Progress In Electromagnetics Research, PIER 89, 199–212, 2009

CLOAK FOR BIANISOTROPIC AND MOVING MEDIA

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Abstract—The case where the background material of a cloak possesses magnetoelectric coupling is investigated in this paper, for examples, the base medium is bianisotropic or an isotropic medium moving with uniform speed which may be comparable with that of light. The specifically proposed constitutive parameters for such kind of cloak show more complicated bianisotropic property, which can not be simply produced by an isotropic medium in a uniform-velocity motion.

1. INTRODUCTION

In recent years, advancement of the invisibility cloaks has taken place in both theoretical [1–8] and experimental [9] domains, which is receiving researchers' tremendous attention all over the world. The development of such unusual devices for the manipulation of electromagnetic waves is based on coordinate transformation [2], which allows calculation of the properties of a hypothetical material [4, 5] in order to achieve a desired propagation of electromagnetic waves. For an ideal invisibility cloak all waves are guided in it around a concealed object without any reflections or scattering [3, 6]. On the other hand, the concepts of coordinate transformation can be employed in a much wider range of physical problems [7, 10–14].

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However, all researches on cloaking reported so far have focused on the background material, which is isotropic or anisotropic. Although Ref. [10] talks about general relativity in electrical engineering, the magneto-electric coupling still vanishes in its discussion on invisibility devices. Bianisotropic media provide the cross-coupling between the electric and magnetic fields, which were observed experimentally in 1960 by Astrov in antiferromagnetic chromium oxide [15]. Since then, many materials have been found to exhibit the magnetoelectric effect [16, 17], which is even more prevalent in artificial composite materials [18–23], or the so-called metamaterials [24–26].

In this paper, we explore the possibility of cloaking for bianisotropic background materials and moving media. In Section 2, all the constitutive parameters of the cloak embedded in a bianisotropic background are determined according to the coordinate transformation, including its permittivity, permeability and magneto-electric couplings. Since almost any medium, when it is in motion, is a bianisotropic medium [16], in Section 3, we deal with the case where there is a cloak fixed in a moving medium. And in Section 4, the potential applications of these cases are discussed.

2. CLOAK FOR BIANISOTROPIC MEDIA

In this section, we review the calculation process of the electromagnetic material property tensors' transformation, which has been presented in [4]; and then we apply the proposed method to an example, where the electric and magnetic fields are coupled to each other. Consider the background medium possesses magnetoelectric coupling or the environment medium moves with a speed which is not negligible compared with the light velocity, the most general form of the constitutive relations in the $\overline{EB}$ representation for a bianisotropic medium is written as:

$$\begin{bmatrix} c\overline{D} \\ \overline{H} \end{bmatrix} = \overline{\overline{C}} \cdot \begin{bmatrix} \overline{E} \\ c\overline{B} \end{bmatrix} \tag{1}$$

where

$$\overline{\overline{C}} = \begin{bmatrix} \overline{\overline{P}} & \overline{\overline{L}} \\ \overline{\overline{M}} & \overline{\overline{Q}} \end{bmatrix} \tag{2}$$

is the constitutive matrix [16]; the elements of which are constitutive parameters; and $c = 3 \times 10^8$ m/s is the velocity of light in vacuum.

In tensor notation, the constitutive relations provide a relation for the excitation tensor $G^{\alpha\beta}$ and the field tensor $F_{\alpha\beta}$, which can be written as [16, 27]:

$$G^{\alpha\beta} = \frac{1}{2} C^{\alpha\beta\mu\nu} F_{\mu\nu}, \tag{3}$$

where

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -cB_z & cB_y \\ -E_y & cB_z & 0 & -cB_x \\ -E_z & -cB_y & cB_x & 0 \end{pmatrix}, \tag{4a}$$

$$G^{\alpha\beta} = \begin{pmatrix} 0 & -cD_x & -cD_y & -cD_z \\ cD_x & 0 & -H_z & H_y \\ cD_y & H_z & 0 & -H_x \\ cD_z & -H_y & H_x & 0 \end{pmatrix}, \tag{4b}$$

and the fourth-rank tensor $C^{\alpha\beta\mu\nu}$ is the constitutive tensor, including permittivity, permeability and bianisotropic properties, which is skew-symmetric with respect to the first pair, as well as the second pair, of indices.

Suppose the space-time coordinate vector is $(x^\alpha) = (ct, x, y, z)$, the Jacobian transformation matrix between the transformed coordinate and the original coordinate is [4]

$$\Lambda_\alpha^{\alpha'} = \frac{\partial x^{\alpha'}}{\partial x^\alpha}. \tag{5}$$

As the Minkowski form of Maxwell equations is form invariant for general space-time transformations, which in tensor notation reads as [16, 27, 28]

$$F_{\alpha\beta,\mu} + F_{\beta\mu,\alpha} + F_{\mu\alpha,\beta} = 0 \tag{6a}$$

$$G^{\alpha\beta}_{,\alpha} = J^\beta \tag{6b}$$

where J^β is the source vector expressed as

$$J^\beta = \begin{pmatrix} c\rho \\ J_x \\ J_y \\ J_z \end{pmatrix}, \tag{7}$$

according to the constitutive relation in Eq. (3), the associated fourth-rank constitutive tensor of the bianisotropic cloak medium becomes [4]

$$C^{\alpha'\beta'\mu'\nu'} = \left|\det\left(\Lambda_\alpha^{\alpha'}\right)\right|^{-1} \Lambda_\alpha^{\alpha'} \Lambda_\beta^{\beta'} \Lambda_\mu^{\mu'} \Lambda_\nu^{\nu'} C^{\alpha\beta\mu\nu}, \tag{8}$$

where the Greek indices run from 0 to 3, in which 0 denotes the time component and 1, 2, 3 denote the three space components.

Since the 6×6 constitutive matrix $\overline{\overline{C}}$ shown in Eq. (2) is a faithful representation of the constitutive tensor, with the relations between the tensor elements of $C^{\alpha\beta\mu\nu}$ and the matrix elements of $\overline{\overline{C}}$ written as [16, 28]:

$$\begin{array}{rl} C^{0i0j} &= p_{ij} \\ C^{ijkl} &= \epsilon_{ijm}\epsilon_{kln}q_{mn} \\ C^{0kij} &= -\epsilon_{ijm}l_{km} \\ C^{ij0k} &= -\epsilon_{ijn}m_{nk} \end{array}, \tag{9}$$

where i, j, k, l, m, n take values from 1 to 3, and ϵ_{ijk} is a Levi-Cevita symbol which is defined in the following way:

$$\epsilon_{ijk} = \begin{cases} +1, & \text{for any even permutation} \\ -1, & \text{for any odd permutation} \\ 0, & \text{if any two indices are equal} \end{cases},$$

the constitutive matrix elements for the bianisotropic cloak can be obtained from newly transformed constitutive tensor $C^{\alpha'\beta'\mu'\nu'}$.

For the stationary ($t' = t$) cylindrical cloak transformation which radially maps points from a radius ρ to a radius ρ' and has the identity mapping for the axial transformation, we have

$$\rho' = a + \frac{b-a}{b}\rho, \quad \theta' = \theta, \quad z' = z \quad (\text{for } \rho < b). \tag{10}$$

Thus Jacobian transformation matrix is [4]

$$\left(\Lambda_j^{i'}\right) = \begin{pmatrix} \frac{\rho'}{\rho} - \frac{ax^2}{\rho^3} & -\frac{axy}{\rho^3} & 0 \\ -\frac{axy}{\rho^3} & \frac{\rho'}{\rho} - \frac{ay^2}{\rho^3} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{11}$$

which since $t' = t$, in four-dimensional space is

$$\left(\Lambda_\alpha^{\alpha'}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\rho'}{\rho} - \frac{ax^2}{\rho^3} & -\frac{axy}{\rho^3} & 0 \\ 0 & -\frac{axy}{\rho^3} & \frac{\rho'}{\rho} - \frac{ay^2}{\rho^3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

Therefore, under such a transformation, the calculated transformation results for those media do not possess magnetoelectric coupling or do not move or change shapes, reduce to the former simpler appearance [4].

Next, we will use the proposed method to calculate the transformed constitutive parameters including magnetoelectric coupling parameters in the following example, where the electric and magnetic fields are coupled to each other.

We discuss the case when the background medium is a stationary ($t' = t$) bianisotropic medium, which means in Eq. (2) $\overline{\overline{L}} \neq \overline{\overline{0}}$, $\overline{\overline{M}} \neq \overline{\overline{0}}$. For examples, the constitutive relations for the bianisotropic medium under consideration in the $\overline{EH}$ representation are in the form:

$$\begin{bmatrix} \overline{D} \\ \overline{B} \end{bmatrix} = \overline{\overline{C}}_{EH} \cdot \begin{bmatrix} \overline{E} \\ \overline{H} \end{bmatrix} \tag{13}$$

where

$$\overline{\overline{C}}_{EH} = \begin{bmatrix} \overline{\overline{\epsilon}} & \overline{\overline{\xi}} \\ \overline{\overline{\zeta}} & \overline{\overline{\mu}} \end{bmatrix} = \begin{bmatrix} \epsilon & 0 & 0 & 0 & \xi & 0 \\ 0 & \epsilon & 0 & -\xi & 0 & 0 \\ 0 & 0 & \epsilon_z & 0 & 0 & 0 \\ 0 & -\xi & 0 & \mu & 0 & 0 \\ \xi & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_z \end{bmatrix}, \tag{14}$$

which indicating a lossless but nonreciprocal medium, where ϵ, ϵ_z, ξ, μ and μ_z are all dimensionless quantities.

For the bianisotropic medium from $\overline{EH}$ into $\overline{EB}$ representation, we obtain from the transformation that:

$$\overline{\overline{P}} = c \begin{pmatrix} \epsilon - \dfrac{\xi^2}{\mu} & 0 & 0 \\ 0 & \epsilon - \dfrac{\xi^2}{\mu} & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}, \tag{15a}$$

$$\overline{\overline{L}} = \overline{\overline{M}} = \begin{pmatrix} 0 & \dfrac{\xi}{\mu} & 0 \\ -\dfrac{\xi}{\mu} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{15b}$$

$$\overline{\overline{Q}} = \frac{1}{c}\begin{pmatrix} \frac{1}{\mu} & 0 & 0 \\ 0 & \frac{1}{\mu} & 0 \\ 0 & 0 & \frac{1}{\mu_z} \end{pmatrix}. \tag{15c}$$

If the y axis is chosen to be the cylinder's axis, instead of Eq. (12), the Jacobian transformation matrix becomes

$$\left(\Lambda_{\alpha}^{\alpha'}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\rho'}{\rho} - \frac{ax^2}{\rho^3} & 0 & -\frac{axz}{\rho^3} \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{axz}{\rho^3} & 0 & \frac{\rho'}{\rho} - \frac{az^2}{\rho^3} \end{pmatrix}. \tag{16}$$

According to the method proposed, for the bianisotropic background medium, its associated constitutive matrix parameters of the cloak become:

$$\epsilon^{i'j'} = \left|\det\left(\Lambda_i^{i'}\right)\right|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \epsilon^{ij}, \tag{17a}$$

$$\mu^{i'j'} = \left|\det\left(\Lambda_i^{i'}\right)\right|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \mu^{ij}, \tag{17b}$$

$$\xi^{i'j'} = \left|\det\left(\Lambda_i^{i'}\right)\right|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \xi^{ij}, \tag{17c}$$

$$\zeta^{i'j'} = \left|\det\left(\Lambda_i^{i'}\right)\right|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \zeta^{ij}, \tag{17d}$$

which written in the constitutive matrix form in the $\overline{EH}$ representation are:

$$\overline{\overline{C'}}_{EH} = \begin{bmatrix} \epsilon'_{xx} & 0 & \epsilon'_{xz} & 0 & \xi'_z & 0 \\ 0 & \epsilon'_{yy} & 0 & -\xi'_z & 0 & \xi'_x \\ \epsilon'_{zx} & 0 & \epsilon'_{zz} & 0 & -\xi'_x & 0 \\ 0 & -\xi'_z & 0 & \mu'_{xx} & 0 & \mu'_{xz} \\ \xi'_z & 0 & -\xi'_x & 0 & \mu'_{yy} & 0 \\ 0 & \xi'_x & 0 & \mu'_{zx} & 0 & \mu'_{zz} \end{bmatrix}, \tag{18}$$

where the elements in the matrix can all be obtained through Eqs. (17a) to (17d).

Consider a slab with bianisotropic media described in Eq. (14) with a cylindrical cloak inside it, whose axis is aong y direction. Both sides of the slab are air, and the wave is incident from one side of the air with the plane of incidence parallel to the x-z plane. In this case, whether in the bianisotropic slab or in the bianisotropic cloak media, the waves can be separated individually into TE and TM waves. For TE wave, the dispersion relation for the bianisotropic medium is:

$$\frac{(\omega\xi - k_z)^2}{\mu} + \frac{k_x^2}{\mu_z} = \omega^2\epsilon, \tag{19}$$

and the relationships between E_y, H_x, and H_z components in the cloak medium are:

$$\epsilon'_{yy}E_y = \left(\xi'_z - \frac{k_z}{\omega}\right)H_x + \left(\frac{k_x}{\omega} - \xi'_x\right)H_z, \tag{20a}$$

$$\left(\xi'_z - \frac{k_z}{\omega}\right)E_y = \mu'_{xx}H_x + \mu'_{xz}H_z, \tag{20b}$$

$$\left(\frac{k_x}{\omega} - \xi'_x\right)E_y = \mu'_{zx}H_x + \mu'_{zz}H_z, \tag{20c}$$

where k_x and k_z are the wave numbers in the x and z directions.

After all the fields inside the slab are obtained as well as in the air for a TE wave, all the amplitude coefficients can be found by matching the boundary conditions for the tangential electric and magnetic fields on the slab boundaries. And the time-averaged power density for the TE wave can be expressed as

$$\left|\langle\overline{S}\rangle\right| = \frac{1}{2}\sqrt{[\mathrm{Re}(E_yH_z^*)]^2 + [\mathrm{Re}(E_yH_x^*)]^2}. \tag{21}$$

Figure 1 shows the comparison when a TE Gaussian beam impinges onto a bianisotropic slab without and with a cloak inside the slab, from which we can see that the region wrapped by the cloak shell is invisible to electromagnetic observations.

3. CLOAK FOR MOVING MEDIA

Such kind of medium as shown in Eq. (14) in previous section can be generated by a uniaxial medium moving uniformly with the velocity $\overline{v}$ along the z direction, whose constitutive relations are as follows:

$$\begin{aligned} \overline{\overline{\epsilon}} &= \begin{pmatrix} \epsilon' & 0 & 0 \\ 0 & \epsilon' & 0 \\ 0 & 0 & \epsilon'_z \end{pmatrix}, \\ \overline{\overline{\mu}} &= \begin{pmatrix} \mu' & 0 & 0 \\ 0 & \mu' & 0 \\ 0 & 0 & \mu'_z \end{pmatrix}, \end{aligned} \tag{22}$$

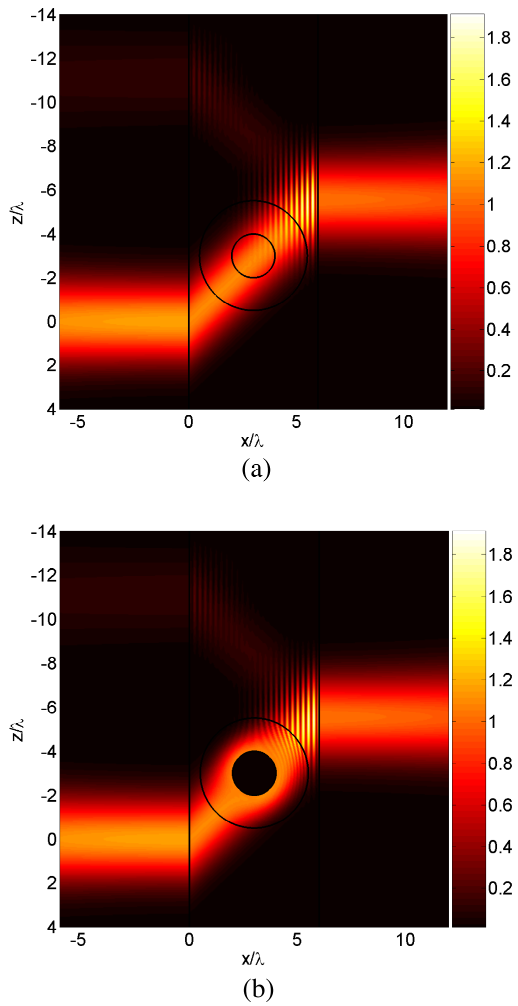

Figure 1. (Color online) Time-averaged power density $|\langle \overline{S} \rangle|$ (mW/m^2) on the x-z plane for a normally incidence of a TE Gaussian beam propagating along $+x$ direction upon a bianisotropic slab of thickness $d = 6\lambda$ with $\epsilon = -24\epsilon_0$, $\epsilon_z = \epsilon_0$, $\mu = -8\mu_0$, $\mu_z = \mu_0$, and $\xi = -15\sqrt{\epsilon_0\mu_0}$ (a) without a cloak (position of cloak is also shown for comparison). (b) with a cloak concealing its inside region.

where the z axis coincides with the optic axis [28, 29]. Thus, the constitutive parameters lying in Eq. (22) become velocity dependent and are related to the ones in Eq. (18) as [28]:

$$\epsilon = \frac{\epsilon' \left(1 - \beta^2\right)}{\left(1 - n^2\beta^2\right)}, \quad \epsilon_z = \epsilon'_z, \tag{23a}$$

$$\mu = \frac{\mu' \left(1 - \beta^2\right)}{\left(1 - n^2\beta^2\right)}, \quad \mu_z = \mu'_z, \tag{23b}$$

$$\xi = \frac{\beta \left(1 - n^2\right)}{c\left(1 - n^2\beta^2\right)}, \tag{23c}$$

where $\beta^2 = \overline{\beta} \cdot \overline{\beta}$, $\overline{\beta} = \hat{z}\beta = \overline{v}/c$, and $n^2 = c^2\mu'\epsilon'$ is the squared refractive index of the moving medium in its rest frame of reference.

Therefore, the bianisotropic slab shown in Fig. 1 could also be a uniaxial moving slab with $\epsilon' = 3\epsilon_0$, $\epsilon'_z = \epsilon_0$, $\mu = \mu_z = \mu_0$, and the velocity $\overline{v}$ directed along the $-z$ direction, whose amplitude is 0.6 times of the speed of light in vacuum. After the motion is taken into account for reconstructing the constitutive matrix, the medium appears to be immobile and bianisotropic to an observer in the laboratory frame [28], thus we can use exactly the same cloak transformation method presented in Section 2 for moving medium once its constitutive matrix has been transformed into the laboratory frame in advance.

One thing has to be noticed in our case is that the cloaked region is stationary in the laboratory frame, although its background is in motion; for example, the cloaked region shown in Fig. 1(b) does not move with the slab if the slab is moving. Therefore, in our situation, the Jacobian transformation matrix keeps unchanged from the one shown in Eq. (16) because the motion of the slab has been equivalently compensated by the bianisotropy.

On the other hand, we expect that the cloak in the moving medium, whose parameters are shown in Eqs. (17a)–(17d) with a complicated bianisotropy, would be described as a homogeneous medium moving with different velocity vectors at different parts of the cloak, which may contain velocity component other than z direction; for example, in Fig. 1(b) the cloak medium may have velocity along x direction as well as z direction. If this works, the bianisotropic cloak may also be generated by a simple medium cooperated with a complicated motion.

From the simplest one, we first assume the unknown medium be isotropic (with permittivity $\tilde{\epsilon}$ and permeability $\tilde{\mu}$ which are both scalar quantities) and in motion with uniform velocity component

$\overline{\beta}_x = \hat{x}\beta_x = \overline{v}_x/c$ and $\overline{\beta}_z = \hat{z}\beta_z = \overline{v}_z/c$ simultaneously. Using Lorentz transformation [28], we obtain its constitutive matrix in the $\overline{EH}$ representation in the laboratory frame as:

$$\overline{\overline{C}}_{EH} = \begin{bmatrix} \epsilon_{xx} & 0 & \epsilon_{xz} & 0 & \xi_z & 0 \\ 0 & \epsilon_{yy} & 0 & -\xi_z & 0 & \xi_x \\ \epsilon_{zx} & 0 & \epsilon_{zz} & 0 & -\xi_x & 0 \\ 0 & -\xi_z & 0 & \mu_{xx} & 0 & \mu_{xz} \\ \xi_z & 0 & -\xi_x & 0 & \mu_{yy} & 0 \\ 0 & \xi_x & 0 & \mu_{zx} & 0 & \mu_{zz} \end{bmatrix}, \tag{24}$$

where

$$\begin{aligned} \epsilon_{xx} &= \frac{\tilde{\epsilon}\left(1-\beta_x^2\tilde{n}^2-\beta_z^2\right)}{1-\beta^2\tilde{n}^2}, \\ \epsilon_{yy} &= \frac{\tilde{\epsilon}\left(1-\beta^2\right)}{1-\beta^2\tilde{n}^2}, \\ \epsilon_{zz} &= \frac{\tilde{\epsilon}\left(1-\beta_x^2-\beta_z^2\tilde{n}^2\right)}{1-\beta^2\tilde{n}^2}, \\ \epsilon_{xz} &= \epsilon_{zx} = \frac{\beta_x\beta_z\tilde{\epsilon}\left(1-\tilde{n}^2\right)}{1-\beta^2\tilde{n}^2}, \\ \mu_{xx} &= \frac{\tilde{\mu}\left(1-\beta_x^2\tilde{n}^2-\beta_z^2\right)}{1-\beta^2\tilde{n}^2}, \\ \mu_{yy} &= \frac{\tilde{\mu}\left(1-\beta^2\right)}{1-\beta^2\tilde{n}^2}, \\ \mu_{zz} &= \frac{\tilde{\mu}\left(1-\beta_x^2-\beta_z^2\tilde{n}^2\right)}{1-\beta^2\tilde{n}^2}, \\ \mu_{xz} &= \mu_{zx} = \frac{\beta_x\beta_z\tilde{\mu}\left(1-\tilde{n}^2\right)}{1-\beta^2\tilde{n}^2}, \\ \xi_x &= \frac{\beta_x\left(1-\tilde{n}^2\right)}{c\left(1-\beta^2\tilde{n}^2\right)}, \\ \xi_z &= \frac{\beta_z\left(1-\tilde{n}^2\right)}{c\left(1-\beta^2\tilde{n}^2\right)}, \end{aligned}$$

in which $\beta^2 = \beta_x^2 + \beta_z^2$ and $\tilde{n}^2 = c^2\tilde{\mu}\tilde{\epsilon}$. The matrix shown in Eq. (24) does have each zero element exactly corresponding to the cloak parameters in that of Eq. (18). However, when we try to seek a group of solutions ($\tilde{\epsilon}$, $\tilde{\mu}$, β_x, and β_z) to satisfy all the elements of cloak parameters, it turns out to be a failure that the cloak material can not be simply produced by an isotropic medium in a uniform-velocity motion. Because in Eq. (24), the permittivity and permeability tensor elements have relations instead of independent from each other, which the required cloak parameters can not catch up with. Although cloak parameters may also satisfy the condition that $\epsilon_{xz} = \epsilon_{zx}$ and $\mu_{xz} = \mu_{zx}$ if undergo a y-symmetrical transformation such as Eq. (16), they can never keep up with the following ones:

$$(\epsilon_{xx} - \epsilon_{yy})(\epsilon_{zz} - \epsilon_{yy}) = \epsilon_{xz}^2, \tag{25a}$$

$$(\mu_{xx} - \mu_{yy})(\mu_{zz} - \mu_{yy}) = \mu_{xz}^2, \tag{25b}$$

which are identical equations for the isotropic moving medium. Therefore, either the medium has to be upgraded or accelerated in order to seek solutions for cloak parameters, which means that much more effort is still needed to realize the material for the design of bianisotropic cloak.

4. DISCUSSION AND APPLICATIONS

The study of cloaking when the background medium is bianisotropic or moving is very useful in practical situations. For example, we may want to conceal an object which is fixed in an environment with its background medium in constantly high-speed motion, such as a satellite in the windy aerosphere or a communication center in the flowing seawater, whose moving environment is not just vacuum. As long as the squared refractive index of the background medium is not equal to unity in its rest frame of reference, it becomes bianisotropic when viewed in the laboratory frame outside the moving environment [28]. If the speed of the moving medium could not be neglected, the effect of bianisotropy of the background medium should be taken into consideration in the cloak design.

5. CONCLUSION

In this paper, we explore the problem in which the background material of the cloak possesses magnetoelectric coupling. The constitutive parameters for the bianisotropic background cloak are given and the scheme of obtaining the bianisotropic parameters can be applied to the design of a cloak fixed in a uniformly moving medium. The bianisotropic cloak parameters, which can not be simply realized by a uniformly moving isotropic medium, needs to be further searched. Potential applications are also discussed.

ACKNOWLEDGMENT

This work is sponsored by the Chinese National Science Foundation under Grant No. 60801005 and 60531020, the Zhejiang Provincial Natural Science Foundation of China under Grant No. R1080320, the ONR under Contract No. N00014-06-1-0001, and the Department of the Air Force under Air Force Contract No. F19628-00-C-0002.

REFERENCES

1. Leonhardt, U., "Optical conformal mapping," *Science*, Vol. 312, 1777, 2006.
2. Pendry, J. B., D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, Vol. 312, 1780, 2006.
3. Cummer, S. A., B.-I. Popa, D. Schurig, D. R. Smith, and J. B. Pendry, "Full-wave simulations of electromagnetic cloaking structures," *Phys. Rev. E*, Vol. 74, 036621, 2006.
4. Schurig, D., J. B. Pendry, and D. R. Smith, "Calculation of material properties and ray tracing in transformation media," *Opt. Express*, Vol. 14, 9794, 2006.
5. Zolla, F., S. Guenneau, A. Nicolet, and J. B. Pendry, "Electromagnetic analysis of cylindrical invisibility cloaks and the mirage effect," *Opt. Lett.*, Vol. 32, 1069, 2007.
6. Chen, H., B.-I. Wu, B. Zhang, and J. A. Kong, "Electromagnetic wave interactions with a metamaterial cloak," *Phys. Rev. Lett.*, Vol. 99, 063903, 2007.
7. Rahm, M., D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, "Design of electromagnetic cloaks and concentrators using form-invariant coordinate transformations of Maxwell's equations," arXiv: 0706.2452v1, 2007.
8. Xi, S., H. Chen, B.-I. Wu, B. Zhang, J. Huangfu, D. Wang, and J. A. Kong, "Effects of different transformations on the performance of cylindrical cloaks," *Journal of Electromagnetic Waves and Applications*, Vol. 22, 1489, 2008.
9. Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, Vol. 314, 977, 2006.
10. Leonhardt, U. and T. G. Philbin, "General relativity in electrical engineering," *New J. Phys.*, Vol. 8, 247, 2006.
11. Zharova, N. A., I. V. Shadrivov, and Y. S. Kivshar, "Inside-out electromagnetic cloaking," *Opt. Express*, Vol. 16, No. 7, 4615, 2008.

12. Zhang, J., Y. Luo, S. Xi, H. S. Chen, L. Ran, B.-I. Wu, and J. A. Kong, "Directive emission obtained by coordinate transformation," *Progress In Electromagnetics Research*, PIER 81, 437, 2008.
13. Duan, Z. Y., B.-I. Wu, J. A. Kong, F. M. Kong, and S. Xi, "Enhancement of radiation properties of a compact planar antenna using transformation media as substrates," *Progress In Electromagnetics Research*, PIER 83, 375, 2008.
14. Silva-Macêdo, J. A., M. A. Romero, and B.-H. V. Borges, "An extended FDTD method for the analysis of electromagnetic field rotations and cloaking devices," *Progress In Electromagnetics Research*, PIER 87, 183, 2008.
15. Astrov, D. N., "The magnetoelectric effect in antiferromagnetics," *Zh. Eksp. Teor. Fiz.*, Vol. 38, 984, 1960.
16. Kong, J. A., "Theorems of bianisotropic media," *Proceedings of the IEEE*, Vol. 60, No. 9, 1036, 1972.
17. Romeo, M., "Transient waves in dissipative magnetoelectric crystals," *Journal of Electromagnetic Waves and Applications*, Vol. 14, No. 3, 301–321, 2000.
18. Lindell, I. V. and A. J. Viitanen, "Eigenwaves in the general uniaxial bianisotropic medium with symmetric parameter dyadics," *Progress In Electromagnetics Research*, PIER 9, 1, 1994.
19. Tretyakov, S. A. and A. A. Sochave, "Novel uniaxial bianisotropic materials: Reflection and transmission in planar structures," *Progress In Electromagnetics Research*, PIER 9, 157, 1994.
20. Chung, C. Y. and K. W. Whites, "Effective constitutive parameters for an artificial uniaxial bianisotropic chiral medium," *Journal of Electromagnetic Waves and Applications*, Vol. 10, No. 10, 1363–1388, 1996.
21. He, S., "Electromagnetic surface waves for some artificial bianisotropic media," *Journal of Electromagnetic Waves and Applications*, Vol. 12, No. 4, 449–466, 1998.
22. Tretyakov, S. A., A. H. Sihvola, A. A. Sochava, and C. R. Simovski, "Magnetoelectric interactions in bi-anisotropic media," *Journal of Electromagnetic Waves and Applications*, Vol. 12, No. 4, 481–497, 1998.
23. Yin, W.-Y. and L.-W. Li, "Effects of surface helical conductances on multiple interactions of composite eccentrically bianisotropic cylinders: The case of TMz-wave incidence — Abstract," *Journal of Electromagnetic Waves and Applications*, Vol. 13, No. 8, 1101–1102, 1999.

24. Xu, W., L. W. Li, H. Y. Yao, T. S. Yeo, and Q. Wu, "Extraction of constitutive relation tensor parameters of SRR structures using transmission line theory," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 1, 13, 2006.
25. Cheng, X., H. Chen, L. Ran, B.-I. Wu, T. M. Grzegorczyk, and J. A. Kong, "Negative refraction and cross polarization effects in metamaterial realized with bianisotropic S-ring resonator," *Phys. Rev. B*, Vol. 76, 024402, 2007.
26. Cheng, X., H. Chen, T. Jiang, L. Ran, and J. A. Kong, "Free space measurement of the cross-polarized transmission band of a bianisotropic left-handed metamaterial," *Appl. Phys. Lett.*, Vol. 92, 174103, 2008.
27. Jackson, J. D., *Classical Electrodynamics*, 3rd edition, Wiley, Beijing reprint, 2004.
28. Kong, J. A., *Electromagnetic Waves Theory*, EMW, Cambridge, MA, 2005.
29. Grzegorczyk, T. M. and J. A. Kong, "Electrodynamics of moving media inducing positive and negative refraction," *Phys. Rev. B*, Vol. 74, 033102, 2006.