\left(1+\nu_1\frac{a_1}{a_o}+x\sin\psi_K\frac{a_1}{a_o}\right)+\frac{(\beta_1\alpha_2-\beta_2\alpha_1)}{x^2}\nu_1 x\sin\psi_K\right]\end{aligned} \tag{53}$$

If the observation angle φ is negative, in order to make use of Jordan's Lemma, the integration path in (43c) must be enclosed to cover the upper half-plane. Thus, we can show that the integral (43c) is equal to $(2\pi i)$ times the sum of the residues at the poles μ_1, μ_2, μ_3, ... which are the zeros of the function $g(\nu)$ defined by

$$g(\nu) = \left[\alpha_2 \left(\frac{\nu}{x}\right)^2 + \gamma_2\right] H_\nu^{(1)}(x) - i\beta_2 H_\nu'^{(1)}(x). \tag{54}$$

Hence, the creeping wave for $\varphi < 0$ yields

$$u^{(2)}(\rho,\varphi) \approx u_i(R_K)\, T_{ec}^{(2)} e^{i\mu_1 \widehat{M_1 M_3}/a} T_{cs}^{(2)} \frac{e^{ikR_3}}{\sqrt{kR_3}}. \tag{55}$$

Here $T_{ec}^{(2)}$ is the transfer coefficient showing the modifications to be considered when the incident field is transformed into a creeping mode at the edge M_1 for $\varphi < 0$ and $T_{cs}^{(2)}$ is surface diffraction coefficient of the surface at $\varphi < 0$. They are found to be

$$\begin{aligned} T_{ec}^{(2)} =& \left(\frac{2}{\pi}\right)^{1/4} e^{-i3\pi/8} \frac{\chi_2^+(w)}{\chi_1^+(w)} \frac{\chi_1^+(\mu_1)}{\chi_2^+(\mu_1)} \sqrt{\frac{H_{\mu_1}^{(1)}(x)}{\beta_2 g'(\mu_1)}} \\ &\times \frac{\sqrt{x}\cos\psi_K}{\left(\alpha_1 \sin^2\psi_K + \gamma_1 + \beta_1 \cos\psi_K\right)(\mu_1 - w)} \\ &\times \left\{(\beta_2\gamma_1 - \beta_1\gamma_2)\left[1 - \frac{a_1}{a_o}(\mu_1 - w)\right] - (\beta_1\alpha_2 - \beta_2\alpha_1)\frac{\mu_1 \sin\psi_K}{x}\right\} \end{aligned} \tag{56}$$

and

$$T_{cs}^{(2)} = \frac{2^{5/4}}{\pi^{1/4}} e^{i\pi/8} \sqrt{\frac{\beta_2}{x H_{\mu_1}^{(1)}(x)\, g'(\mu_1)}}, \tag{57}$$

respectively.

3.2. The Case in Which the Edge is Illuminated by a Creeping Mode

3.2.1. Creeping Modes Generated by the Incident Field

When $\psi_K > \pi/2$, at a certain point M_4 of the scatterer, the line between the source and M_4 will be tangent to the scatterer where a surface diffraction process occurs and excites creeping modes propagating towards the edge point M_1 (see Fig. 3). If the observation point lies in the region $\psi < \pi/2$ the term $u^{(1)}(\rho,\varphi)$ cannot be detectable as reflected field. The evaluation of this integral yields

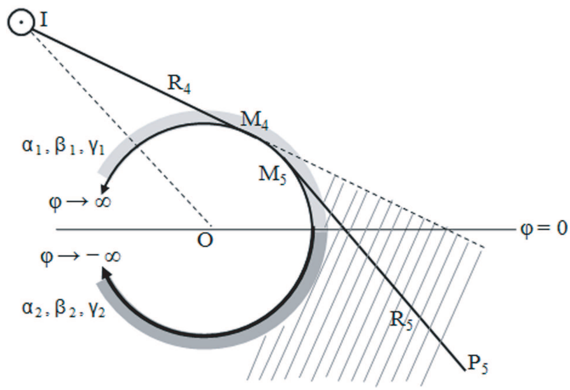

Figure 4. Surface diffraction in creeping mode illumination.

$$u_{sc}^{(1)}(\rho,\varphi) = u_i(M_4)\, T_{sc}^{(1)}(M_4)\, e^{i\nu_1 \widehat{M_4M_5}/a} T_{cs}^{(1)}(M_5)\, \frac{e^{ikR_5}}{\sqrt{kR_5}} \tag{58}$$

Here, the meanings of M_4, M_5 and R_5 are shown in Fig. 4 and the surface diffraction coefficients $T_{cs}^{(1)} = T_{sc}^{(1)}$ are given by (52).

3.2.2. Edge Excited Direct Ray

In order to obtain edge excited fields in the region $\rho > a$ when the edge is illuminated by a creeping mode, we have to reconsider the integral (45b). The analysis in Subsubsection 3.1.2 can be carried over to the present case provided that (22a) is taken into account for $Q^-(\nu)$ and the solution for $\psi_K > \pi/2$ is valid in (23). So, by direct application of the above analysis we obtain the result

$$u_{sc}^{(2)}(\rho,\varphi) = u_i(L)\, T_{sc}^{(1)} e^{i\nu_1 \widehat{LM_1}/a} T_{ce} \frac{e^{ik\tilde{R}_1}}{\sqrt{k\tilde{R}_1}} \tag{59}$$

where $T_{sc}^{(1)}$ is defined as in (52) while

$$\begin{aligned} T_{ce} & \left(\frac{2}{\pi}\right)^{1/4} e^{i5\pi/8} \frac{\chi_2^+(\nu_1)}{\chi_1^+(\nu_1)} \frac{\chi_2^+(x\sin\psi)}{\chi_1^+(x\sin\psi)} \sqrt{\frac{H_{\nu_1}^{(1)}(x)}{\beta_1 f'(\nu_1)}} \\ & \times \frac{\cos\psi}{(\alpha_1 \sin^2\psi + \gamma_1 + \beta_1\cos\psi)} \frac{\sqrt{x}}{(x\sin\psi + \nu_1)} \\ & \times \left[(\beta_2\gamma_1 - \beta_1\gamma_2)\left(1 + v\frac{a_1}{a_o} x\sin\psi + \frac{a_1}{a_o}\nu_1\right) + (\beta_1\alpha_2 - \beta_2\alpha_1)\frac{\nu_1\sin\psi}{x}\right]. \end{aligned} \tag{60}$$

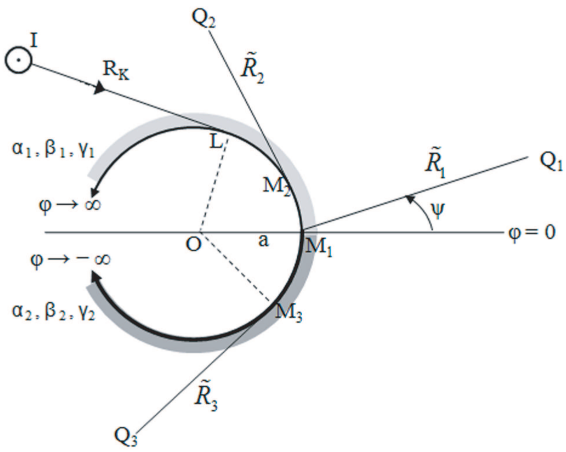

Figure 5. Creeping mode illumination.

Here, the meanings of L, M_1 and $\tilde{R}_1$ is shown in Fig. 5. T_{ce} is the transfer coefficient related to the transformation of the creeping mode into an edge excited diffracted ray occurring at the edge M_1. Notice that this result satisfies the reciprocity principle, since T_{ce} is equal to T_{ec} provided that ψ_K is replaced by ψ.

3.2.3. Creeping Mode Excited by the Edge

The creeping mode generated by the edge can easily be obtained by carrying over the same analysis as in § 3.1.3 to the present case provided that (22a) is taken into account for $Q^-(\nu)$ and the solution for $\psi_K > \pi/2$ is valid in (23). The solution for $\psi > \pi/2$ can be written in form

$$u_{sc}^{(2)}(\rho,\varphi) = u_i(L)\, T_{sc}^{(1)} e^{i\nu_1 \widehat{LM_1}/a} T_{cc}^{(1)} e^{i\nu_1 \widehat{M_1M_2}/a} T_{cs}^{(1)} \frac{e^{ik\tilde{R}_2}}{\sqrt{k\tilde{R}_2}} \tag{61}$$

with $T_{cc}^{(1)}$ given by

$$\begin{aligned} T_{cc}^{(1)} \; = \; & -\left[\frac{\chi_2^+(\nu_1)}{\chi_1^+(\nu_1)}\right]^2 \frac{H_{\nu_1}^{(1)}(x)}{2\nu_1\beta_1 f'(\nu_1)} \\ & \times\left[(\beta_2\gamma_1-\beta_1\gamma_2)\left(1+\frac{a_1}{a_o}2\nu_1\right)+(\beta_1\alpha_2-\beta_2\alpha_1)\left(\frac{\nu_1}{x}\right)^2\right] \end{aligned} \tag{62}$$

is the transfer coefficient which dictates the modifications that the incident creeping mode suffers at the edge M_1 during its transformation into an edge excited creeping mode.

On the other hand, when $\varphi < 0$, in order to make use of Jordan's Lemma, the integration path in (43c) must be enclosed to cover the upper half-plane. Thus, we can show that the integral (43c) is equal to $(2\pi i)$ times the sum of the residues at the poles μ_1, μ_2, μ_3, ... which are the zeros of the function $g(\nu)$ defined by (54). Hence, the creeping wave for $\varphi < 0$ yields

$$u_{sc}^{(2)}(\rho, \varphi) = u_i(L)\, T_{sc}^{(1)} e^{i\nu_1 \widehat{LM_1}/a} T_{cc}^{(2)} e^{i\nu_1 \widehat{M_1 M_3}/a} T_{cs}^{(2)} \frac{e^{ik\tilde{R}_3}}{\sqrt{k\tilde{R}_3}} \tag{63}$$

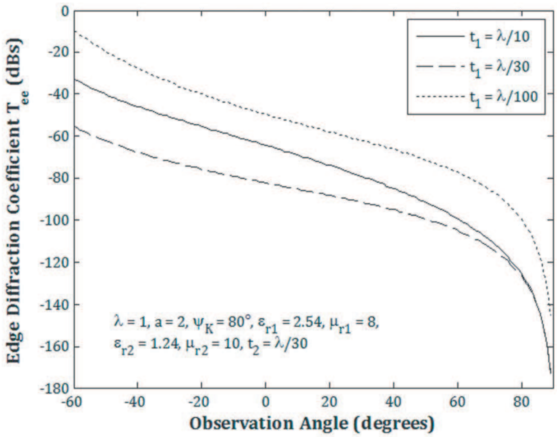

Figure 6. Variation of T_{ee} with respect to the coating thickness t_1.

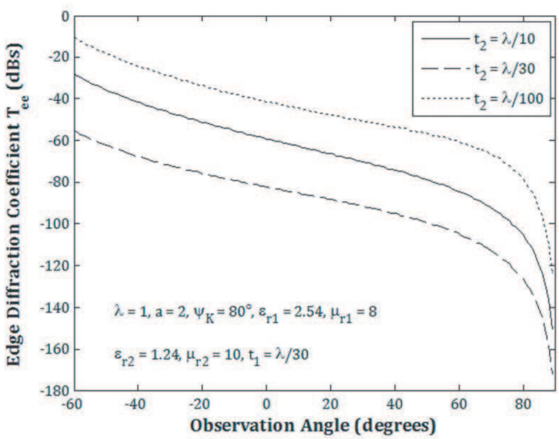

Figure 7. Variation of T_{ee} with respect to the coating thickness t_2.

with $T_{cc}^{(2)}$ given by

$$T_{cc}^{(2)} = \frac{\chi_2^+(\nu_1)}{\chi_1^+(\nu_1)} \frac{\chi_1^+(\mu_1)}{\chi_2^+(\mu_1)} \sqrt{\frac{H_{\nu_1}^{(1)}(x) H_{\mu_1}^{(1)}(x)}{\beta_1\beta_2 f'(\nu_1) g'(\mu_1)}} \frac{1}{(\mu_1-\nu_1)} \times\left\{(\beta_2\gamma_1-\beta_1\gamma_2)\left[1+\frac{a_1}{a_o}(\nu_1-\mu_1)\right]-\frac{\mu_1\nu_1}{x}(\beta_1\alpha_2-\beta_2\alpha_1)\right\}. \quad (64)$$

4. COMPUTATIONAL RESULTS

For numerical purposes, the solution of $f(\nu) = 0$, $g(\nu) = 0$ and the numerical values of $f'(\nu_1)$ and $g'(\mu_n)$ are required. By applying a similar procedure described in [21], the first zero of $f(\nu)$ can be obtained as:

$$\nu_1 = x - \tau_1 \left(\frac{2}{x}\right)^{1/3} e^{i\pi/3} \quad (65)$$

with

$$\tau_1 \sim -1.019 + \frac{e^{i5\pi/6}}{1.019\beta_1}\left(\frac{x}{2}\right)^{1/3}\left\{\frac{\alpha_1}{x^2}\left[x+1.019\left(\frac{x}{2}\right)^{1/3} e^{i\pi/3}\right]+\gamma_1\right\}. \quad (66)$$

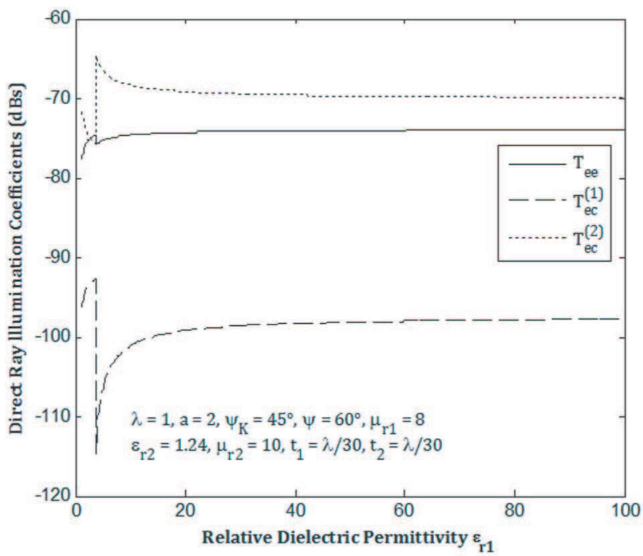

Figure 8. Variation of direct ray illumination coefficients with respect to the relative permittivity ε_{r1}.

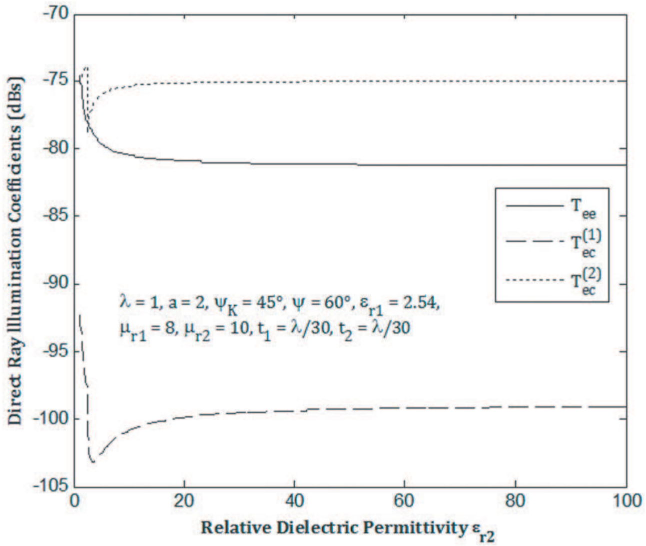

Figure 9. Variation of direct ray illumination coefficients with respect to the relative permittivity ε_{r2}.

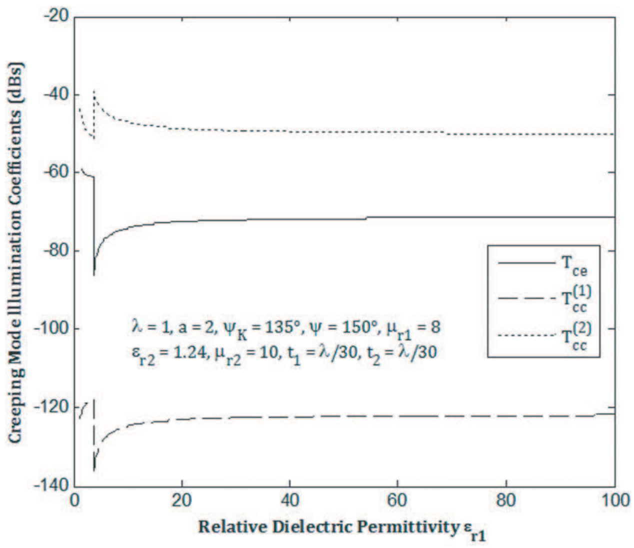

Figure 10. Variation of creeping mode illumination coefficients with respect to the relative permittivity ε_{r1}.

Similarly the first zero of $g(\nu)$ can be obtained as

$$\mu_1 = x - \tau_2 \left(\frac{2}{x}\right)^{1/3} e^{i\pi/3} \tag{67}$$

with

$$\tau_2 \sim -1.019 + \frac{e^{i5\pi/6}}{1.019\beta_2}\left(\frac{x}{2}\right)^{1/3}\left\{\frac{\alpha_2}{x^2}\left[x + 1.019\left(\frac{x}{2}\right)^{1/3} e^{i\pi/3}\right] + \gamma_2\right\}. \tag{68}$$

In Figs. 6–10, the dependences of the coefficients T_{ee}, $T_{ec}^{(1,2)}$, T_{ce} and $T_{cc}^{(1,2)}$ on various parameters are shown.

5. CONCLUDING REMARKS

In this paper, the diffraction of electromagnetic waves generated by a line source by a cylindrically curved metallic sheet which is partially coated by dielectric layers of different physical properties and thicknesses is investigated rigorously. The problem is formulated in an infinitely extended angular space from which explicit asymptotic expressions for the diffraction coefficients are obtained through the asymptotic solution of a Hilbert problem.

For $\alpha/\beta \to 0$ and $\gamma/\beta \to \eta$ which corresponds to the case where coatings are modelled with the first order impedance boundary conditions ($\eta = Z/Z_o$), we get

$$\frac{\chi_1(\nu)}{\chi_2(\nu)} = \frac{\beta_2}{\beta_1} G(\nu)$$

where $G(\nu)$ is the same function as defined in Equation (6a) of the paper of Büyükaksoy and Uzgören which is published in 1987 [5]. In this case, all the coefficients defined in Section 3 coincide with the ones given in [5].

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