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TEMPORAL SOLITONS OF MODIFIED COMPLEX GINZBURG LANDAU EQUATION

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Abstract—In this paper we have reported soliton solution of one dimensional modified complex Ginzburg Landau equation. The parametric region where such soliton solution is possible is also identified.

1. INTRODUCTION

Investigations on nonlinear wave equations have been a fascinating topic both for mathematicians and physicists because of their numerous application potential in diverse areas [1–15, 29–33]. In particular, the nonlinear Schrödinger equation (NLSE) and the complex Ginzburg Landau equation (CGLE) and their modified versions have drawn tremendous attention during last three decades [2–8]. These equations describe a variety of physical phenomena in plasmas, optical waveguides and fibers, Bose Einstein condensation, phase transitions, bimolecule dynamics, open flow motions, spatially extended nonequilibrium systems etc [1]. Large volume of analytical investigations have been carried out to find the soliton solutions of these equations which are localized waves that show particle like behaviour i.e., their forms are preserved in space or in time or both in space and time resulting in respectively spatial, temporal or spatiotemporal solitons. Finding soliton solutions of nonlinear wave equations representing various physical phenomena have been the major thrust of theoretical research [2–10]. Nonlinear equations representing Hamiltonian systems are not always integrable. Though the standard NLSE is exactly integrable by the method of inverse scattering transform (IST) yielding soliton solution in compact form, higher order nonlinear Schrödinger equations (HONLSE) are in general not integrable. The non Hamiltonian systems or the dissipative systems are more complicated as these systems are free to exchange energy with external sources. Soliton solutions for such systems can not be found out in a closed form. Therefore, both for nonintegrable Hamiltonian systems and dissipative systems, various methods have been developed to get approximate solutions. These include the Painleve analysis, Hirota bilinear method, Ablowitz-Kaup-Newell-Segur (AKNS) technique, Darboux-Backlund transform, collective variable (CV) approach, Lagrangian variational method etc. [9–20].

Recently, based on variational iteration, a very simple yet effective method was proposed by He [21] which has the capacity to solve a large class of nonlinear problems. This method has been successfully employed for the determination of limit cycles in self excited systems modeled by van der Pol oscillator [22] and for finding the solitary wave solution of the Zakharov equation [15]. The one dimensional (1D) CGLE [23] is a generic equation which describes dissipative systems near a subcritical bifurcation to traveling waves. The 1D CGLE possesses a rich variety of solutions such as pulses (solitary waves), breathing solitons, pulsating, erupting and creeping solitons, multisolitons, fronts (shock waves), sinks (propagating hole with negative asymptotic group velocity), sources (propagating hole with positive asymptotic group velocity), periodic and quasi periodic solutions, periodic unbounded solutions [24]. For some dissipative systems, 1D CGLE needs to be modified to include nonlinear gradient terms resulting in 1D modified CGLE (1DMCGLE) [25]. For example, to observe the influences of vacuum dissipation effects on the collective motion on top of a super fluid covariant nondissipative chaotic background, the 1D MCGLE needs to be solved. The complex field $\Psi(z,\, t)$ of a 1D MCGLE is represented by

$$i\frac{\partial\Psi}{\partial t}+p\frac{\partial^2\Psi}{\partial x^2}+q\left|\Psi\right|^2\Psi=c\frac{\frac{\partial\Psi}{\partial x}\frac{\partial\Psi^*}{\partial x}}{\Psi^*}+d\,\nabla^2\left(\sqrt{\Psi\Psi^*}\right)\sqrt{\Psi/\Psi^*}+i\gamma\Psi \tag{1}$$

where the operator $\nabla^2 \equiv \frac{\partial^2}{\partial x^2}$ and the system parameters p, q, c, d and γ may be real, complex or a combination of the two. Different combinations of the above system parameters describe different types of wave propagation in different physical systems. In systems modeled by complex values of p, q, c, d and γ, Yomba and Kofane [26], using a combination of Painleve analysis and Hirota bilinear technique have found pulses, fronts, periodic unbounded waves, sources and sink solutions. Mohamadau et al. [27] have studied the modulation instabilities in the 1DMCGLE and reported the existence of several special soliton solutions with complex p, q, c, d and γ. They obtained explicit expressions for fixed amplitude, arbitrary amplitude and chirp free solutions. Recently, using the method of paraxial ray approximation, Hong [28] has reported the existence of a family of stationary solitons in a system modeled by real values of p, q, c, d and purely imaginary γ. These solitons have been found to be robust against small perturbations in positive dispersion, positive nonlinearity and negative dispersion, negative nonlinearity regimes. The purpose of the present work is to obtain the soliton solution of the 1D MCGLE employing the variational iteration method for the system represented by real values of p, q, c, d and γ.

2. MATHEMATICAL METHOD

The one dimensional MCGLE can be recasted in the following form

$$\begin{aligned} i\frac{\partial\Psi}{\partial t}+p\frac{\partial^2\Psi}{\partial x^2}+q\left|\Psi\right|^2\Psi = c\frac{\frac{\partial\Psi}{\partial x}\frac{\partial\Psi^*}{\partial x}}{\Psi^*}+d\,&\left[\frac{1}{2}\frac{\partial^2\left(\Psi\Psi^*\right)}{\partial x^2}\Psi\Psi^*\right.\\ &\left.-\frac{1}{4}\left\{\frac{\partial\left(\Psi\Psi^*\right)}{\partial x}\right\}^2\right]\frac{1}{\Psi\Psi^{*2}}+i\gamma\Psi \end{aligned} \tag{2}$$

We look for the solitary wave solution of the above equation in the form

$$\Psi\left(x,\,t\right)=f\left(\xi\right)\,e^{i(mx-nt)},\quad \xi=x-Ut \tag{3}$$

where $f\{\xi\}$ is a real function, U represents the wave speed, m and n are constants. We substitute (3) into (2) to get following complex equation for f,

$$\begin{aligned} &-iU\frac{\partial f}{\partial\xi}+nf+p\left(\frac{\partial^2 f}{\partial\xi^2}+2im\frac{\partial f}{\partial\xi}-m^2f\right)+qf^3\\ &=c\frac{\left(\frac{\partial f}{\partial\xi}\right)^2+m^2f^2}{f}+d\frac{\partial^2 f}{\partial\xi^2}+i\gamma\,f \end{aligned} \tag{4}$$

The real and imaginary parts of Equation (4) are separated which yield the following equations.

$$nf+p\left(\frac{\partial^2 f}{\partial\xi^2}-m^2f\right)+q\,f^3=c\frac{\left(\frac{\partial f}{\partial\xi}\right)^2+m^2f^2}{f}+d\frac{\partial^2 f}{\partial\xi^2} \tag{5a}$$

and

$$(2pm-U)\,\frac{\partial f}{\partial \xi}=\gamma\, f \tag{5b}$$

Equation (5a), can be integrated once to get

$$\frac{(p-d)}{2}\left(\frac{\partial f}{\partial \xi}\right)^2+\frac{1}{2}\left\{n-m^2\,(p+c)-\frac{c\gamma^2}{(2pm-U)^2}\right\}f^2+\frac{q}{4}f^4=K, \tag{6}$$

where K is a constant. The constancy of Equation (6) is utilized in constructing a stationary integral J as follows

$$J=\int\limits_0^\infty\left[\frac{(p-d)}{2}\left(\frac{\partial f}{\partial \xi}\right)^2+\frac{1}{2}\left\{n-m^2\,(p+c)-\frac{c\gamma^2}{(2pm-U)^2}\right\}f^2+\frac{q}{4}f^4\right]d\xi. \tag{7}$$

The shape function of the solitary wave $f\,(\xi)$ is assumed to be of the form

$$f\,(\xi)=A\,Sech\,(B\xi)\,, \tag{8}$$

where A and B are constants. Substitution of solution (8) in Equation (7) yields the following expression

$$J=\frac{(p-d)\,A^2B}{6}+\frac{\left\{n-m^2\,(p+c)-\dfrac{c\gamma^2}{(2pm-U)^2}\right\}A^2}{2B}+\frac{qA^4}{6B}. \tag{9}$$

The value of the unknown constants A and B can be easily obtained by imposing the stationary condition on J with respect to A and B, thus requiring $\frac{\partial J}{\partial A}=0$ and $\frac{\partial J}{\partial B}=0$, resulting in, after some simple algebra, a set of simultaneous equations of A and B as follows

$$(p-d)\,B^2+3\left\{n-m^2\,(p+c)-\frac{c\gamma^2}{(2pm-U)^2}\right\}+2qA^2\;=\;0. \tag{10}$$

$$(p-d)\,B^2-3\left\{n-m^2\,(p+c)-\frac{c\gamma^2}{(2pm-U)^2}\right\}-qA^2\;=\;0. \tag{11}$$

These equations can easily be solved to obtain A and B as follows

$$A=\sqrt{\frac{2}{q}\left\{\frac{c\gamma^2}{(2pm-U)^2}+m^2\,(p+c)-n\right\}}, \tag{12a}$$

and

$$B = \sqrt{\frac{\left\{n - m^2\left(p+c\right) - \dfrac{c\gamma^2}{\left(2pm-U\right)^2}\right\}}{p-d}}. \tag{12b}$$

Thus, the solitary wave solution of the 1D MCGLE turns out to be

$$\Psi\left(x,\,t\right) = \sqrt{\frac{2}{q}\left\{\frac{c\gamma^2}{\left(2pm-U\right)^2} + m^2\left(p+c\right) - n\right\}}\, Sech\left\{\sqrt{\frac{\left\{n - m^2\left(p+c\right) - \dfrac{c\gamma^2}{\left(2pm-U\right)^2}\right\}}{p-d}}\cdot\xi\right\}\cdot e^{i(mx-nt)},$$

and the above solution is obtainable only if

$$\sqrt{\frac{\left\{n - m^2\left(p+c\right) - \dfrac{c\gamma^2}{\left(2pm-U\right)^2}\right\}}{p-d}} > 0$$

3. CONCLUSION

In conclusion, following the variational iteration approach, we have found soliton solution of one dimensional modified complex Ginzburg Landau equation. The parametric regime where such solution is possible has been also identified.

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