# SCATTERING FIELD FOR THE ELLIPSOIDAL TARGETS IRRADIATED BY AN ELECTROMAGNETIC WAVE WITH ARBITRARY POLARIZING AND PROPAGATING DIRECTION 

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#### Abstract

Obtaining scattering field for an ellipsoid irradiated by electromagnetic wave with arbitrary polarizing and propagating direction is a hard topic that has caused large attention in the world. Literatures relative to it are seldom found. In this paper, the scattering field for an ellipsoid is presented by utilizing the scales transformation of electromagnetic field and coordinate system rotation, as the incident wave irradiating the target with arbitrary polarizing and propagating direction. The result obtained is in good agreement with that in the reference when all the scale factors changes into 1 . We take a conductor ellipsoid as an example, simulations both for ellipsoid and plant leaf are presented respectively by way of choosing the different


scale factor. Results show that the scattering field is sensitively affected by polarization of the incident wave and varies not too greatly with the incident wave and changes with the observing point. At some points the scattering energy arrives to its maximum.

## 1. INTRODUCTION

In recent years, the scholars and engineers in the world pay their large attention to investigation of combination electromagnetic scattering both for leaves of vegetation and chaffs etc. [1-6]. In literature $[7,8]$, the deciduous leaves are modeled as disks and needles to study electromagnetic wave from the randomly oriented dielectric scatters utilizing the Helmholtz integrated equation and digital arithmetic. In literatures [9] and [10], the echo power of electromagnetic wave from the finite long fiberglass coated with aluminum is investigated by regarding it as a scattering model for chaffs. In reality, the more exact model for pant leaves should be the ellipse disk and the better model for chaffs should be the ellipse cylinder. In rectangular coordinate system, when the three semi axes $a, b$ and $c$ for an elliptical sphere are given with different sizes, the elliptical sphere is different in size and shape respectively. For example, when $a \gg b \approx c$, the elliptical sphere is approximately the leaf or a finite long cylinder both locate along $x$-axis. Thus investigation the scattering wave from an elliptical sphere both has theoretical and engineering value, as the incident wave propagating and polarizing in arbitrary. Literatures [11-13] present the solution of scattering field for a sphere when the incident wave propagating in $z$-axis and polarizing along $x$-axis. In [14], the scattering field from a sphere is presented when the target irradiated by electromagnetic wave with arbitrary propagating direction and polarization. In $[15,16]$, the scales transformation of electromagnetic theory and curve-face coordinate system are used respectively to investigate the scattering characteristics of an elliptical sphere on specific terms of electromagnetic wave propagating in $z$-axis and polarized in $x$-axis. The scattering characteristic for other target made-up by conductor and media is researched [20-24] with digital algorithm and Integral Equation method.

This paper is organized as follows. The solution of scattering field from a elliptical target is first presented assuming that the incident wave propagates and polarization in arbitrary. Then the agreement between this obtained solution and that from a sphere in reference is demonstrated. Finally we take the conductor elliptical sphere as an example, partial simulations based on the obtained results are
presented. The scattering field versus the ellipse sphere size, azimuth angle and operating frequency etc. is discussed. The method used in this paper is valid to investigate the coated elliptical sphere and has the characteristic of briefness in calculation and distinctness in physics significance.

## 2. SCATTERING FOR ELLIPSOID IRRADIATED BY AN ELECTROMAGNETIC WAVE WITH ARBITRARY POLARIZING AND PROPAGATING DIRECTION

### 2.1. The Procedure of Solving the Problem

It is assumed that there is an ellipsoid locating in the origin of the rectangular system $\sum$ as illustrated in Fig. 1. We use $a, b$ and $c$ respectively to denote its three semi axes. This ellipsoid can be changed into a sphere and the incident wave also be changed into a another one $[17,18]$ in a new coordinates $\sum_{m}$ by utilizing the scales transformation, measurement criterion changes with the direction. By using the literatures $[14,18]$, we can obtain the solution of scattering field from the sphere and then transform this solution into the initial system $\sum$, the solution of scattering field from an ellipsoid is obtained as the incident wave freewill irradiating the target. The concrete process of obtaining this problem is demonstrated as Fig. 1 to Fig. 4. In coordinates $\sum$, there is a elliptical target with the semi axes $a, b, c$. There are angle $\theta_{0}$ between the incident wave direction and $z$-axis, angle $\phi_{0}$ between the polarizing direction and $x$ axis. This two parameters can entirely determine the incident wave propagation [14] in coordinates $\sum$. After scales transformation, the target in coordinates $\sum_{m}$ be changed into a sphere with radius of 1 m , the original parameters $\theta_{0}, \phi_{0}$ changed into $\theta_{0 m}$ and $\phi_{0 m}$ as shown in


Figure 1. Incident wave irradiat- Figure 2. Incident wave irradiating on an ellipse target. ing on spherical target.


Figure 3. Coordinates $\Sigma_{m}$ and Figure 4. Incident wave irradiatcoordinates $\Sigma^{\prime}$. ing on a sphere.

Fig. 2. All the physics qualities in $\sum_{m}$ are symbolized with subscript ' $m$ '. We get another coordinate system $\sum$ ' by first rotating $x_{m}$ an angle $\phi_{0 m}$ around $z_{m}$ axis and then rotating $z_{m}$ an angle $\theta_{0 m}$ around $x_{m}$ axis as shown in Fig. 3 and Fig. 4. Now the incident wave propagates along $z^{\prime}$-axis and polarizes along $x^{\prime}$-axis which irradiating on a sphere. All the physics qualities in $\sum^{\prime}$ are symbolized with superscript "'". Finally the solution of scattering field from the ellipse target is obtained after a series of athwart transformation.

### 2.2. The Scattering Field from an Elliptical Sphere

Let a plane electromagnetic wave in coordinates $\sum$ being the form

$$
\begin{equation*}
E=E_{0} e^{-j k \cdot r} \tag{1}
\end{equation*}
$$

Its propagating direction goes away from $z$-axis an angle $\theta_{0}$ and its polarization goes away from $x$-axis an angle $\phi_{0}$. We know from literatures $[17,18]$ that the measurement standard for length is changed as the scale factor $a, b, c$ are introduced. The incident wave in coordinates $\sum_{m}$ has been changed into the following

$$
\begin{equation*}
E_{m}=E_{0 m} e^{-j k_{m} \cdot r_{m}} \tag{2}
\end{equation*}
$$

where,

$$
\begin{aligned}
E_{0 m} & =E_{0} / q_{0}, \quad k_{m}=k / q_{0}, \\
q_{0} & =\left(\frac{\sin ^{2} \theta_{0} \cos ^{2} \varphi_{0}}{a^{2}}+\frac{\sin ^{2} \theta_{0} \sin ^{2} \varphi_{0}}{b^{2}}+\frac{\cos ^{2} \theta_{0}}{c^{2}}\right)^{1 / 2} .
\end{aligned}
$$

The trigonometric functions of azimuth angle are

$$
\begin{align*}
& \sin \theta_{0 m}=\frac{\sin \theta_{0} g_{0}}{q_{0}}, g_{0}=\left(\frac{\cos ^{2} \varphi_{0}}{a^{2}}+\frac{\sin ^{2} \varphi_{0}}{b^{2}}\right)^{1 / 2}, \cos \theta_{0 m}=\frac{\cos \theta_{0} / c}{q_{0}} \\
& \sin \varphi_{0 m}=\frac{\sin \varphi_{0} / b}{g_{0}}, \cos \varphi_{0 m}=\frac{\cos \varphi_{0} / a}{g_{0}} \tag{3}
\end{align*}
$$

Another coordinate system $\sum^{\prime}$ is constructed by first rotating $x_{m}$ an angle $\phi_{0 m}$ around $z_{m}$ axis and then rotating $z_{m}$ an angle $\theta_{0 m}$ around $x_{m}$ axis as shown in Fig. 3. The incident wave in $\sum^{\prime}$ coordinates propagates along $z^{\prime}$-axis and polarizes along $x^{\prime}$-axis which irradiating on a sphere, namely

$$
\begin{equation*}
E_{i}^{\prime}=E_{0}^{\prime} e^{-j k^{\prime} z^{\prime}} \tag{4}
\end{equation*}
$$

where, $E_{0}^{\prime}=E_{0 m}=E_{0} / q_{0}, k^{\prime}=k_{m}=k / q_{0}$. The scattering field from the target in the new coordinates can be written as $[4,6,11]$ :

$$
\begin{align*}
& E_{\theta^{\prime} s}=\frac{j E_{0}^{\prime} e^{-j k^{\prime} r^{\prime}}}{k^{\prime} r^{\prime}} \cos \varphi^{\prime} s_{2}\left(\theta^{\prime}\right)  \tag{5}\\
& E_{\varphi^{\prime} s}=\frac{-j E_{0}^{\prime} e^{-j k^{\prime} r^{\prime}}}{k^{\prime} r^{\prime}} \sin \varphi^{\prime} s_{1}\left(\theta^{\prime}\right)
\end{align*}
$$

where

$$
\begin{aligned}
s_{2}\left(\theta^{\prime}\right) & =\sum_{n=1}^{\infty} a_{n}\left[c_{n} \tau_{n}+d_{n} \pi_{n}\right], \quad s_{1}\left(\theta^{\prime}\right)=\sum_{n=1}^{\infty} a_{n}\left[\pi_{n} c_{n}+d_{n} \tau_{n}\right], \\
\tau_{n} & =\frac{d P_{n}^{1}\left(\cos \theta^{\prime}\right)}{d \theta^{\prime}}, \quad \pi_{n}=\frac{P_{n}^{1}\left(\cos \theta^{\prime}\right)}{\sin \theta^{\prime}} \\
c_{n} & =-\frac{\hat{J}_{n}^{\prime}\left(k^{\prime}\right)}{\hat{H}_{n}^{\prime}\left(k^{\prime}\right)}, \quad d_{n}=-\frac{\hat{J}_{n}\left(k^{\prime}\right)}{\hat{H}_{n}\left(k^{\prime}\right)}, \quad a_{n}=\frac{2 n+1}{n(n+1)}
\end{aligned}
$$

We know from literatures [14,18] that the above trigonometric functions as

$$
\begin{align*}
\sin \theta^{\prime}= & \sqrt{A^{2}+B^{2}}, \cos \varphi^{\prime}=A / \sqrt{A^{2}+B^{2}}, \sin \varphi^{\prime}=B / \sqrt{A^{2}+B^{2}} \\
\cos \theta^{\prime}= & \sin \theta_{0 m} \sin \theta_{m} \sin \left(\varphi_{0 m}+\varphi_{m}\right)+\cos \theta_{0 m} \cos \theta_{m} \\
d \theta^{\prime}= & \frac{\binom{\left[\sin \theta_{0 m} \cos \theta_{m} \sin \left(\phi_{0 m}+\phi_{m}\right)-\cos \theta_{0 m} \sin \theta_{m}\right] d \theta_{m}}{+\sin \theta_{0 m} \sin \theta_{m} \cos \left(\phi_{0 m}+\phi_{m}\right) d \phi_{m}}}{-\sqrt{A^{2}+B^{2}}}  \tag{6}\\
A= & \cos \varphi_{0 m} \sin \theta_{m} \cos \varphi_{m}-\sin \varphi_{0 m} \sin \theta_{m} \sin \varphi_{m} \\
B= & \cos \theta_{0 m} \sin \varphi_{0 m} \sin \theta_{m} \cos \varphi_{m}-\sin \theta_{0 m} \cos \theta_{m} \\
& +\cos \theta_{0 m} \cos \varphi_{0 m} \sin \theta_{m} \sin \varphi_{m}
\end{align*}
$$

In expression (6), the qualities with subscript 'om' are relative the azimuth angles $\theta_{0}, \phi_{0}$ and the equalities subscript ' $m$ ' express the variable corresponding to angles $\theta, \phi$. We name the transformations from (1) to (5) as the positive transformation. The scattering field from an ellipse target irradiating by the incident wave with arbitrary polarization and propagation will be presented by the contradictorily transformation in the following. We know from [7] the following relation of electromagnetic field between two coordinates $\sum^{\prime}$ and coordinates $\sum_{m}$

$$
\begin{equation*}
\mathbf{E}_{s m}=\mathbf{P}_{s} \mathbf{E}_{s}^{\prime} \quad \mathbf{P}_{s}=\mathbf{P}_{m}^{-1} \mathbf{A}_{0}^{-1} \mathbf{P}_{1} \tag{7}
\end{equation*}
$$

We know from $[10,11]$ the following relation of electromagnetic field between the coordinates $\sum$ and coordinates $\sum_{m}$

$$
\begin{equation*}
\mathbf{E}_{\mathbf{s}}=\mathbf{Q}^{-1} \mathbf{E}_{\mathbf{s} m} \tag{8}
\end{equation*}
$$

The relation of electromagnetic field between coordinates $\sum$ and $\sum^{\prime}$ is obtained by putting (7) into (8)

$$
\begin{equation*}
\mathbf{E}_{\mathbf{s}}=\mathbf{T} \mathbf{E}_{s}^{\prime} \tag{9}
\end{equation*}
$$

$\mathbf{T}=\mathbf{Q}^{-\mathbf{1}} \mathbf{P}_{\mathbf{m}}^{-\mathbf{1}} \mathbf{A}_{\mathbf{0}}^{\mathbf{- 1}} \mathbf{P}_{\mathbf{1}}$. The elements of the matrix in (9) can be seen in the reference. The scattering field in $\sum$ coordinates is presented by putting (5) into (9), namely

$$
\begin{align*}
E_{\theta s} & =T_{22} E_{\theta^{\prime} s}+T_{23} E_{\phi^{\prime} s}  \tag{10}\\
E_{\varphi s} & =T_{32} E_{\theta^{\prime} s}+T_{33} E_{\phi^{\prime} s}
\end{align*}
$$

(10) is the expression of the scattering field from an ellipse target irradiating by the incident wave in arbitrary direction. For the sake of briefness, all the physic qualities in above be expressed with that in $\sum$ coordinates. Followings are the concrete expressions.

$$
\begin{gathered}
r^{\prime}=r q, \quad k^{\prime} r^{\prime}=k r \\
T_{22}=q_{21} p_{12}+q_{22} p_{22}, \quad T_{23}=q_{21} p_{13}+q_{22} p_{23} \\
T_{32}=q_{31} p_{12}+q_{32} p_{22}+q_{33} p_{32}, \quad T_{33}=q_{31} p_{13}+q_{32} p_{23}+q_{33} p_{33} \\
q_{21}=\frac{\sin 2 \theta}{2 q}\left[g^{2}-\frac{1}{c^{2}}\right], \quad q_{22}=\frac{g}{c q} \\
q_{31}=\frac{\sin \theta \sin 2 \varphi}{2 q}\left[\frac{1}{b^{2}}-\frac{1}{a^{2}}\right], q_{32}=\frac{\cos \theta \sin 2 \varphi}{2 c q g}\left[\frac{1}{b^{2}}-\frac{1}{a^{2}}\right], q_{33}=\frac{1}{a b g} \\
q=\left(\frac{\sin ^{2} \theta \cos ^{2} \varphi}{a^{2}}+\frac{\sin ^{2} \theta \sin ^{2} \varphi}{b^{2}}+\frac{\cos ^{2} \theta}{c^{2}}\right)^{1 / 2}, g=\left(\frac{\cos ^{2} \varphi}{a^{2}}+\frac{\sin ^{2} \varphi}{b^{2}}\right)^{1 / 2}
\end{gathered}
$$

$$
\begin{aligned}
& p_{12}=-\cos \theta \sin \theta_{m} \cos \theta_{0}-\sin \theta_{0} \cos \theta \cos \theta_{m} \sin \varphi_{m} \\
& +\cos \varphi_{0} \sin \theta \cos \theta_{m} \cos \varphi_{m} \cos \varphi-\sin \theta_{0} \cos \varphi_{0} \sin \theta_{m} \sin \theta \sin \varphi \\
& -\sin \theta_{0} \sin \theta_{m} \sin \theta \cos \varphi \sin \varphi_{0}-\sin \theta \cos \theta_{m} \cos \varphi_{m} \sin \varphi \sin \varphi_{0} \\
& +\cos \theta_{0} \sin \varphi_{0} \cos \theta_{m} \sin \varphi_{m} \sin \theta \cos \varphi \\
& +\cos \varphi_{0} \cos \theta_{m} \sin \varphi_{m} \sin \theta \sin \varphi \cos \theta_{0} \\
& p_{13}=\sin \theta_{m} \sin \theta \sin \varphi \sin \varphi_{0}-\cos \varphi_{0} \sin \varphi_{m} \sin \theta \cos \varphi \\
& -\sin \theta_{0} \cos \theta \cos \varphi_{m}+\cos \varphi_{0} \cos \varphi_{m} \sin \theta \sin \varphi \cos \theta_{0} \\
& +\cos \varphi_{m} \sin \theta \cos \varphi \cos \theta_{0} \sin \varphi_{0} \\
& p_{22}=\sin \theta_{m} \sin \theta \cos \theta_{0}-\sin \theta_{0} \cos \varphi_{0} \cos \theta \sin \theta_{m} \sin \varphi \\
& +\cos \varphi_{0} \cos \theta \cos \theta_{m} \sin \varphi_{m} \sin \varphi \cos \theta_{0} \\
& +\cos \theta \cos \theta_{m} \sin \varphi_{m} \cos \varphi \cos \theta_{0} \sin \varphi_{0} \\
& -\sin \theta_{0} \cos \theta \sin \theta_{m} \cos \varphi \sin \varphi_{0}+\cos \varphi_{0} \cos \theta \cos \theta_{m} \cos \varphi_{m} \cos \varphi \\
& +\sin \theta_{0} \cos \theta_{m} \sin \varphi_{m} \sin \theta-\cos \theta \cos \theta_{m} \cos \varphi_{m} \sin \varphi \sin \varphi_{0} \\
& p_{23}=\cos \theta \sin \varphi_{m} \sin \varphi \sin \varphi_{0}-\cos \varphi_{0} \cos \theta \sin \varphi_{m} \cos \varphi \\
& +\sin \theta_{0} \cos \varphi_{m} \sin \theta+\cos \varphi_{0} \cos \theta \cos \varphi_{m} \sin \varphi \cos \theta_{0} \\
& +\cos \theta \cos \varphi_{m} \cos \varphi \cos \theta_{0} \sin \varphi_{0} \\
& p_{32}=\cos \varphi_{0} \cos \theta_{0} \cos \theta_{m} \sin \varphi_{m} \cos \varphi-\cos \varphi_{0} \sin \theta_{0} \sin \theta_{m} \cos \varphi \\
& -\cos \varphi_{0} \cos \theta_{m} \cos \varphi_{m} \sin \varphi+\sin \theta_{0} \sin \theta_{m} \sin \varphi \sin \varphi_{0} \\
& -\cos \theta_{m} \cos \varphi_{m} \cos \varphi \sin \varphi_{0}-\cos \theta_{0} \cos \theta_{m} \sin \varphi_{m} \sin \varphi \sin \varphi_{0} \\
& p_{33}=\cos \varphi_{0} \cos \varphi_{m} \cos \theta_{0} \cos \varphi+\cos \varphi_{0} \sin \varphi_{m} \sin \varphi \\
& +\sin \varphi_{m} \cos \varphi \sin \varphi_{0}-\cos \varphi_{m} \cos \theta_{0} \sin \varphi \sin \varphi_{0} \\
& \sin \theta_{m}=\frac{\sin \theta g}{q}, \quad g=\left(\frac{\cos ^{2} \varphi}{a^{2}}+\frac{\sin ^{2} \varphi}{b^{2}}\right)^{1 / 2}, \cos \theta_{m}=\frac{\cos \theta / c}{q} \\
& \sin \varphi_{m}=\frac{\sin \varphi / b}{g}, \quad \cos \varphi_{m}=\frac{\cos \varphi / a}{g}
\end{aligned}
$$

It is concluded that the elements of the transformation matrix are completely determined by the parameters $\theta_{0}, \varphi_{0}$ and $\theta, \varphi$ in $\Sigma$ coordinates. It is obviously difficulty of trying to express these elements into simple ones. This situation demonstrates the complexity of investigating the scattering wave from an ellipse target be irradiated by a wave at freewill. In fact the fussy expressions in above adapt to computer simulation, namely if the incident azimuth $\theta_{0}, \varphi_{0}$ is given, the middle azimuth $\theta_{0 m}, \varphi_{0 m}$ can be determined, if the observing azimuth $\theta, \varphi$ and distance $r$ are given, these parameters $\theta_{m}, \varphi_{m} r^{\prime}$ and $\theta^{\prime}, \varphi^{\prime}$ are also known entirely.

## 3. DISCUSSION AND SIMULATIONS

When the incident azimuth satisfy $\theta_{0}=\varphi_{0}=0$, we know from expression (3) that $\theta_{0 m}=0, \varphi_{0 m}=0$, the elements of the matrix can be written as

$$
\begin{aligned}
& p_{12}=-\cos \theta \sin \theta_{m}+\sin \theta \cos \theta_{m} \cos \varphi_{m} \cos \varphi+\cos \theta_{m} \sin \varphi_{m} \sin \theta \sin \varphi \\
& p_{13}=-\sin \varphi_{m} \sin \theta \cos \varphi+\cos \varphi_{m} \sin \theta \sin \varphi \\
& p_{22}=\sin \theta_{m} \sin \theta+\cos \theta \cos \theta_{m} \sin \varphi_{m} \sin \varphi+\cos \theta \cos \theta_{m} \cos \varphi_{m} \cos \varphi \\
& p_{23}=-\cos \theta \sin \varphi_{m} \cos \varphi+\cos \theta \cos \varphi_{m} \sin \varphi \\
& p_{32}=\cos \theta_{m} \sin \varphi_{m} \cos \varphi-\cos \theta_{m} \cos \varphi_{m} \sin \varphi \\
& p_{33}=\cos \varphi_{m} \cos \varphi+\sin \varphi_{m} \sin \varphi
\end{aligned}
$$

When the scale factors satisfy that $a=1, b=1, c=1$, we obtain $\theta_{m}=$ $\theta, \phi_{m}=\phi, q=1, g=1, q_{21}=0, q_{22}=1, q_{31}=0, q_{32}=0, q_{33}=1$. It is further obtained that $r^{\prime}=r, k^{\prime}=k ; p_{12}=0, p_{13}=0, p_{22}=1$, $p_{23}=0, p_{32}=0, p_{33}=1$ and $T_{22}=1 T_{23}=0 T_{32}=0 T_{33}=1$. We thus acquire from (6) that $\phi^{\prime}=\phi, \theta^{\prime}=\theta$. Followings are gotten after putting the above results into expression (10)

$$
\begin{equation*}
E_{\theta s}=E_{\theta^{\prime} s}, \quad E_{\varphi s}=E_{\phi^{\prime} s} \tag{11}
\end{equation*}
$$

The ellipse target has been changed into a sphere with radius of 1 m as all the scale factors are 1. Expression (11) is inevitable which demonstrates the correctness of this algorithm used in this paper. In the process of frontal derivations, the configuration and type etc. of medium of the target are not confined concretely, so the obtained result (10) is valid to investigate the scattering characteristics for medium ellipsoidal target, conductor ellipsoidal target and multilayer medium ellipsoidal target. This result has established a base in theory for investigating the scattering wave from an ellipse disk, chaff and leaves of plant. For the sake of briefness, we take the conductor ellipsoidal target as an example. The parameters be used in the partial simulations are respectively $f=0.6 \mathrm{GHz}$, and $a=1.1 \mathrm{~m}, b=1.3 \mathrm{~m}$, $c=1.5 \mathrm{~m}$. Following are the results.

In Fig. 5, the polarizing angle is $\pi / 3$ and in Fig. 6, tthe incident wave propagates along $z$-axis. It is concluded that the scattering field generally varies not much with the incident wave, but in the specific observing point the scattering field changes greatly. When the incident angle is given, the scattering field varies obviously with the polarizing angle. Fig. 7 and Fig. 8 demonstrate that when the incident wave polarized state is given, the energy of scattering wave is very great in a observing area and is very small in the rest area, which provides a theory for target hide and target identification.

Although the leaves of plant are various shapes and sizes, they are ellipse disk approximately in shape and its size generally is several


Figure 5. Scattering field varies with incident angle.


Figure 6. Scattering field varies with polarizing angle.


Figure 7. Scattering field varies with observing point.


Figure 8. Scattering field varies with observing point.
centimeters. It is assumed in the following that $2 c=7 \mathrm{~cm}, 2 b=3 \mathrm{~cm}$, $2 a=1 \mathrm{~mm}, f=60 \mathrm{GHz}$. This kind of target is a very good scattering model for the leaf situated in the $y-z$ plane. Following are its partial simulations. The observing point in Fig. 9 and Fig. 10 is same one which indicates that the polarization of incident wave influence thescattering field greatly, the scattering effect arrives its large value as the incident wave irradiating the target vertically. The size of scattering field is about at the same centimeter scale in Fig. 11 and


Figure 9. Scattering field versus incident angle.


Figure 10. Scattering field versus incident angle.

Fig. 12. The scattering field in Fig. 8 versus the angle $\phi$ sensitively and at the angles $\phi=0$ or $\pi$, the scattering field arrive its maximum. This is comprehensible that the scattering is induced by the faradic current on the leaf. In fact if we use the parameters of $a \approx b \gg c$, this will be a good scattering model for a chaff and a finite long cylinder. We no longer investigate it detail here.


Figure 11. Scattering field versus observing point.


Figure 12. Scattering field versus observing point.

## 4. CONCLUSION

Obtaining the analytic solution of scattering wave from an elliptical target irradiated by electromagnetic wave in arbitrary direction is a Gordian knot. Although it has caused the large attention of scientists in the world, the relative literatures to the problem are seldom published. In this paper, the elliptical target is first reshaped into a
spherical target by utilizing the scale transformation of electromagnetic theory. Then we use the coordinates rotation and literature to obtain the solution of scattering wave from the reshaped target. Finally the solution of scattering field from an elliptical target irradiated by the incident wave in freewill is presented by the counter-transformation, counter-rotation transformation and counter-scales transformation. Obtained results show that the solution of scattering wave of an elliptical target is changed into that of the spherical target as all the scale are one and target changed into a sphere. This is in good agreement with that in the literature. We take a conductor ellipsoid as an example to establish a model both for elliptical target and plant leaf by using the different scale factor. Simulations both for two models are presented respectively. Results show that the scattering field is sensitively affected by polarization of the incident wave and varies not too greatly with the incident wave and changes with the observing point. At some points the energy of scattering wave arrives to its maximum. The method used in this paper has the characteristic of briefness, perspicuity in physics. The result obtained is convenient for computer simulation and has a broad usage in the fields such as complex scattering of chaff, plant leaves and target identification etc.

## ACKNOWLEDGMENT

This work is supported by the National Natural Science Foundation of China (60741003), the Natural Science Foundation of Shaanxi Province (2005A10), the Natural Science Foundation of Shaanxi Education Office (06JK162) and the Natural Science Foundation of Xian Yang Teacher's University (05XSYK102).

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