# CONCURRENT NEURO-FUZZY SYSTEMS FOR RESONANT FREQUENCY COMPUTATION OF RECTANGULAR, CIRCULAR, AND TRIANGULAR MICROSTRIP ANTENNAS 

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#### Abstract

A method based on concurrent neuro-fuzzy system (CNFS) is presented to calculate simultaneously the resonant frequencies of the rectangular, circular, and triangular microstrip antennas (MSAs). The CNFS comprises an artificial neural network (ANN) and an adaptive-network-based fuzzy inference system (ANFIS). In a CNFS, neural network assists the fuzzy system continuously (or vice versa) to compute the resonant frequency. The resonant frequency results of CNFS for the rectangular, circular, and triangular MSAs are in very good agreement with the experimental results available in the literature.


## 1. INTRODUCTION

MSAs are used in a broad range of applications from communication systems to biomedical systems, primarily due to their simplicity, conformability, low manufacturing cost, light weight, low profile, reproducibility, reliability, and ease in fabrication and integration with microwave integrated circuit or monolithic microwave integrated circuit components [1-3]. Accurate determination of resonant frequency is
important in the design of MSAs because of their narrow bandwidth. Several methods [1-36] have been proposed and used to calculate the resonant frequency of the rectangular, circular, and triangular MSAs. These methods have different levels of complexity and require vastly different computational efforts. The analytical methods use simplifying physical assumptions, but generally offer simple and analytical solutions that are well suited for an understanding of the physical phenomena and for antenna computer-aided design (CAD). However, these methods are not suitable for many structures, in particular, if the thickness of the substrate is not very thin. Most of the limitations of analytical methods can be overcome by using the numerical methods. The numerical methods are mathematically complex, take tremendous computational efforts, still can not make a practical antenna design feasible within a reasonable period of time, require strong background knowledge and have time-consuming numerical calculations which need very expensive software packages. So, they are not very attractive for the interactive CAD models.

The resonant frequencies of MSAs were calculated in [37] by using a neuro-fuzzy network. In [37], the number of rules and the premise parameters of fuzzy inference system (FIS) were determined by the fuzzy subtractive clustering method and then the consequent parameters of each output rule were determined by using linear least squares estimation method. The training data sets were obtained by numerical simulations using a moment-method code based on electric field integral equation approach. To validate the performances of the neuro-fuzzy network, a set of further moment-method simulations was realized and presented to the neuro-fuzzy network.

In our previous works [38-50], the methods based on genetic algorithm (GA) [38,39], tabu search algorithm (TSA) [40, 41], ANN [42-45], and ANFIS [46-50] were used for calculating the resonant frequencies of various MSAs. It is well known that ANFIS can only produce single output. However, in [51,52], more outputs were calculated by using multiple ANFIS. In general, in the literature, each different parameter of each different MSA was computed by using a different individual ANN [53-55] or ANFIS model [56-60]. Single neural models were proposed in $[61,62]$ for simultaneously calculating the resonant frequencies of the rectangular, circular, and triangular MSAs. The results of single neural models [61,62] are not in very good agreement with the experimental results available in the literature $[4,5,8,12,13,16,19,22,23,30,31]$. For this reason, a hybrid method [63] based on a combination of ANN with ANFIS has been presented to improve the performance of single ANN models. In [63], the optimal values for the premise and
consequent parameters of ANFIS were obtained by the hybrid learning (HL) algorithm [64,65], and the ANN was trained with bayesian regularization (BR) algorithm [66]. In previous works [67-72], we successfully also used ANNs and ANFISs for computing accurately the various parameters of the transmission lines and for target tracking.

In this paper, a method based on CNFS [73,74] is presented for computing simultaneously and accurately the resonant frequencies of the rectangular, circular, and triangular MSAs. The CNFS used in this paper comprises an ANN $[75,76]$ and an ANFIS $[64,65]$. In the CNFS, the ANN assists the ANFIS continuously (or vice versa) to calculate the resonant frequency. The ANN is a computational system inspired by the structure, processing method, and learning ability of a biological brain. The ANFIS is a class of adaptive networks which are functionally equivalent to FISs. The ANN and ANFIS are very powerful approaches for building complex and nonlinear relationship between a set of input and output data. The high-speed real-time computation features of the ANN and ANFIS recommend their use in antenna CAD programs. The main advantage of the method proposed here is that the single CNFS model is used to simultaneously calculate the resonant frequencies of all three different types of MSAs including the rectangular, circular, and triangular MSAs.

In this paper, the next section briefly describes the resonant frequency computation of the MSAs and the CNFS. The application of the CNFS to the resonant frequency computation is given in the following section. The results are then presented and conclusion is made.

## 2. RESONANT FREQUENCY OF MICROSTRIP ANTENNAS (MSAs)

It is clear from the literature [1-36] that the resonant frequencies of the rectangular, circular, and triangular MSAs are determined by the substrate thickness $h$ and relative dielectric constant $\varepsilon_{r}$, the mode numbers $m$ and $n$, and the dimensions of the patch (the patch width $W$ and the patch length $L$ for the rectangular MSA, the patch radius $a$ for the circular MSA, and the side length $s$ for the triangular MSA). To compute simultaneously the resonant frequencies of the rectangular, circular, and triangular MSAs by using the CNFS model, the areas of the circular and triangular patches are equated to that of the rectangular MSA. The following formulas are then used for the equivalent dimensions of the circular and triangular patches with
reference to Figure 1

$$
\begin{array}{lll}
W=\frac{\pi a}{2} & \text { and } L=2 a & \text { for the circular MSA } \\
W=\frac{s}{2} & \text { and } L=d & \text { for the triangular MSA } \tag{2}
\end{array}
$$

where $d$ is the height of the triangular patch. It is evident from Eqns. (1) and (2) that multiplying $W$ by $L$ is equal to the area of the corresponding patch.


Figure 1. Diagram for equating the patch area of the circular and triangular MSAs with the rectangular MSA.

In the calculation of the resonant frequencies by using the single CNFS model, first the equivalent values of $W$ and $L$ for the circular and triangular MSAs should be obtained by using Eqns. (1) and (2). The resonant frequencies of the rectangular, circular, and triangular MSAs are then determined by $W, L, h, \varepsilon_{r}, m$, and $n$. The fundamental modes for the rectangular and circular MSAs are $\mathrm{TM}_{10}(m=1$ and $n=0)$ and $\mathrm{TM}_{11}(m=n=1)$, respectively. These modes are widely used in MSA applications.

## 3. CONCURRENT NEURO-FUZZY SYSTEM (CNFS)

The ANN and ANFIS can simulate and analysis the mapping relation between the input and output data through a learning algorithm. In
practice, ANNs and ANFISs can only approximate a system up to a certain degree. Therefore, it is always possible to further improve the output of ANN or ANFIS by using other appropriate tools.

There are many methods to combine FISs and ANNs in the literature $[73,74]$. These methods can be broadly classified into three categories: the cooperative neuro-fuzzy systems, the concurrent neuro-fuzzy systems (CNFSs), and the integrated (fused) neuro-fuzzy systems $[73,74]$. In this paper, a CNFS is used. The CNFS used in this paper comprises an ANN and an ANFIS. In the CNFS, ANN assists ANFIS continuously (or vice versa) to compute the resonant frequency.

In this paper, we present two CNFSs, called CNFS \# 1 and CNFS $\# 2$, to calculate the resonant frequencies of rectangular, circular, and triangular MSAs. In the CNFS \# 1, first the resonant frequencies are computed by using ANN, and then the inaccuracies in the ANN computation are corrected by the ANFIS. In the CNFS \# 2, first the resonant frequencies are computed by using ANFIS, and then the inaccuracies in the ANFIS computation are corrected by the ANN. The CNFSs \# 1 and \# 2 can be illustrated simply as shown in Figure 2.

(a) CNFS \# 1

(b) CNFS \# 2

Figure 2. CNFS models for resonant frequency calculation of rectangular, circular, and triangular MSAs.

For the ANN used in CNFS \# 1, the inputs are $W, L, h, \varepsilon_{r}, m$, and $n$, and the output is the resonant frequencies $f_{A N N}$ calculated by using ANN. For the ANFIS used in CNFS \# 1, the input is $f_{A N N}$, and the output is the resonant frequencies $f_{A N F I S}$ calculated by using

ANFIS.
For the ANFIS used in CNFS $\# 2$, the inputs are $W, L, h, \varepsilon_{r}, m$, and $n$, and the output is the resonant frequencies $f_{A N F I S}$ calculated by using ANFIS. For the ANN used in CNFS \# 2, the input is $f_{A N F I S}$, and the output is the resonant frequencies $f_{A N N}$ calculated by using ANN.

In the following sections, the ANN and the ANFIS used in CNFSs \# 1 and \# 2 are described briefly. The details on the ANN and ANFIS, and their training algorithms can be found in the previously published works of the authors [50, 77].

### 3.1. Artificial Neural Network (ANN)

An ANN is a highly simplified model of the biological structures found in a human brain $[75,76]$. In the course of developing an ANN model, the architecture of ANN and the learning algorithm are the two most important factors. ANNs have many structures and architectures [75,76]. The class of ANN and/or architecture selected for a particular model implementation depends on the problem to be solved. After several experiments using different architectures coupled with different training algorithms, in this paper, the multilayered perceptron (MLP) neural network architecture [75, 76] is used in calculating the resonant frequencies of MSAs. MLPs have a simple layer structure in which successive layers of neurons are fully interconnected, with connection weights controlling the strength of the connections. The MLP comprises an input layer, an output layer, and a number of hidden layers. MLPs can be trained using many different learning algorithms. In this paper, five different learning algorithms, BR [66], Levenberg-Marquardt (LM) [78], scaled conjugate gradient (SCG) [79], quasi-Newton (QN) [80], and conjugate gradient of Fletcher-Reeves (CGF) [81], are used to train the MLPs.

### 3.2. Adaptive-Network-Based Fuzzy Inference System (ANFIS)

The ANFIS is a class of adaptive networks which are functionally equivalent to FISs [64,65]. The FIS forms a useful computing framework based on the concepts of fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning. The selection of the FIS is the major concern in the design of an ANFIS. In this paper, the first-order Sugeno fuzzy model is used to generate fuzzy rules from a set of input-output data pairs. Among many FIS models, the Sugeno fuzzy model is the most widely applied one for its high interpretability and computational
efficiency, and built-in optimal and adaptive techniques. The grid partitioning method [65] is used for the fuzzy rule extraction.

The ANFIS architecture consists of five layers: fuzzy layer, product layer, normalized layer, de-fuzzy layer, and summation layer. In the fuzzy layer, the crisp input values are converted to the fuzzy values by the membership functions (MFs). After, in the product layer, "and" operation is performed between the fuzzy values by using production so as to determine the weighting factor of each rule. Then, the normalized weighting factors are calculated in the normalized layer. In the de-fuzzy layer, the output rules are constructed. Finally, each rule is weighted by own normalized weighting factor and the output of the ANFIS is calculated by summing of all rule outputs in the summation layer.

The main objective of the ANFIS is to optimize the parameters of the fuzzy system parameters by applying a learning algorithm using input-output data sets. The parameter optimization is done in a way such that the error measure between the target and the actual output is minimized. During the learning process of the ANFIS, the premise parameters in the fuzzy layer and the consequent parameters in the de-fuzzy layer are tuned until the desired response of the FIS is achieved. In this paper, five different optimization algorithms, leastsquares (LSQ) algorithm [82-84], nelder-mead (NM) algorithm [85, 86], GA [87, 88], HL algorithm [64, 65], and particle swarm optimization (PSO) [89, 90], are used to determine the optimum values of the fuzzy system parameters and adapt the FISs.

## 4. APPLICATION OF CNFS TO THE RESONANT FREQUENCY COMPUTATION

In this paper, the CNFSs \# 1 and \# 2 have been used to calculate simultaneously the resonant frequencies of the rectangular, circular, and triangular MSAs. In CNFSs \# 1, first, the resonant frequencies are computed by using ANN models. Then, the resonant frequencies computed by ANN models are used in training the ANFISs. The ANN and ANFIS models in CNFSs \# 1 are trained by (BR, LM, SCG, QN, and CGF) and (LSQ) algorithms, respectively.

In CNFSs \# 2, first, the resonant frequencies are computed by using ANFIS models. Then, the resonant frequencies computed by ANFIS models are used in training the ANNs. The ANFIS and ANN models in CNFSs \# 2 are trained by (LSQ, NM, GA, HL, and PSO) and (LM) algorithms, respectively.

The accuracy of a properly trained ANN and ANFIS depends on the accuracy and the effective representation of the data used
for their training. A good collection of the training data, i.e., data which is well-distributed, sufficient, and accurately simulated, is the basic requirement to obtain an accurate model. There are two types of data generators for antenna applications. These data generators are the measurement and simulation. The selection of a data generator depends on the application and the availability of the data generator. The training and test data sets used in this paper have been obtained from the previous experimental works published by 11 sources $[4,5,8,12,13,16,19,22,23,30,31]$, and are given in Tables 1, 2, and 3 for the rectangular, circular, and triangular MSAs, respectively. Total 68 data sets are listed in Tables 1-3. 54 data sets are used to train the CNFSs \# 1 and \#2, and the remaining 14 data sets, marked with an asterisk in Tables $1-3$, are used for testing. The equivalent values of $W$ and $L$ for the circular and triangular MSAs are calculated by using Eqns. (1) and (2). The input and output data sets are scaled between 0 and 1 before training.

Currently, there is no deterministic approach that can optimally determine the number of hidden layers and the number of neurons for ANNs. A common practice is to take a trial and error approach which adjusts the hidden layers to strike a balance between memorization and generalization. The training algorithms, and the number of neurons in the first and second hidden layers and training epochs for neural models used in CNFSs \# 1 and \# 2 are given in Tables 4 and 5. The tangent sigmoid function is used in the hidden layers. The linear activation function is used in the output layer. Initial weights of the neural models are set up randomly.

In the design of ANFIS, it is very important to determine the MFs. However, no common approach is available for determining these functions. A careful determination of MFs has to be performed in each problem. In some cases, they are attained subjectively as a model for human concepts. In other cases, they are based on statistical or/and empirical distributions, heuristic determination, reliability with respect to some particular problem, or theoretical demands. In this paper, MFs are selected heuristically and verified empirically. Therefore, the optimal fuzzy MF configuration which gives the best result is chosen for the resonant frequency calculation.

For the ANFIS used in CNFS \# 1, the MF for the input variable $f_{A N N}$ is the generalized bell. For the ANFIS used in CNFS $\# 2$, the MFs for the input variables $W, L, h, \varepsilon_{r}, m$, and $n$ are the gaussian, generalized bell, triangular, triangular, generalized bell, and gaussian, respectively. The training algorithms, and the number of MFs, epochs, rules, premise parameters, and consequent parameters of ANFIS used in CNFSs \# 1 and \# 2 are given in Tables 4 and 5.

Table 1. Resonant frequencies of rectangular MSAs for $\mathrm{TM}_{10}(m=1$ and $n=0$ ) mode.

| Patch <br> No | $W$ <br> $(\mathrm{~cm})$ | $L$ <br> $(\mathrm{~cm})$ | $h$ <br> $(\mathrm{~cm})$ | $\varepsilon_{r}$ | $f_{M E}$ <br> Measured <br> $(\mathrm{MHz})[30,31]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.850 | 1.290 | 0.017 | 2.22 | 7740 |
| $2^{*}$ | 0.790 | 1.185 | 0.017 | 2.22 | 8450 |
| 3 | 2.000 | 2.500 | 0.079 | 2.22 | 3970 |
| 4 | 1.063 | 1.183 | 0.079 | 2.55 | 7730 |
| 5 | 0.910 | 1.000 | 0.127 | 10.20 | 4600 |
| 6 | 1.720 | 1.860 | 0.157 | 2.33 | 5060 |
| $7^{*}$ | 1.810 | 1.960 | 0.157 | 2.33 | 4805 |
| 8 | 1.270 | 1.350 | 0.163 | 2.55 | 6560 |
| 9 | 1.500 | 1.621 | 0.163 | 2.55 | 5600 |
| $10^{*}$ | 1.337 | 1.412 | 0.200 | 2.55 | 6200 |
| 11 | 1.120 | 1.200 | 0.242 | 2.55 | 7050 |
| 12 | 1.403 | 1.485 | 0.252 | 2.55 | 5800 |
| 13 | 1.530 | 1.630 | 0.300 | 2.50 | 5270 |
| 14 | 0.905 | 1.018 | 0.300 | 2.50 | 7990 |
| 15 | 1.170 | 1.280 | 0.300 | 2.50 | 6570 |
| $16^{*}$ | 1.375 | 1.580 | 0.476 | 2.55 | 5100 |
| 17 | 0.776 | 1.080 | 0.330 | 2.55 | 8000 |
| 18 | 0.790 | 1.255 | 0.400 | 2.55 | 7134 |
| 19 | 0.987 | 1.450 | 0.450 | 2.55 | 6070 |
| $20^{*}$ | 1.000 | 1.520 | 0.476 | 2.55 | 5820 |
| 21 | 0.814 | 1.440 | 0.476 | 2.55 | 6380 |
| 22 | 0.790 | 1.620 | 0.550 | 2.55 | 5990 |
| 23 | 1.200 | 1.970 | 0.626 | 2.55 | 4660 |
| 24 | 0.783 | 2.300 | 0.854 | 2.55 | 4600 |
| $25^{*}$ | 1.256 | 2.756 | 0.952 | 2.55 | 3580 |
| 26 | 0.974 | 2.620 | 0.952 | 2.55 | 3980 |
| 27 | 1.020 | 2.640 | 0.952 | 2.55 | 3900 |
| 28 | 0.883 | 2.676 | 1.000 | 2.55 | 3980 |
| 29 | 0.777 | 2.835 | 1.100 | 2.55 | 3900 |
| 30 | 0.920 | 3.130 | 1.200 | 2.55 | 3470 |
| $31^{*}$ | 1.030 | 3.380 | 1.281 | 2.55 | 3200 |
| 32 | 1.265 | 3.500 | 1.281 | 2.55 | 2980 |
| 33 | 1.080 | 3.400 | 1.281 | 2.55 | 3150 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

*Test data sets.

Table 2. Resonant frequencies of circular MSAs for $\mathrm{TM}_{11}(m=n=1)$ mode.

| Patch | $a$ | $h$ |  | $f_{M E}$ <br> No |
| :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{cm})$ | $(\mathrm{cm})$ |  | Measured <br> $(\mathrm{MHz})$ |  |
| 1 | 6.800 | 0.08000 | 2.32 | $835^{\square}$ |
| $2^{*}$ | 6.800 | 0.15900 | 2.32 | $829^{\square}$ |
| 3 | 6.800 | 0.31800 | 2.32 | $815^{\square}$ |
| 4 | 5.000 | 0.15900 | 2.32 | $1128^{\Delta}$ |
| 5 | 3.800 | 0.15240 | 2.49 | $1443^{\nabla}$ |
| 6 | 4.850 | 0.31800 | 2.52 | $1099^{x}$ |
| $7^{*}$ | 3.493 | 0.15880 | 2.50 | $1570^{\downarrow}$ |
| 8 | 1.270 | 0.07940 | 2.59 | $4070^{\downarrow}$ |
| 9 | 3.493 | 0.31750 | 2.50 | $1510^{\downarrow}$ |
| 10 | 4.950 | 0.23500 | 4.55 | 825 |
| 11 | 3.975 | 0.23500 | 4.55 | 1030 |
| 12 | 2.990 | 0.23500 | 4.55 | 1360 |
| $13^{*}$ | 2.000 | 0.23500 | 4.55 | 2003 |
| 14 | 1.040 | 0.23500 | 4.55 | 3750 |
| 15 | 0.770 | 0.23500 | 4.55 | 4945 |
| 16 | 1.150 | 0.15875 | 2.65 | $4425 \dagger$ |
| 17 | 1.070 | 0.15875 | 2.65 | $4723 \dagger$ |
| 18 | 0.960 | 0.15875 | 2.65 | $5224 \dagger$ |
| $19^{*}$ | 0.740 | 0.15875 | 2.65 | $6634 \dagger$ |
| 20 | 0.820 | 0.15875 | 2.65 | $6074 \dagger$ |
|  |  |  |  |  |

$\square$ These frequencies measured by Dahele and Lee [12]; ${ }^{\Delta}$ this frequency measured by Dahele and Lee [13]; ${ }^{\nabla}$ this frequency measured by Carver [8]; ${ }^{x}$ this frequency measured by Antoszkiewicz and Shafai [22]; these frequencies measured by Howell [5]; †these frequencies measured by Itoh and Mittra [4]; the remainder measured by Abboud et al. [19]. *Test data sets.

Table 3. Resonant frequencies of triangular MSAs for various modes.

| Mode | $s$ <br> $(\mathrm{~cm})$ | $h$ <br> $(\mathrm{~cm})$ | $\varepsilon_{r}$ | $f_{M E}$ <br> Measured <br> $(\mathrm{MHz})$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{TM}_{10}$ | 4.1 | 0.070 | 10.50 | $1519^{+}$ |
| $\mathrm{TM}_{11}{ }^{*}$ | 4.1 | 0.070 | 10.50 | $2637^{+}$ |
| $\mathrm{TM}_{20}$ | 4.1 | 0.070 | 10.50 | $2995^{+}$ |
| $\mathrm{TM}_{21}$ | 4.1 | 0.070 | 10.50 | $3973^{+}$ |
| $\mathrm{TM}_{30}$ | 4.1 | 0.070 | 10.50 | $4439^{+}$ |
| $\mathrm{TM}_{10}$ | 8.7 | 0.078 | 2.32 | $1489^{+}$ |
| $\mathrm{TM}_{11}$ | 8.7 | 0.078 | 2.32 | $2596^{+}$ |
| $\mathrm{TM}_{20}$ | 8.7 | 0.078 | 2.32 | $2969^{+}$ |
| $\mathrm{TM}_{21}{ }^{*}$ | 8.7 | 0.078 | 2.32 | $3968^{+}$ |
| $\mathrm{TM}_{30}$ | 8.7 | 0.078 | 2.32 | $4443^{+}$ |
| $\mathrm{TM}_{10}$ | 10.0 | 0.159 | 2.32 | 1280 |
| $\mathrm{TM}_{11}$ | 10.0 | 0.159 | 2.32 | 2242 |
| $\mathrm{TM}_{20}$ | 10.0 | 0.159 | 2.32 | 2550 |
| $\mathrm{TM}_{21}$ | 10.0 | 0.159 | 2.32 | 3400 |
| $\mathrm{TM}_{30} *$ | 10.0 | 0.159 | 2.32 | 3824 |

${ }^{+}$These frequencies measured by Chen et al. [23]; the remainder measured by Dahele and Lee [16]. *Test data sets.

Table 4. Parameter values of ANN and ANFIS models used in CNFSs \# 1 for resonant frequency computation of rectangular, circular, and triangular MSAs.

| CNFS \# 1 Models | ANN Models |  |  | ANFIS Models |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Training <br> Algorithms | Number of Neurons in Hidden Layers | Number of Training Epochs | Training <br> Algorithm | Number of MFs | Number of Training Epochs | Number of Rules | Number of Premise Parameters | Number of Consequent Parameters |
| $\mathrm{ANN}_{\text {BR }}+\mathrm{ANFIS}_{\text {LSQ }}$ | BR | $6 \times 12$ | 381 | LSQ | 30 | 1770 | 30 | 90 | 60 |
| $\mathrm{ANN}_{\text {LM }}+\mathrm{ANFIS}_{\text {LSQ }}$ | LM | $6 \times 12$ | 652 | LSQ | 27 | 261 | 27 | 81 | 54 |
| $\mathrm{ANN}_{\text {SCG }}+\mathrm{ANFIS}_{\text {LSQ }}$ | SCG | $12 \times 10$ | 50000 | LSQ | 15 | 250 | 15 | 45 | 30 |
| $\mathrm{ANN}_{\mathrm{QN}}+\mathrm{ANFIS}_{\text {LSQ }}$ | QN | $3 \times 12$ | 100000 | LSQ | 17 | 35 | 17 | 51 | 34 |
| $\mathrm{ANN}_{\text {CGF }}+$ ANFIS ${ }_{\text {LSQ }}$ | CGF | $12 \times 11$ | 50000 | LSQ | 20 | 275 | 20 | 60 | 40 |

Table 5. Parameter values of ANFIS and ANN models used in CNFSs \# 2 for resonant frequency computation of rectangular, circular, and triangular MSAs.

| CNFS \# 2 Models | ANFIS Models |  |  |  |  |  | ANN Models |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Training Algorithms | Number of MFs | Number of Training Epochs | Number of Rules | Number of Premise Parameters | Number of Consequent Parameters | Training Algorithm | Number of Neurons in Hidden Layers | Number of <br> Training <br> Epochs |
| $\mathrm{ANFIS}_{\text {LSQ }}+\mathrm{ANN}_{\text {LM }}$ | LSQ | 2, 2, 3, 2, 2, 8 | 350 | 384 | 47 | 2688 | LM | 5x9 | 2267 |
| $\mathrm{ANFIS}_{\text {NM }}+\mathrm{ANN}_{\text {LM }}$ | NM | 2,2,3,2,2,8 | 500 | 384 | 47 | 2688 | LM | $6 \times 6$ | 3396 |
| $\mathrm{ANFIS}_{\mathrm{GA}}+\mathrm{ANN}_{\mathrm{LM}}$ | GA | 2, 2, 3, 2, 2, 8 | 50 | 384 | 47 | 2688 | LM | $6 \times 12$ | 4034 |
| $\mathrm{ANFIS}_{\mathrm{HL}}+\mathrm{ANN}_{\mathrm{LM}}$ | HL | 2,2,3, 2, 2, 8 | 50 | 384 | 47 | 2688 | LM | 6x6 | 10872 |
| $\mathrm{ANFIS}_{\text {PSO }}+\mathrm{ANN}_{\text {LM }}$ | PSO | 2, 2, 3, 2, 2, 8 | 10 | 384 | 47 | 2688 | LM | 9x4 | 5941 |

## 5. RESULTS AND CONCLUSIONS

The resonant frequencies computed by using CNFSs \# 1 and \# 2 for the rectangular, circular, and triangular MSAs are given in Tables 6 and 7 , respectively. For comparison, the resonant frequency results $f_{A N N}$ and $f_{A N F I S}$ obtained by using the single ANN models in CNFSs \# 1 and the single ANFIS models in CNFSs \# 2 are also given in Tables 6 and 7, respectively. The sum of the absolute errors between the theoretical and experimental results for every model is listed in Tables 6 and 7.

It is clear from Tables 6 and 7 that the results of CNFSs \# 1 and \# 2 show better agreement with the experimental results as compared to the results of the single ANN and ANFIS models. A significant improvement is obtained in the ANN and ANFIS results. The very good agreement between the measured values and our computed resonant frequency values supports the validity of CNFSs \# 1 and \# 2. When the performances of CNFS \# 1 and \# 2 models are compared with each other, the best result is obtained from the CNFS \# 1 model, which comprises an ANN trained by the BR algorithm and an ANFIS trained by the LSQ algorithm, as shown in Tables 6 and 7. It needs to be emphasized that better results may be obtained from CNFSs \# 1 and \# 2 either by choosing different training and test data sets from the ones used in the paper or by supplying more input data set values for training. Better results can also be obtained by using different CNFS \# 1 and \# 2 models for each different MSA.

The results obtained by using the single neural models [61,62] and the hybrid method [63] are given in Table 8. $f_{E D B D}, f_{D B D}, f_{B P}$, and $f_{P T S}$ in Table 8 represent, respectively, the resonant frequency values computed by using the single neural model trained with extended delta-bar-delta, delta-bar-delta, back propagation, and parallel tabu

Table 6. Comparison of measured and calculated resonant frequencies obtained by using single ANN and CNFS \# 1 models presented in this paper for rectangular, circular, and triangular MSAs.


Table 7. Comparison of measured and calculated resonant frequencies obtained by using single ANFIS and CNFS \# 2 models presented in this paper for rectangular, circular, and triangular MSAs.


Table 8. Resonant frequencies obtained from hybrid method and single neural models for rectangular, circular, and triangular MSAs.

| Types of MSAs | $f_{M E}$ <br> Measured | Hybrid Method [63] | Single Neural Models [61, 62] |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $f_{\text {EDBD }}$ [61] | $f_{\text {DBD }}$ [61] | $f_{B P}$ [61] | $f_{P T S}$ [62] |
| Rectangular | 7740 | 7743.8 | 7935.5 | 7890.1 | 7858.6 | 7847.4 |
|  | 8450 | 8455.5 | 8328.2 | 8226.0 | 8233.1 | 8148.6 |
|  | 3970 | 3971.1 | 4046.4 | 4023.0 | 4075.4 | 3971.5 |
|  | 7730 | 7726.6 | 7590.1 | 7567.3 | 7616.8 | 7881.6 |
|  | 4600 | 4598.8 | 4604.8 | 4573.9 | 4592.4 | 4603.4 |
|  | 5060 | 5057.5 | 4934.2 | 4914.0 | 4930.3 | 4969.4 |
|  | 4805 | 4842.6 | 4699.2 | 4684.8 | 4703.3 | 4879.0 |
|  | 6560 | 6559.8 | 6528.6 | 6502.8 | 6516.5 | 6635.8 |
|  | 5600 | 5594.7 | 5503.2 | 5473.3 | 5449.0 | 5516.3 |
|  | 6200 | 6181.3 | 6176.6 | 6142.6 | 6147.2 | 6205.7 |
|  | 7050 | 7048.9 | 7099.6 | 7064.3 | 7132.9 | 7113.8 |
|  | 5800 | 5800.7 | 5805.6 | 5768.8 | 5765.7 | 5794.3 |
|  | 5270 | 5277.6 | 5287.7 | 5260.3 | 5254.0 | 5313.0 |
|  | 7990 | 7991.6 | 7975.5 | 7881.8 | 8002.2 | 7776.6 |
|  | 6570 | 6570.1 | 6674.8 | 6632.8 | 6682.7 | 6481.9 |
|  | 5100 | 5097.8 | 5311.8 | 5293.2 | 5291.4 | 5191.4 |
|  | 8000 | 7998.0 | 7911.1 | 7841.6 | 7942.5 | 7893.0 |
|  | 7134 | 7134.9 | 7183.2 | 7162.1 | 7215.9 | 7267.0 |
|  | 6070 | 6072.6 | 6173.0 | 6155.1 | 6170.2 | 6030.4 |
|  | 5820 | 5863.2 | 5931.0 | 5918.0 | 5924.5 | 5780.3 |
|  | 6380 | 6380.3 | 6424.0 | 6417.5 | 6430.7 | 6500.0 |
|  | 5990 | 5990.0 | 5866.1 | 5873.9 | 5870.5 | 6004.0 |
|  | 4660 | 4659.2 | 4699.0 | 4728.3 | 4718.9 | 4562.8 |
|  | 4600 | 4606.3 | 4459.1 | 4517.1 | 4519.2 | 4591.2 |
|  | 3580 | 3600.5 | 3659.8 | 3655.7 | 3644.6 | 3685.2 |
|  | 3980 | 3972.1 | 3952.9 | 3982.6 | 3975.9 | 3948.5 |
|  | 3900 | 3907.0 | 3905.4 | 3930.0 | 3922.2 | 3891.4 |
|  | 3980 | 3984.5 | 3938.8 | 3970.7 | 3965.3 | 3969.4 |
|  | 3900 | 3894.7 | 3825.5 | 3851.1 | 3845.9 | 3893.0 |
|  | 3470 | 3472.5 | 3481.4 | 3466.2 | 3458.4 | 3456.9 |
|  | 3200 | 3194.9 | 3230.3 | 3184.7 | 3178.0 | 3167.0 |
|  | 2980 | 2979.4 | 3036.1 | 2965.6 | 2961.2 | 3035.5 |
|  | 3150 | 3148.5 | 3191.2 | 3140.4 | 3134.0 | 3135.3 |
| Errors |  | 204 | 2392 | 2427 | 2372 | 2239 |
| Circular | 835 | 834.6 | 822.9 | 793.9 | 818.4 | 848.5 |
|  | 829 | 823.3 | 820.2 | 792.4 | 817.4 | 850.4 |
|  | 815 | 815.8 | 814.5 | 789.4 | 815.5 | 857.9 |
|  | 1128 | 1128.0 | 1108.1 | 1092.1 | 1034.6 | 1102.3 |
|  | 1443 | 1444.4 | 1430.4 | 1452.1 | 1449.4 | 1437.0 |
|  | 1099 | 1097.7 | 1109.6 | 1095.0 | 1039.6 | 1128.5 |
|  | 1570 | 1570.3 | 1565.5 | 1584.6 | 1623.9 | 1566.9 |
|  | 4070 | 4070.7 | 4144.3 | 4149.5 | 4191.4 | 4092.4 |
|  | 1510 | 1510.0 | 1561.7 | 1573.2 | 1602.6 | 1506.0 |
|  | 825 | 824.8 | 882.4 | 892.2 | 889.1 | 895.6 |
|  | 1030 | 1030.5 | 1028.0 | 1064.5 | 1040.5 | 1003.5 |
|  | 1360 | 1360.3 | 1312.6 | 1352.6 | 1347.8 | 1330.5 |
|  | 2003 | 2034.8 | 1979.1 | 1960.6 | 1975.4 | 2030.8 |
|  | 3750 | 3749.5 | 3732.2 | 3716.4 | 3615.9 | 3777.1 |
|  | 4945 | 4942.9 | 4965.5 | 4934.6 | 5024.6 | 4927.9 |
|  | 4425 | 4424.2 | 4428.7 | 4424.1 | 4457.4 | 4371.7 |
|  | 4723 | 4720.4 | 4712.3 | 4700.1 | 4728.3 | 4707.2 |
|  | 5224 | 5225.3 | 5198.0 | 5166.0 | 5188.2 | 5256.3 |
|  | 6634 | 6671.8 | 6662.5 | 6584.8 | 6636.8 | 6616.4 |
|  | 6074 | 6071.5 | 6045.0 | 5983.3 | 6011.9 | 6095.7 |
| Errors |  | 91 | 462 | 727 | 922 | 508 |
| Triangular | 1519 | 1517.5 | 1527.0 | 1540.8 | 1557.6 | 1510.8 |
|  | 2637 | 2635.8 | 2623.5 | 2632.3 | 2609.5 | 2633.1 |
|  | 2995 | 2996.8 | 2983.3 | 2990.3 | 2976.9 | 2989.7 |
|  | 3973 | 3973.0 | 3992.1 | 3954.0 | 4005.6 | 3974.8 |
|  | 4439 | 4438.6 | 4424.5 | 4410.7 | 4410.8 | 4440.3 |
|  | 1489 | 1488.8 | 1503.7 | 1541.3 | 1557.4 | 1493.2 |
|  | 2596 | 2595.8 | 2600.6 | 2526.8 | 2597.7 | 2609.4 |
|  | 2969 | 2969.5 | 2986.6 | 3007.1 | 2995.8 | 2969.6 |
|  | 3968 | 3977.4 | 3945.6 | 3891.2 | 3854.8 | 3969.1 |
|  | 4443 | 4442.3 | 4440.2 | 4472.3 | 4505.0 | 4443.5 |
|  | 1280 | 1280.4 | 1257.7 | 1318.2 | 1250.8 | 1276.1 |
|  | 2242 | 2242.4 | 2224.1 | 2194.6 | 2134.9 | 2227.7 |
|  | 2550 | 2549.4 | 2500.9 | 2505.3 | 2512.2 | 2553.1 |
|  | 3400 | 3398.2 | 3416.5 | 3491.5 | 3497.7 | 3395.5 |
|  | 3824 | 3831.5 | 3861.2 | 3768.4 | 3784.5 | 3821.6 |
| Errors |  | 27 | 272 | 622 | 729 | 68 |
| Total absolute errors for the three MSAs in one model |  | 322 | 3126 | 3776 | 4023 | 2815 |

Table 9. Resonant frequencies obtained from the conventional methods for rectangular MSAs.

| $\begin{aligned} & \text { Patch } \\ & \text { No } \end{aligned}$ | $f_{M E}$ Measured | [5] | [6] | [8] | [1] | [2] | [11] | [15] | [17] | [25] | [31] | [36] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 7740 | 7804 | 7697 | 7750 | 7791 | 7635 | 7737 | 7763 | 7720 | 7717 | 412 | 7765 |
| 2 | 8450 | 8496 | 8369 | 8431 | 8478 | 8298 | 8417 | 8446 | 8396 | 8389 | 488 | 8451 |
| 3 | 3970 | 4027 | 3898 | 3949 | 3983 | 3838 | 3951 | 3950 | 3917 | 3887 | 510 | 3977 |
| 4 | 7730 | 7940 | 7442 | 7605 | 7733 | 7322 | 7763 | 7639 | 7551 | 7376 | 1610 | 7730 |
| 5 | 4600 | 4697 | 4254 | 4407 | 4641 | 4455 | 4979 | 4729 | 4614 | 4430 | 113 | 4618 |
| 6 | 5060 | 5283 | 4865 | 4989 | 5070 | 4741 | 5101 | 4958 | 4924 | 4797 | 1621 | 5077 |
| 7 | 4805 | 5014 | 4635 | 4749 | 4824 | 4520 | 4846 | 4724 | 4688 | 4573 | 1460 | 4830 |
| 8 | 6560 | 6958 | 6220 | 6421 | 6566 | 6067 | 6729 | 6382 | 6357 | 6114 | 2550 | 6563 |
| 9 | 5600 | 5795 | 5270 | 5424 | 5535 | 5158 | 5625 | 5414 | 5374 | 5194 | 1769 | 5535 |
| 10 | 6200 | 6653 | 5845 | 6053 | 6201 | 5682 | 6413 | 5987 | 5988 | 5735 | 2860 | 6193 |
| 11 | 7050 | 7828 | 6566 | 6867 | 7052 | 6320 | 7504 | 6682 | 6769 | 6433 | 4792 | 7030 |
| 12 | 5800 | 6325 | 5435 | 5653 | 5801 | 5259 | 6078 | 5552 | 5586 | 5326 | 3259 | 5787 |
| 13 | 5270 | 5820 | 4943 | 5155 | 5287 | 4762 | 5572 | 5030 | 5081 | 4842 | 3383 | 5273 |
| 14 | 7990 | 9319 | 7334 | 7813 | 7981 | 6917 | 8885 | 7339 | 7570 | 6822 | 8674 | 8101 |
| 15 | 6570 | 7412 | 6070 | 6390 | 6550 | 5794 | 7076 | 6135 | 6264 | 5951 | 5486 | 6543 |
| 16 | 5100 | 5945 | 4667 | 4993 | 5092 | 4407 | 5693 | 4678 | 4830 | 4338 | 5437 | 5193 |
| 17 | 8000 | 8698 | 6845 | 7546 | 7519 | 6464 | 8447 | 6889 | 7160 | 6367 | 8067 | 7948 |
| 18 | 7134 | 7485 | 5870 | 6601 | 6484 | 5525 | 7342 | 5904 | 6179 | 5452 | 7242 | 7169 |
| 19 | 6070 | 6478 | 5092 | 5660 | 5606 | 4803 | 6317 | 5125 | 5341 | 4735 | 6103 | 6026 |
| 20 | 5820 | 6180 | 4855 | 5423 | 5352 | 4576 | 6042 | 4886 | 5100 | 4513 | 5875 | 5817 |
| 21 | 6380 | 6523 | 5101 | 5823 | 5660 | 4784 | 6453 | 5122 | 5396 | 4729 | 6546 | 6515 |
| 22 | 5990 | 5798 | 4539 | 5264 | 5063 | 4239 | 5804 | 4550 | 4830 | 4196 | 5976 | 6064 |
| 23 | 4660 | 4768 | 3746 | 4227 | 4141 | 3526 | 4689 | 3770 | 3949 | 3479 | 4600 | 4613 |
| 24 | 4600 | 4084 | 3201 | 3824 | 3615 | 2938 | 4209 | 3168 | 3446 | 2921 | 4603 | 4550 |
| 25 | 3580 | 3408 | 2668 | 3115 | 2983 | 2485 | 3430 | 2670 | 2845 | 2461 | 3574 | 3628 |
| 26 | 3980 | 3585 | 2808 | 3335 | 3162 | 2590 | 3668 | 2790 | 3015 | 2572 | 3955 | 3956 |
| 27 | 3900 | 3558 | 2785 | 3299 | 3133 | 2573 | 3629 | 2771 | 2987 | 2555 | 3895 | 3907 |
| 28 | 3980 | 3510 | 2753 | 3294 | 3112 | 2522 | 3626 | 2721 | 2966 | 2509 | 3982 | 3922 |
| 29 | 3900 | 3313 | 2608 | 3147 | 2964 | 2364 | 3473 | 2554 | 2823 | 2356 | 3903 | 3747 |
| 30 | 3470 | 3001 | 2358 | 2838 | 2675 | 2146 | 3129 | 2317 | 2549 | 2137 | 3493 | 3381 |
| 31 | 3200 | 2779 | 2183 | 2623 | 2474 | 1992 | 2889 | 2151 | 2357 | 1983 | 3197 | 3123 |
| 32 | 2980 | 2684 | 2102 | 2502 | 2370 | 1936 | 2752 | 2086 | 2259 | 1924 | 2982 | 2972 |
| 33 | 3150 | 2763 | 2168 | 2600 | 2453 | 1982 | 2863 | 2139 | 2338 | 1972 | 3160 | 3096 |
| Errors |  | 13136 | 24097 | 11539 | 12322 | 30669 | 8468 | 22572 | 18148 | 30504 | 56698 | 1393 |

Resonant frequencies and errors are in MHz .
search algorithms. In the hybrid method, the resonant frequencies were obtained by using ANFIS after ANN. The ANN and ANFIS were trained by BR and HL algorithms, respectively. Therefore, the hybrid method can also be called as a CNFS. It is apparent from Tables 6 and 8 that the results of all single ANN models in CNFSs \# 1 are better than those predicted by the single neural models in [61, 62]. When the performances of the single ANN models in CNFSs \# 1 and the single ANFIS models in CNFSs \# 2 are compared with each other, the best result is obtained from the single ANFIS model trained by the LSQ algorithm, as shown in Tables 6 and 7. The LSQ is a very powerful algorithm that allows us to design highly accurate and efficient ANFIS. Better results can also be obtained by using a different individual ANN

Table 10. Resonant frequencies obtained from the conventional methods and the methods based on GA and TSA for circular MSAs.

| Patch <br> No | $f_{M E}$ <br> Measured | $[8]$ | $[5]$ | $[9]$ | $[19]$ | $[18]$ | $[24]$ | $[28]$ | $[29]$ | $[41]$ | $[39]$ | $[32]$ | $[33]$ |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 835 | 845 | 849 | 840 | 842 | 844 | 838 | 841 | 840 | 843 | 840 | 842 | 839 |
| 2 | 829 | 842 | 849 | 833 | 837 | 839 | 831 | 836 | 832 | 838 | 831 | 837 | 833 |
| 3 | 815 | 834 | 849 | 821 | 826 | 829 | 819 | 826 | 818 | 828 | 815 | 827 | 824 |
| 4 | 1128 | 1141 | 1154 | 1127 | 1133 | 1136 | 1124 | 1132 | 1125 | 1135 | 1123 | 1133 | 1129 |
| 5 | 1443 | 1445 | 1466 | 1427 | 1436 | 1439 | 1423 | 1435 | 1423 | 1438 | 1432 | 1436 | 1431 |
| 6 | 1099 | 1115 | 1142 | 1098 | 1105 | 1109 | 1095 | 1105 | 1091 | 1107 | 1100 | 1107 | 1103 |
| 7 | 1570 | 1565 | 1580 | 1545 | 1555 | 1559 | 1541 | 1554 | 1539 | 1558 | 1550 | 1556 | 1550 |
| 8 | 4070 | 4203 | 4290 | 4145 | 4175 | 4187 | 4134 | 4173 | 4120 | 4183 | 4168 | 4179 | 4163 |
| 9 | 1510 | 1539 | 1580 | 1513 | 1522 | 1529 | 1509 | 1523 | 1498 | 1524 | 1510 | 1525 | 1520 |
| 10 | 825 | 818 | 833 | 818 | 827 | 827 | 816 | 825 | 817 | 824 | 823 | 827 | 823 |
| 11 | 1030 | 1014 | 1037 | 1016 | 1027 | 1027 | 1013 | 1026 | 1013 | 1024 | 1022 | 1028 | 1023 |
| 12 | 1360 | 1339 | 1379 | 1344 | 1358 | 1360 | 1340 | 1359 | 1336 | 1355 | 1352 | 1361 | 1355 |
| 13 | 2003 | 1972 | 2061 | 1990 | 2009 | 2012 | 1984 | 2012 | 1966 | 2007 | 2002 | 2015 | 2009 |
| 14 | 3750 | 3627 | 3963 | 3749 | 3744 | 3737 | 3739 | 3752 | 3634 | 3750 | 3750 | 3750 | 3751 |
| 15 | 4945 | 4722 | 5353 | 5001 | 4938 | 4922 | 4987 | 4943 | 4817 | 4948 | 4945 | 4932 | 4944 |
| 16 | 4425 | 4461 | 4695 | 4399 | 4413 | 4437 | 4388 | 4422 | 4328 | 4422 | 4413 | 4423 | 4415 |
| 17 | 4723 | 4776 | 5046 | 4712 | 4723 | 4749 | 4699 | 4731 | 4630 | 4730 | 4722 | 4733 | 4725 |
| 18 | 5224 | 5289 | 5625 | 5223 | 5226 | 5257 | 5209 | 5237 | 5121 | 5231 | 5224 | 5237 | 5231 |
| 19 | 6634 | 6733 | 7297 | 6679 | 6644 | 6684 | 6661 | 6658 | 6499 | 6634 | 6636 | 6652 | 6651 |
| 20 | 6074 | 6125 | 6585 | 6063 | 6047 | 6084 | 6046 | 6061 | 5920 | 6046 | 6043 | 6057 | 6054 |
| Errors |  |  |  |  |  | 965 | 3341 | 337 | 253 | 383 | 380 | 253 | 1047 |
|  | 253 | 207 | 275 | 235 |  |  |  |  |  |  |  |  |  |

Resonant frequencies and errors are in MHz.

Table 11. Resonant frequencies obtained from the conventional methods and the methods based on GA and TSA for triangular MSAs.

| Mode | $f_{M E}$ <br> Measured | $[1]$ | $[7]$ | $[20]$ | $[21]$ | $[23]$ | $[26]$ | $[27]$ | $[38]$ | $[40]$ | $[34]$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{TM}_{10}$ | 1519 | 1725 | 1498 | 1494 | 1577 | 1509 | 1511 | 1541 | 1501 | 1501 | 1505 |
| $\mathrm{TM}_{11}$ | 2637 | 2988 | 2594 | 2588 | 2731 | 2614 | 2617 | 2669 | 2601 | 2600 | 2629 |
| $\mathrm{TM}_{20}$ | 2995 | 3450 | 2995 | 2989 | 3153 | 3018 | 3021 | 3082 | 3003 | 3002 | 3013 |
| $\mathrm{TM}_{21}$ | 3973 | 4564 | 3962 | 3954 | 4172 | 3993 | 3997 | 4077 | 3972 | 3971 | 3985 |
| $\mathrm{TM}_{30}$ | 4439 | 5175 | 4493 | 4483 | 4730 | 4528 | 4532 | 4623 | 4504 | 4503 | 4519 |
| $\mathrm{TM}_{10}$ | 1489 | 1627 | 1500 | 1480 | 1532 | 1498 | 1486 | 1481 | 1489 | 1488 | 1485 |
| $\mathrm{TM}_{11}$ | 2596 | 2818 | 2599 | 2564 | 2654 | 2595 | 2573 | 2565 | 2579 | 2577 | 2580 |
| $\mathrm{TM}_{20}$ | 2969 | 3254 | 3001 | 2961 | 3065 | 2996 | 2971 | 2962 | 2978 | 2976 | 2969 |
| $\mathrm{TM}_{21}$ | 3968 | 4304 | 3970 | 3917 | 4054 | 3963 | 3931 | 3918 | 3940 | 3937 | 3940 |
| $\mathrm{TM}_{30}$ | 4443 | 4880 | 4501 | 4441 | 4597 | 4494 | 4457 | 4443 | 4468 | 4464 | 4456 |
| $\mathrm{TM}_{10}$ | 1280 | 1413 | 1299 | 1273 | 1340 | 1296 | 1280 | 1280 | 1281 | 1281 | 1277 |
| $\mathrm{TM}_{11}$ | 2242 | 2447 | 2251 | 2206 | 2320 | 2244 | 2217 | 2218 | 2219 | 2218 | 2224 |
| $\mathrm{TM}_{20}$ | 2550 | 2826 | 2599 | 2547 | 2679 | 2591 | 2560 | 2561 | 2562 | 2562 | 2555 |
| $\mathrm{TM}_{21}$ | 3400 | 3738 | 3438 | 3369 | 3544 | 3428 | 3387 | 3387 | 3389 | 3389 | 3397 |
| $\mathrm{TM}_{30}$ | 3824 | 4239 | 3898 | 3820 | 4019 | 3887 | 3840 | 3841 | 3843 | 3842 | 3835 |
| Errors |  | 5124 | 424 | 326 | 1843 | 408 | 314 | 590 | 273 | 273 | 233 |

Resonant frequencies and errors are in MHz .
or ANFIS model for each different MSA.
In order to make a further comparison, the resonant frequency results of conventional methods [1, 2, 5-9, 11, 15, 17-21, 23-29, 31$34,36]$ and the methods based on GA $[38,39]$ and TSA [40,41] for the rectangular, circular, and triangular MSAs are given in Tables $9-$ 11. The sum of the absolute errors between the experimental results and the theoretical results in Tables $9-11$ for every method is also given in Tables $9-11$. It can be clearly seen from Tables $9-11$ that the conventional methods and the methods based on GA and TSA give comparable results. Some cases are in good agreement with measurements, and others are far off. It should be noted that the conventional methods and the methods based on GA and TSA were used to compute the resonant frequencies of each different MSA. However, the CNFSs \# 1 and \# 2 presented in this paper are valid for the resonant frequency computation of all three different types of MSAs including the rectangular, circular, and triangular MSAs.

As a result, a method based on CNFS is used to accurately and simultaneously compute the resonant frequencies of the rectangular, circular, and triangular MSAs. The CNFS comprises an ANN and an ANFIS. The ANNs are trained with BR, LM, SCG, QN, and CGF algorithms. The LSQ, NM, GA, HL, and PSO are used to identify the parameters of ANFIS. In order to verify the validity and accuracy of the CNFS models for resonant frequency computation, comprehensive comparisons are made. The results of CNFS models are in very good agreement with the measurements. A significant improvement is obtained in the ANN and ANFIS results. The proposed method is not limited to the calculation of the resonant frequency of MSAs. This method can easily be applied to other antenna and microwave circuit problems. Accurate, fast, and reliable CNFS models can be developed from measured/simulated antenna data. Once developed, these CNFS models can be used in place of computationally intensive numerical models to speed up antenna design. We expect that the CNFS will find wide applications in solving antenna and microwave integrated circuit problems.

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