## A NOVEL DUAL-BAND BANDPASS FILTER USING GENERALIZED TRISECTION STEPPED IMPEDANCE RESONATOR WITH IMPROVED OUT-OF-BAND PER-FORMANCE

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Abstract—This paper presents the synthesis of a novel dual-band bandpass filter with improved out-of-band performance. The proposed circuit is constructed by cascading a dual-band filter using trisection stepped impedance resonators (SIRs) and an L-C ladder lowpass filter using open-circuited stubs. The dual-band trisection SIR can provide the desired dual-band response, and the lowpass filter can improve the out-of-band performance by suppressing the harmonics and spurious responses. The proposed filter has been fabricated and measured. Simulation and measurement results are found to be in good agreement.

## 1. INTRODUCTION

The requirements of microwave bandpass filters for modern wireless and mobile communication systems have become more stringent. Compact, small size and good performances are often the typical requirements of filters. Compared to single-band filter, dual-band and multi-band planar filters are more popular due to their advantages of compact size, ease of integration and fabrication by using printed circuit technology for commercial applications [1,2].

Recently, more researchers pay their attention to exploring dualband and multi-band filters [3–6]. In [3], a dual-band filter which was a mixture of shunt stub bandpass filter and shunt serial resonator bandstop filter was presented. The two passbands of the bandpass filter was implemented by using the bandstop filter to split the wide

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passband and result in a dual-band response. Dual-band resonators were not used in [3]. In [4], a design method of dual-band filter realized by distributed circuits was introduced. Series and parallel open stubs were used as the resonators to fulfill the dual-band characteristics and two dual-band inverters were proposed, which could be easily merged with adjacent resonators to reduce the circuit size. Based on [4], a dual-band bandpass filter using shorted stepped impedance resonators (SIRs) was developed in [5]. But both [3] and [4] did not suppress the harmonics to improve the out-of-band performance. In [6], a dual band bandpass filter using trisection SIRs was proposed. By appropriately selecting the impedance ratio and length ratio of the SIR, the dualpassband response was generated by tuning the harmonic frequencies. In [7], an open-loop resonator bandpass filter based on trisection SIR was utilized to suppress the second and third harmonics, but it is not used for dual-band bandpass filter.

This paper presents the synthesis of a novel dual-band bandpass filter with improved out-of-band performance. The proposed circuit is constructed by cascading a dual-band filter using trisection SIRs and an L-C ladder lowpass filter using open-circuited stubs. The dual-band trisection SIR can provide the desired dual-band response, and the lowpass filter can improve the out-of-band performance by suppressing the harmonics and spurious responses. The filter has been fabricated and measured.

### 2. FILTER DESIGN

### 2.1. Dual-band Generalized Trisection Stepped Impedance Resonator

Stepped-impedance resonator (SIR) is widely used as a basic element in filter design. Fig. 1 shows a short-ended trisection SIR with characteristic impedance  $Z_{31}$ ,  $Z_{32}$  and  $Z_{33}$  and the electrical lengths are  $\theta_{31}$ ,  $\theta_{32}$  and  $\theta_{33}$ , respectively. In most previous SIRs, their variables satisfy  $Z_{31} = Z_{33}$ ,  $\theta_{31} = \theta_{33}$  and  $\theta_{32} = 2\theta_{31}$ . But in this section, we assume all variables are arbitrary in our generalized trisection SIR.  $\theta_{31}$ ,



Figure 1. The structure of generalized trisection SIR.

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 $\theta_{32}$  and  $\theta_{33}$  are the electrical lengths at the center frequency  $f_1$  of the first passband. Then, the resonant condition can be expressed as

$$Y_{in} = j \frac{NY_{in}}{DY_{in}} = 0 \tag{1}$$

where

$$NY_{in} = Z_{33}Z_{32} - Z_{33}Z_{31}\tan\left(\theta_{32}\frac{f}{f_1}\right)\tan\left(\theta_{31}\frac{f}{f_1}\right) \\ -Z_{32}Z_{31}\tan\left(\theta_{33}\frac{f}{f_1}\right)\tan\left(\theta_{31}\frac{f}{f_1}\right) - Z_{32}^2\tan\left(\theta_{33}\frac{f}{f_1}\right)\tan\left(\theta_{32}\frac{f}{f_1}\right) \tag{2}$$
$$DY_{in} = Z_{33}^2Z_{31}\tan\left(\theta_{33}\frac{f}{f_1}\right)\tan\left(\theta_{32}\frac{f}{f_1}\right)\tan\left(\theta_{32}\frac{f}{f_1}\right) \\ -Z_{33}Z_{32}^2\tan\left(\theta_{32}\frac{f}{f_1}\right) - Z_{33}^2Z_{32}\tan\left(\theta_{33}\frac{f}{f_1}\right) - Z_{33}Z_{32}Z_{31}\tan\left(\theta_{31}\frac{f}{f_1}\right) \tag{3}$$

The susceptance slope parameter at resonant frequency  $f_r$  of trisection SIR can be expressed as

$$b = \frac{f_r}{2} \operatorname{Im}\left(\frac{\partial Y_{in}}{\partial f}\right)|_{f=f_r} = \frac{N_b}{D_b}$$
(4)

where

$$N_{b} = f_{r} \sec^{2} \left( \theta_{33} \frac{f_{r}}{f_{1}} \right) \left( Z_{32}^{2} Z_{33}^{2} \theta_{33} + Z_{32}^{4} \theta_{33} \tan^{2} \left( \theta_{32} \frac{f_{r}}{f_{1}} \right) \right. \\ \left. + Z_{31}^{2} Z_{32}^{2} \theta_{33} \tan^{2} \left( \theta_{31} \frac{f_{r}}{f_{1}} \right) + Z_{32}^{3} Z_{33} \theta_{32} \sec^{2} \left( \theta_{32} \frac{f_{r}}{f_{1}} \right) \right. \\ \left. + Z_{31}^{2} Z_{32} Z_{33} \theta_{32} \tan^{2} \left( \theta_{31} \frac{f_{r}}{f_{1}} \right) \sec^{2} \left( \theta_{32} \frac{f_{r}}{f_{1}} \right) \right. \\ \left. + Z_{31}^{2} Z_{32}^{2} Z_{33} \theta_{31} \sec^{2} \left( \theta_{31} \frac{f_{r}}{f_{1}} \right) \sec^{2} \left( \theta_{32} \frac{f_{r}}{f_{1}} \right) \right. \\ \left. + Z_{31}^{2} Z_{33}^{2} \theta_{33} \tan^{2} \left( \theta_{31} \frac{f_{r}}{f_{1}} \right) \tan^{2} \left( \theta_{32} \frac{f_{r}}{f_{1}} \right) \right. \\ \left. + 2 Z_{31} Z_{32}^{3} \theta_{33} \tan^{2} \left( \theta_{31} \frac{f_{r}}{f_{1}} \right) \tan^{2} \left( \theta_{32} \frac{f_{r}}{f_{1}} \right) \right.$$
 (5)

and

$$D_{b} = 2f_{1}Z_{33} \left( Z_{31}Z_{33} \tan\left(\theta_{31}\frac{f_{r}}{f_{1}}\right) \tan\left(\theta_{32}\frac{f_{r}}{f_{1}}\right) \tan\left(\theta_{33}\frac{f_{r}}{f_{1}}\right) - Z_{32}Z_{33} \tan\left(\theta_{33}\frac{f_{r}}{f_{1}}\right) - Z_{31}Z_{32} \tan\left(\theta_{31}\frac{f_{r}}{f_{1}}\right) - Z_{31}Z_{32} \tan\left(\theta_{31}\frac{f_{r}}{f_{1}}\right) \right)^{2} (6)$$

Let  $f_2$  be defined as the center frequency of the second band, and a is the frequency ratio of  $f_2$  to  $f_1$ . The slope parameters at  $f_1$ and  $f_2$  are  $b_1$  and  $b_2$ , respectively. The resonant conditions and the susceptance slope parameters at two resonant frequencies,  $f_1$  and  $f_2$ can be rewritten as

$$NY_{in}|_{\frac{f_r}{f_1}=1} = 0$$
 (7)

$$NY_{in}\big|_{\frac{f_r}{f_1}=a} = 0 \tag{8}$$

$$b_1 = \frac{N_{b1}}{D_{b1}} \Big|_{\frac{f_r}{f_1} = 1} \tag{9}$$

$$b_2 = \frac{N_{b2}}{D_{b2}}|_{\frac{f_r}{f_1} = a} \tag{10}$$

Based on the classical Chebyshev and Butterworth filter synthesis method, for the resonator at input and output, the susceptance slope parameters can be expressed as [4]

$$b_{input} = \frac{g_0 g_1}{\Delta} \tag{11}$$

$$b_{output} = \frac{g_n g_{n+1}}{\Delta} \tag{12}$$

where  $\Delta$  is the fractional bandwidths of the passband.

Thus, for the dual-bandpass resonator at input and output, the susceptance slope parameter can be written as

$$b_{input1} = \frac{g_0 g_1}{\Delta_1} \tag{13}$$

$$b_{output1} = \frac{g_n g_{n+1}}{\Delta_1} \tag{14}$$

$$b_{input2} = \frac{g_0 g_1}{\Delta_2} \tag{15}$$

$$b_{output2} = \frac{g_n g_{n+1}}{\Delta_2} \tag{16}$$

where  $\Delta_1$  and  $\Delta_2$  are the fractional bandwidths of the first and the second passband. For Butterworth and Chebyshev filter of odd-order,



Figure 2. The structure of J-Inverter.

the slope parameter at input and output are equal with  $g_0g_1 = g_ng_{n+1}$ :

$$b_1 = \frac{g_0 g_1}{\Delta_1} \tag{17}$$

$$b_2 = \frac{g_0 g_1}{\Delta_2} \tag{18}$$

There are 6 unknown variables and 4 simultaneous equations. We can first preset 2 variables as certain values of convenience at  $f_1$ . Then substituting (17)–(18) into (7)–(10), the other unknown variables can be derived by using some optimization techniques.

The dual-band admittance inverter between adjacent resonators can be realized by a transmission line with two open stubs shown in Fig. 2. Using the ABCD matrix of this structure and an ideal Jinverter, the parameters of this inverter can be solved as

$$\theta_{J1} = \theta_{J2} = \frac{n\pi}{a+1}, \quad n = 1, 2, 3...$$
(19)

$$Z_{J1} = \frac{1}{J|\sin\theta_{J1}|}$$
(20)

$$Z_{J2} = \frac{\tan^2 \theta_{J1}}{J|\sin \theta_{J1}|} \tag{21}$$

where the characteristic impedances of transmission line and open stubs are  $Z_{J1}$  and  $Z_{J2}$  and their corresponding electrical lengths are  $\theta_{J1}$  and  $\theta_{J2}$  as defined at resonant frequency  $f_1$ , respectively.

#### 2.2. Out-of-band Performance Improvement

In filter design, improving out-of-band performance is a challenging issue. In this section, we consider the method to improve the out-of band performance of dual-band bandpass filter. One useful method is to push the higher-order harmonics and spurious responses away to much higher frequencies. But fixing the  $f_1$  and  $f_2$  of dualband limits the flexibility of pushing away those undesired responses. Another method to improve out-of-band performance is to suppress the undesired harmonics and spurious responses. This is achievable by using a lowpass filter to attenuate those unwanted responses. Therefore, the proposed filter is constructed by cascading the dualband generalized trisection SIR filter and the lowpass filter. The lowpass filter which covers the first and second passbands of the generalized trisection SIR filter can improve out-of-band performance effectively.

The prototype of the lowpass filter is of L-C ladder type, and it can be implemented by open-circuited stubs [2], as shown in Fig.3. A shunt capacitor and a series inductor are realized by

$$\omega_c C = \frac{1}{Z_{oC}} \tan\left(\frac{2\pi}{\lambda_{gC}} l_c\right) \quad \text{for} \quad l < \lambda_g/4, \tag{22}$$

$$\omega_c L = Z_{oL} \sin\left(\frac{2\pi}{\lambda_{gL}} l_L\right) \tag{23}$$

The terms on the left of (22) and (23) are the susceptance of shunt capacitor and the reactance of series inductor. The terms on the right represent the input susceptance of the open stub and the input reactance of the series line, which have characteristic impedances  $Z_{oC}$ and  $Z_{oL}$ , and physical lengths  $l_C$  and  $l_L$ .  $\lambda_{gL}$  and  $\lambda_{gC}$  are guided wavelengths of high- and low- impedance line at cut-off frequency  $\omega_c$ .

In this type of filter, high-impedance lines  $(Z_{oL})$  are used for series inductors, while the open stubs are realized by low-impedance lines  $(Z_{oC})$ . From (22) and (23), the physical lengths of the high-  $(l_L)$  and



Figure 3. The prototype of proposed L-C ladder open-circuited lowpass filter.

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low- impedance  $(l_C)$  lines can be calculated by

$$l_C = \frac{\lambda_{gC}}{2\pi} \tan^{-1}(\omega_c C Z_{oC}) \tag{24}$$

$$l_L = \frac{\lambda_{gL}}{2\pi} \sin^{-1} \left( \frac{\omega_c L}{Z_{oL}} \right) \tag{25}$$

To compensate for the parasitic susceptance from the two adjacent high-impedance lines, the  $l_C$  should be adjusted to satisfy

$$\omega C = \frac{1}{Z_{oC}} \tan\left(\frac{2\pi}{\lambda_{gC}}l_C\right) + 2 * \frac{1}{Z_{oL}} \tan\left(\frac{\pi}{\lambda_{gL}}l_L\right)$$
(26)

### 3. SIMULATION AND MEASUREMENT

The proposed filter in this paper is simulated and fabricated on a substrate with relative dielectric constant  $\varepsilon_r = 2.44$  and substrate height h = 0.635 mm.

For the dual-band trisection SIR, both passband bandwidths are chosen as 1 GHz. The center frequency of the first band  $f_1$  is set as 1.2 GHz, while the second band is set as 3 GHz. Thanks to the extra parameters available, we can set  $Z_3$  and  $\theta_3$  for convenience and choose  $g_1 = 1.4142$ . By using optimization techniques, we can find out the dimensions of trisection dualband SIR. The calculated dimensions are listed in Table 1.

Table 1. Calculated dimensions of trisection SIR filter.

		W (mm)	L (mm)
$Z_{31}$ ( $\Omega$ )	69.16	1.04	8.63
$\theta_{31}$ (deg)	21.75		
$Z_{32}$ ( $\Omega$ )	17.50	F 1F	0.00
$\theta_{32}$ (deg)	7.61	(.1)	2.83
$Z_{33}$ ( $\Omega$ )	50	1 70	91 70
$\theta_{33}$ (deg)	80	1.79	31.70
$Z_{J1}(\Omega)$	63.95	1.10	25.4
$\theta_{J1}$ (deg)	51.43	1.19	25.4
$50 \Omega$	$W = 1.79 \mathrm{mm}$		

For the lowpass filter, we choose Chebyshev lowpass prototype, and passband ripple is 0.05 dB. The cut-off frequency is set as 4 GHz.

The high- and low-impedance lines for inductance and capacitance are 100  $\Omega$  ( $Z_{oL}$ ) and 25  $\Omega$  ( $Z_{oC}$ ). Based on (25)–(26), the calculated results are in Table 2.

Figure 4(a) shows the response of the calculated trisection dualband SIR. It can be seen that the center frequency of the first band is at 1.2 GHz, and the center frequency of the second band is at 3 GHz. Fig. 4(b) shows the response of the calculated lowpass filter whose cut-off frequency is at 4 GHz.

The trisection dual-band SIR and lowpass filter are cascaded in

Characteristic impedance $(\Omega)$	$Z_{oC} = 25$	Zo = 50	$Z_{oL} = 100$
Guided wavelengths (mm)	$\lambda_{gC} = 50.76$	52.54	$\lambda_{gL} = 54.80$
Microstrip line widths (mm)	$W_{C} = 4.65$	1.79	$W_{L} = 0.47$
5th-order Chebyshev lowpass T-element values (0.1 dB ripple)	$C_2 = C_4 = 1.094 \text{ pF}$		$L_1 = L_5 = 1.986 \text{ nH}$ $L_3 = 3.637 \text{ nH}$
Calculated lengths (mm)	$l_{C2} = l_{C4} = 4.87$		$l_{L1} = l_{L5} = 4.56$ $l_{L3} = 10.06$

 Table 2. Calculated dimensions of lowpass filter.



Figure 4. The responses of the trisection SIR filter and the lowpass filter.

Fig. 5. To reduce the size, the SIR has been bent. Fig. 5 also shows the finalized dimensions of the proposed filter, after tuning and optimization. Fig. 6 shows its simulation and measurement results. From Fig. 6, two passbands are seen at 1.2 and 3 GHz, and simulation and measurement results are found to be in good agreement. It is also observed that the undesired responses in out-of-band have been suppressed effectively.



Figure 5. The dimensions of proposed filter using trisection SIRs (Units: mm).



Figure 6. Simulation and measurement results of the proposed filter in Fig. 5.

# 4. CONCLUSION

This paper has presented the synthesis of a novel dual-band bandpass filter with improved out-of-band performance. The proposed circuit has been constructed by cascading a dual-band filter using generalized trisection SIRs and an L-C ladder lowpass filter using open-circuited stubs. The dual-band trisection SIR can provide the desired dualband response, and the lowpass filter can improve the out-of-band performance by suppressing the harmonics and spurious responses. The filter has been fabricated and measured. Simulation and measurement results are found to be in good agreement.

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