Progress In Electromagnetics Research Letters, Vol. 6, 157–164, 2009

COMPACT 2-D FULL-WAVE ORDER-MARCHING TIME-DOMAIN METHOD WITH A MEMORY-REDUCED TECHNIQUE

W. Shao, S.-J. Lai, and T. Z. Huang

Institute of Applied Physics
University of Electronic Science and Technology of China
Chengdu 610054, P. R. China

Abstract—This paper describes a memory-reduced (MR) compact two-dimensional (2-D) order-marching time-domain (OMTD) method for full-wave analyses. To reduce memory requirements in the OMTD method, the divergence theorem is introduced to obtain a memory-efficient matrix equation. A lossy microstrip line is presented to validate the accuracy and efficiency of our algorithm.

1. INTRODUCTION

The finite-difference time-domain (FDTD) method has been widely used for numerical analysis because of its accuracy and simplicity [1–11]. A compact two-dimensional (2-D) scheme is applied to full-wave analysis of uniform and infinitely long transmission lines to reduce memory requirement and shorten computation time [12, 13]. However, in case of fine grid division, the Courant-Friedrich-Lewy (CFL) stability condition imposes tiny time steps and results in a long solution time. To eliminate the CFL stability condition, a new order-marching time-domain (OMTD) algorithm was introduced [14]. This unconditionally stable scheme with weighted Laguerre polynomials does not have to deal with time steps and may be computationally much more efficient than the FDTD method which requires time steps to get the solution. [15, 16] combined the OMTD method with compact 2-D full-wave scheme for lossy transmission lines.

Although the OMTD method solves the temporal variables analytically, it results in an implicit relation and has to perform the matrix inversion. The memory storage requirements and computation

Corresponding author: W. Shao (shao-wei@yeah.net).

time of the OMTD method are dependent on the produced sparse matrix equation.

In this paper, a memory-reduced (MR) technique presented in [17, 18] is applied to the compact 2-D OMTD method. By substituting a Maxwell's divergence relationship for one of the curl difference equation, the memory storage of nonzero unknowns is reduced by 4/27 and 1/3 of electric field components do not need to summate from the order 0 to $m-1$.

2. MATHEMATICAL FORMULATION

The electromagnetic field components expressed in real variables for any phase constant β in z-direction satisfy [13, 19]

$$\{E_x, E_y, H_z(x, y, z, t)\} = \{e_x, e_y, h_z(x, y, t)\} \cdot j_0 e^{-j_0 \beta z} \tag{1}$$

$$\{H_x, H_y, E_z(x, y, z, t)\} = \{h_x, h_y, e_z(x, y, t)\} \cdot e^{-j_0 \beta z} \tag{2}$$

where $j_0 = \sqrt{-1}$. If the partial derivative with respect to z is replaced with $-j_0\beta$, taking e_x and h_x for example, the 3-D differential Maxwell's equations yield

$$\frac{\partial e_x}{\partial t} = \frac{1}{\varepsilon}\left[\frac{\partial h_z}{\partial y} + \beta h_y - \sigma e_x\right] \tag{3}$$

$$\frac{\partial h_x}{\partial t} = \frac{1}{\mu}\left[\beta e_y - \frac{\partial e_z}{\partial y}\right] \tag{4}$$

where ε is the electric permittivity, μ is the magnetic permeability, σ is the conductor conductivity. The other four equations can be similarly constructed.

In charge-free regions, the divergence of $\boldsymbol{D}$ can be chosen to replace (3)

$$\nabla \cdot \boldsymbol{D} = \frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} - \beta e_z = 0 \tag{5}$$

Since the Laguerre polynomials $L_n(st)$ are orthogonal with respect to the weighting function e^{-t}, an orthogonal set $\{\varphi_0, \varphi_1, \varphi_2, \ldots\}$ is chosen as the basis functions

$$\varphi_n(st) = e^{-st/2} L_n(st) \tag{6}$$

where $s > 0$ is a time scale factor. Using these entire-domain temporal basis functions, the fields can be expanded as

$$\{e_x, e_y, e_z, h_x, h_y, h_z(x, y, t)\} = \sum_{n=0}^{N_L} \left\{e_x^n, e_y^n, e_z^n, h_x^n, h_y^n, h_z^n(x, y)\right\} \varphi_n(st) \tag{7}$$

Using a Galerkin's testing procedure in time domain and central difference in space domain, and eliminating magnetic fields, with reference to [15], we get

$$e_x^m|_{i,j} - e_x^m|_{i-1,j} + \frac{\Delta x_i}{\Delta y_j} e_y^m|_{i,j} - \frac{\Delta x_i}{\Delta y_j} e_y^m|_{i,j-1} - \Delta x_i \beta e_z^m|_{i,j} = 0 \tag{8}$$

$$\begin{aligned}
&-C_x^h|_{i-1,j} e_y^m|_{i-1,j} + \left(\frac{1+2\sigma_{i,j}/(s\varepsilon_{i,j})}{C_x^e|_{i,j}} + C_x^h|_{i-1,j} + C_x^h|_{i,j} + \frac{2\beta^2 \Delta\bar{x}_i}{s\mu_{i,j}} \right) e_y^m|_{i,j} \\
&-C_x^h|_{i,j} e_y^m|_{i+1,j} + C_y^h|_{i-1,j} e_x^m|_{i-1,j} - C_y^h|_{i-1,j} e_y^m|_{i-1,j+1} - C_y^h|_{i,j} e_x^m|_{i,j} \\
&+C_y^h|_{i,j} e_x^m|_{i,j+1} + \beta \Delta\bar{x}_i C_y^h|_{i,j} e_z^m|_{i,j} - \beta \Delta\bar{x}_i C_y^h|_{i,j} e_z^m|_{i,j+1} \\
&= 2\beta \Delta\bar{x}_i \sum_{k=0}^{m-1} h_x^k|_{i,j} - \frac{2}{C_x^e|_{i,j}} \sum_{k=0}^{m-1} e_y^k|_{i,j} - 2 \sum_{k=0}^{m-1} \left(h_z^k|_{i,j} - h_z^k|_{i-1,j} \right)
\end{aligned} \tag{9}$$

$$\begin{aligned}
&-C_x^e|_{i,j} C_x^h|_{i-1,j} e_z^m|_{i-1,j} - C_y^e|_{i,j} C_y^h|_{i,j-1} e_z^m|_{i,j-1} + \Bigg[1 + \frac{2\sigma_{i,j}}{s\varepsilon_{i,j}} \\
&+C_x^e|_{i,j} C_x^h|_{i,j} + C_x^e|_{i,j} C_x^h|_{i-1,j} + C_y^e|_{i,j} C_y^h|_{i,j} + C_y^e|_{i,j} C_y^h|_{i,j-1} \Bigg] e_z^m|_{i,j} \\
&-C_x^e|_{i,j} C_x^h|_{i,j} e_z^m|_{i+1,j} - C_y^e|_{i,j} C_y^h|_{i,j} e_z^m|_{i,j+1} - \frac{2\beta}{s\mu_{i-1,j}} C_x^e|_{i,j} e_x^m|_{i-1,j} \\
&+\tfrac{2\beta}{s\mu_{i,j}} C_x^e|_{i,j} e_x^m|_{i,j} - \frac{2\beta}{s\mu_{i,j-1}} C_y^e|_{i,j} e_y^m|_{i,j-1} + \frac{2\beta}{s\mu_{i,j}} C_y^e|_{i,j} e_y^m|_{i,j} \\
&= -2C_x^e|_{i,j} \sum_{k=0}^{m-1} \left(h_y^k|_{i,j} - h_y^k|_{i-1,j} \right) + 2C_y^e|_{i,j} \sum_{k=0}^{m-1} \left(h_x^k|_{i,j} - h_x^k|_{i,j-1} \right) - 2\sum_{k=0}^{m-1} e_z^k|_{i,j}
\end{aligned} \tag{10}$$

where

$$C_x^e|_{i,j} = 2/(s\varepsilon_{i,j}\Delta\bar{x}_i) \tag{11}$$
$$C_y^e|_{i,j} = 2/(s\varepsilon_{i,j}\Delta\bar{y}_j) \tag{12}$$
$$C_x^h|_{i,j} = 2/(s\mu_{i,j}\Delta x_i) \tag{13}$$
$$C_y^h|_{i,j} = 2/(s\mu_{i,j}\Delta y_j) \tag{14}$$

where Δx_i and Δy_j are the lengths of the lattice edge where the electric fields are located; $\Delta\bar{x}_i$ and $\Delta\bar{y}_j$ are the distances between the adjacent center nodes where magnetic fields are located.

From (8), we can see that $e_x^m|_{i,j}$ has a relationship only with adjacent four electric field components, not eight in the traditional

OMTD method. Therefore, the proposed scheme results in a reduction of nonzero element storage by 4/27, and does not need to summate from order 0 to $m-1$. After obtaining $\left\{e_x^0, e_y^0, e_z^0\right\}^T$, we can solve (8)–(10) in an order-marching procedure recursively for a given β and calculate the expansion coefficients of the basis functions. Thus, we can obtain the time-domain electromagnetic fields from (6) and (7).

In the presence of conductors, the divergence of the electric flux is not zero and (5) is not satisfied any more. Different from the reduced FDTD method described in [17], which is an explicit time-domain algorithm, the MR-OMTD method presents an implicit relationship. At conductor interfaces and in conductors, therefore, we can use formulations of the traditional OMTD method described in [15].

3. NUMERICAL EXAMPLES

In this paper, the excitation source is chosen as

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{g}(\boldsymbol{r})\delta(t) \tag{15}$$

where the temporal variation $\delta(t)$ is a Dirac pulse, the spatial variation $\boldsymbol{g}(\boldsymbol{r})$ is a quasi-static finite-difference solution of transverse electric fields in a transmission line, and $\boldsymbol{r} = x\boldsymbol{a}_x + y\boldsymbol{a}_y$ is the position vector on cross section.

A microstrip line, shown in Fig. 1, is considered as a numerical example to validate the proposed method. The microstrip line has a lossless isotropic dielectric substrate with the thickness $d = 10\,\mu\text{m}$ and $\varepsilon_r = 3.3$, a gold strip with the width $W = 22\,\mu\text{m}$, the thickness $t_1 = 3\,\mu\text{m}$, $h = 80d$, $M = 30W$, the finite conductivity $\sigma = 3.9 \times 10^7\,\text{S/m}$,

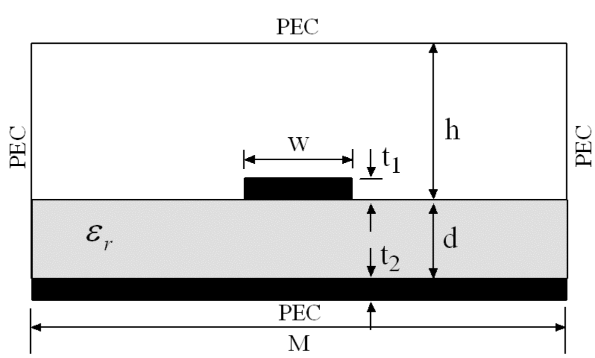

Figure 1. Cross section of a lossy microstrip line.

and a gold ground with the thickness $t_2 = 5\,\mu\text{m}$. The boundary walls are assumed to be perfect electric conductors (PECs).

In order to consider the conductor loss, the electromagnetic fields in the strip are analyzed and fine grid spacing is taken inside the strip because of the influence of the skin depth. Graded grid division is adopted, and the minimum grid spacing is 1/3 of the skin depth. With a time series voltage from the OMTD method, a corresponding attenuation constant α for a given β can be obtained [20]. Fig. 2 shows the attenuation constants of the lossy microstrip line. The results from the compact 2-D MR-OMTD method are in good agreement with the results from the compact 2-D FDTD and OMTD methods.

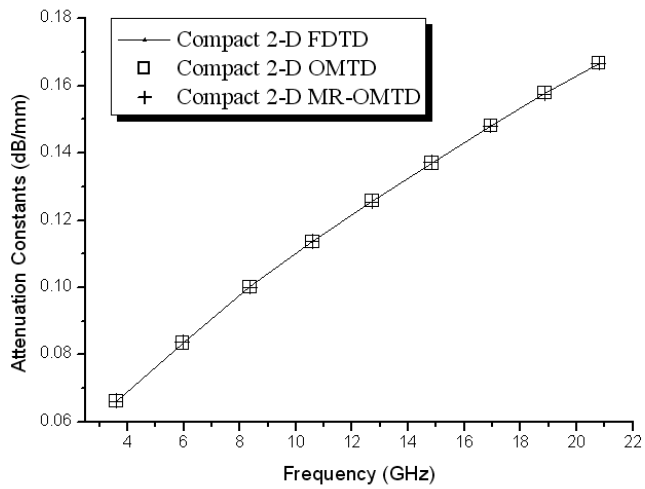

Figure 2. Attenuation constants versus frequency for the lossy microstrip line.

Table 1 shows a comparison of the computing time for the three compact 2-D methods for $\beta = 0.1482\,\text{rad/mm}$. The two OMTD methods consume much less CPU time than the FDTD method, in which the CFL stability condition imposes a tiny time step tied to the fine grid division. In addition, compared with the traditional OMTD method, the MR-OMTD method shows improvement in the computation efficiency because of its reduction in memory requirement. All calculations are performed on an Intel Core2 2.1-GHz machine.

Table 1. Comparison of the computing time for the microstrip line.

	FDTD	OMTD	MR-OMTD
CPU Time (s)	1018	92	73

4. CONCLUSIONS

This paper describes a compact 2-D full-wave MR-OMTD method to study the attenuation constants of lossy transmission lines. With the divergence theorem, the memory storage of nonzero unknowns in the matrix is reduced by 4/27 and 1/3 of electric field components do not need to summate from the order 0 to $m-1$. The formulations of the field components at conductor interfaces and in conductors are the same as the traditional OMTD method. In the numerical example, the proposed method yields results that show improvement in computation efficiency compared with the traditional 2-D OMTD method.

ACKNOWLEDGMENT

This work is supported by the National Natural Science Foundation of China (10771031), 973 programs (2008CB317110) and the Youth Technology Foundation of UESTC (JX0734).

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