A Grating-Coupled Plasmon Index Sensor Observing the 0th Reflection Carefully and Sensibly for Recognizing the Resonance Curve with the Absorption Center: The Existing State of Affairs Aiming for Seven-Digit Resolution

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Abstract—First of all we inform the audiences that this article is a Review Paper (RP) for the PIERS17 Proceedings Paper (Zheng et al. [17]). The reason why we publish this RP is that: although the paper [17] reported important ideas and simulation facts, details of the contents were insufficient, and the audiences of the report were not satisfied. This was because the page number was limited, and we saved the number of pages. However, because the contents of [17] are important, we decided to publish an RP, which would provide additional explanations or give considerations supporting main issues. Now, we start the abstract from Section 1. Here, we mention historical topics and RP-related things. In Section 2, we explain the problem of diffraction by a conically-mounted metal grating. To save the page number, we skip the method of solution. In Section 3, we explain our method in noise free case. We show the high precision of the quadratic (or parabola) approximation. We define the workspace (WS: relation between index range of a sample and a proper azimuth angle) and one-to-one correspondence between sample index and resonance angle. In Section 4, we try our method in a noisy environments. A curve-fitting procedure and three types of noise filters work to find satisfactory solutions. That is, in both 3. and 4., the resolution of the index is 7-digit usually, which is our target from the beginning. We think that the introduction of AI or statistical processing would increase the stability of the result. In Section 5, we mention some of future works. In APPENDIX A, we explain the method: How to find the azimuth angle, which we need in solving the diffraction problems by conically-mounted grating.

1. INTRODUCTION

When a metal grating is illuminated by a TM($p$)-polarized light, absorption of the incident light occurs at a specific angle of incidence $\theta_r$, which is called resonance angle. This phenomenon is referred to as plasmon resonance absorption (PRA), which is caused by the excitation of plasmon surface waves and is accompanied by an abrupt change (usually a dip) of the reflection efficiency [1–5]. It is known that the resonance angle is a sensitive function of the refractive index of the liquid material that occupies $V_1$, the half space whose lower surface borders are on the grating surface.

By the way, it is also widely known that metal gratings have the problem of Wood’s anomaly, and again, in the TM incidence, the anomaly spoils the performance of the gratings. It was discovered by Wood in 1902 [6] and the discussions on the character of anomaly and countermeasures by Rayleigh [7] and Fano [8], and others continued until recently. The recent discussions included the similarity between the plasmon and the anomaly [2, 9]. In this connection, we remember that the last 20 or 30 years of the
20th Century were the beginning of the Nano-Technology (Nanotech, Nanotechnologies). The following is a famous story: Prof. Taniguchi (Science Univ. Tokyo) used the word “Nano-Technology” for the first time [10], and he said “the processing accuracy in the 21st Century must be 1 nm”. We should say: a quick temper might be the only defect in his character. Anyway, we should utilize one edge or a single part of the Nanotech in designing a prototype. Because the present article is something like a draft of the prototype of our sensor kept in a computer, we can try our ideas. If we get good results, we will equip that function in an actual prototype.

Now, we go back to the original way, and let us confirm our target. We already mentioned it in the title, and here we would like to provide additional explanations. We are a small group participating in the issue of grating-coupled refractive index sensors from the viewpoint of Computational Electromagnetics (CEM). We guess that a huge number of researchers and students are working in Nanotechnologies. A considerable number of them are developing index sensors [11, 12] as an application of the PRA event. Although the application seems easy, actual development of an index sensor requires much time and effort. The source of this issue might be the high specification in resolution.† We hope to achieve 7-digit resolution in the refractive index. This requires no less than 6 or 7 significant figures of the resonance angle \( \theta_r \) in degrees. Thus, we face many problems in finding the precise location of \( \theta_r \). Some of the representative causes are: lack of instrumental resolution (IR), the efficiency curves beyond our control, inconvenient reflected beams (in strength, half-width, etc.), etc. Therefore, it is not an easy work to develop a required index sensor. Although we basically follow the above phenomenon, we should find and use some innovative idea to attain our goal. In the last few years (from 2017 through 19), we have found a couple of important points from the results of computer simulations, the points which help us to achieve our target (7-digit resolution). We think publication of the points together with related knowledge may help the people working in sensor application of Plasmonics. We do not know how helpful we could be, but we would like to do our best.

We may have to explain our computational method for solving the diffraction problems. This is called Yasuura’s method of modal expansion (YM). Basically, this is a simple and fast method, but the present problem is formulated in pseudo 3D, and the computer program must be complicated. Fortunately, one of the authors (Y.O.) published a joint-authorship chapter recently on introduction to the YM with applications to grating problems [13]. In addition, a couple of former colleagues published a paper on their status in sensor development [14]. These two articles must be good for interested readers. In particular, the latter is convenient for newcomers to the CEM because: (a) Their problem is common to ours in the present article; (b) Most of the variables and symbols are also common. Hence, we decided that: after formulating the EM problem in Section 2, we move to Section 3 without explaining our method of solution. Although the method is rather complicated, it is not difficult at all. In addition to the two references above, some of our recent publications in PIERS [15–18] may serve well. As for our rating of the YM, please consult the footnote,‡

Before closing this section, we would like to provide some additional explanation and our opinion. As we pointed out, the proceedings paper [17] is too short and vague sometimes. We think that it is for a short range. Instead, we hope that this article has an opposite nature: enough space and free from confusion or doubt. We hope this and endeavor to attain.

2. FORMULATION OF THE DIFFRACTION PROBLEM

Figure 1(a) shows the geometry of conical diffraction by a metal grating. We employ a grating with sinusoidal surfaces \( S_1 \) and \( S_2 \) given by

\[
\begin{align*}
z_1 &= h \sin 2\pi x/d; \\
z_2 &= z_1(x) - e,
\end{align*}
\]

where \( (x, z_1) \) and \( (x, z_2) \) are points (or straight lines in \( Y \)-direction) on the surfaces \( S_1 \) and \( S_2 \). The grating shape is assumed sinusoidal and can be replaced by a Fourier grating in a completed version.†

We have heard 7- or 8-digit resolution (minimum indicate value 1E(−6) or 1E(−7)) was required by Clinical Medicine, Physiology, and others. Comparison with the light speed, we supposed that 8-digit would be a very high target. Thus we took a refractometer type 7-digit sensor, which might be a little more realistic and could be our goal.

† The power of the YM is average so long as usual modal functions (MF) (e.g., Floquet) are used as MF. But, if one uses MF whose combinations can represent the boundary condition easier, the person gets good solution with a small number of MF. The reason why we choose the YM is that we have used it for more than 40 years.
Figure 1. (a) Conical diffraction of a TM (p) polarized incident light by a metal grating. $V_1$ is a semi-infinite region over the grating surface and is occupied by a sample material whose relative index is $n_1$. $V_2$ is a gold over-coating (thickness $e$) and $V_3$ is a holographic grating made of aluminum. The surfaces $S_1$ and $S_2$ are corrugated sinusoidally in $X$ with a period $d$ and a depth $2h$. The plane of incidence makes an azimuth angle $\phi$ with the direction of periodicity $X$. $\theta$ is an incident zenith angle and the electric field of the incident light is parallel to the plane. If $e$ is sufficiently thick (a few hundred nm), we can neglect the aluminum layer together with the boundary $S_2$ (b) Definition of a polarization angle $\delta$.

We assume that the incident wavelength $\lambda$ is a little greater than the grating period $d$, so that there is only one propagating order ($m = 0$) in the majority of the zenith angle range. The incident light is given by

$$\begin{bmatrix} E^i \\ H^i \end{bmatrix}(r) = \begin{bmatrix} e^i \\ h^i \end{bmatrix} \exp(i k^i \cdot r - i \omega t).$$

(2)

Here, $e^i$ and $h^i$ are electric- and magnetic-field amplitudes of the incident light [See Figure 1(b)]. They satisfy

$$h^i = (\omega \mu_0)^{-1} k^i \times e^i,$$

(3)

and $k^i$ is the incident wavevector given by

$$k^i = n_1 k (\sin \theta \cos \phi, \sin \theta \sin \phi, -\cos \theta) = (\alpha_0, \beta, -\gamma_0)$$

(4)

with $\theta$ being the zenith angle between $k^i$ and the grating normal $u_Z$. The $e^i$ and $h^i$ carry the information of polarizations: TM(p) and TE(s). To show this clearly, we define two unit vectors that span a plane orthogonal to the incident wavevector $k^i$:

$$e^{TE} = (\sin \phi, -\cos \phi, 0), \quad e^{TM} = (\cos \theta \cos \phi, \cos \theta \sin \phi, \sin \theta).$$

(5)

Combining the two vectors in Eq. (5), we represent the electric-field amplitude as

$$e^i = e^{TE} \cos \delta + e^{TM} \sin \delta,$$

(6)

where $\delta$ is a polarization angle (see Fig. 1(b)). We set $\delta = \pi/2$ in the following because we use a TM polarized incident wave. The time factor $\exp(-i \omega t)$ will be suppressed throughout.

Now we can state the problem to seek the diffracted electric- and magnetic-fields in the three regions:

**Problem 1.** Find the diffracted waves $(E_R, H_R)$ in $V_R$ ($R = 1, 2, 3$) that satisfy the following conditions (In conical mounting, the EM fields in each region have both TM- and TE-components, e.g., $E_1 = E_1^{TM} + E_1^{TE}$):
(a) The homogeneous Helmholtz equation in each region;  
(b) The 1D radiation condition in \( V_1 \) and in \( V_3 \);  
(c) Periodicity conditions that: any component \( f(X, Y, Z) \) of the diffracted waves satisfies the relation  
\[
    f(X + d, Y, Z) = \exp(i\alpha_0 d) f(X, Y, Z),
\]
and the phase constant in \( Y \) is \( \beta \) constantly;  
(d) The boundary conditions on \( S_1 \) and \( S_2 \) that the total tangential components of electric- and  
magnetic-fields are continuous across the two boundaries.

We mentioned in Section 1 that we employ the YM in solving **Prob. 1** and that we do not go  
deeper into the method in this article. We hope that interested readers get necessary information from  
the references cited at the end of Section 1.

3. NUMERICAL EXAMPLES AND DISCUSSION (I): WITHOUT NOISE

3.1. Physical Parameters in Numerical Analysis and Examples of Typical Plasmon  
Excitation

Here we list the physical parameters employed in our numerical analysis. The majority of the parameters  
are taken from catalogues of optical devices and systems published by Nanotech Companies. The others  
are similar to catalogue data, but slightly modified sometimes to satisfy our convenience. The grating  
parameters\( \parallel \) are \( 2h = 61 \text{ nm} \) (depth), \( d = 556 \text{ nm} \) (period), and \( e = 44 \text{ nm} \) (gold layer thickness). The  
incident light is a monochromatic TM plane wave with \( \lambda = 660 \text{ nm} \). Refractive indices of gold and  
aluminum at this wavelength are \( n_2 = 0.1355 + i3.4679 \) and \( n_3 = 1.3517 + i7.1150 \). We postulate that  
these values are correct throughout the whole effective digits: That is, if we use a 7 significant digits  
system, we put \( n_2 = 0.135500 + i3.467900 \), and so on. In this connection, let us define a parameter (a  
rough approximation of the sample index \( n_1 \)), which will play an important role in our method. Here,  
rough means 4- or 5-digit, and we take 4 in this paper: if \( n_1 = 1.234567 \), then \( n_{1, \text{rough}} = 1.235 \).

\[\parallel\] This original was a UV grating made of Al with an Au over-coating. The original grooving was 1,800 g/mm. We have neglected the  
harmonics from the original profile because the optimization of parameters belongs to our future issues. By the way, we should check  
the postulates or assumptions relating to the metal index value and, if necessary, cope with a problem caused by the assumptions [18].  
This report mentioned four sources [19–22] of Au indices.

\[\parallel\] Let the solution in \( V_1 \) be \( \mathbf{E}_1 = \mathbf{E}_1^\text{TM} + \mathbf{E}_1^\text{TE} \). Then, this is the coefficient of the 0th modal function of \( \mathbf{E}_1^\text{TM} \).

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Figure 2. Typical resonance absorption in planer and conical mounting: (a) Planer mounting (TM incidence, \( n_1 = 1.0 \), and \( \phi = 0^\circ \)); (b) Conical mounting (TM incidence, \( n_1 = 1.0 \), and \( \phi = 30^\circ \)).

Figures 2(a) and 2(b) show typical PRA’s: 2(a) planer mounting; and 2(b) conical mounting. In  
Fig. 2(a), we observe the 0th reflection efficiency \( \rho_0(\theta) = \rho_0^\text{TM}(\theta) = |A_{1,0}^\text{TM}(\theta)|^2 \), where \( \theta = \theta^i \) and \( A_{1,0}^\text{TM} \) is  
one of the modal coefficients.\( ^* \) Because this is a scalar 2D problem, we have no mode conversion here.
The dip at $\theta = 8.2^\circ$ shows the PRA, and the peak at $10.6^\circ$ is caused by the $-1$ order cutoff. Fig. 2(b) shows what happens when the resonance absorption occurs in conical mounting. The resonance angle is $\theta_r = 9.6^\circ$, and the cutoff is $12.8^\circ$. The biggest difference from Fig. 2(a) is the appearance of the TE$_0$-mode near the resonance angle. Because the incident light is polarized in TM, this means occurrence of the TM-TE mode conversion \[23, 24\]. Hence, this is a quasi 3D vector problem. Probably, the mode conversion occurs for satisfaction of the boundary conditions: because a greater part of the TM component is lost by the resonance absorption, the TE component appears to supply the deficiency. Note that the zenith angles corresponding to the extremes $\{\min(\rho_{0\,TM}^0)\,\min(\rho_0), \max(\rho_{0\,TE}^0)\}$ are slightly different from each other. The degree of difference depends on a variety of conditions.

### 3.2. Computation Results of a Resonance Curve near $\theta_r$ and a Quadratic Approximation

We have found through a systematic numerical survey that the curve of 0th-order TM reflection efficiency $\rho_{0\,TM}^0$ carries most of the information about the location of $\theta_r$, the location closely related to $n_1$. On the contrary, we found almost no interesting result from the reflected TE$_0$ mode in the present setting. Because we often mention a distinctive part of the efficiency curve including $\theta = \theta_r$, we designate that part as a resonance curve (RC)* in the following discussions. Now, we show an example of the RC and an approximation by a quadratic (or parabola, second-order polynomial) curve, which also plays an important role in our method.

Both Figs. 3(a) and 3(b) illustrate two curves: the RC near the $\theta_r$ (blue solid curve) and its approximation by a quadratic function (red broken curve). Note that the former is a numerical solution of Prob 1 obtained by the YM, while the latter is generated from the former by using the following conjecture (C) and procedures (P).

**Figure 3.** A resonance curve (RC) and its approximation ($n_{1,\,\text{rough}} = 1.306, \phi = 29.73^\circ, n_1 = 1.306000$): (a) shows the RC obtained by the YM (blue, solid curve) and an approximation by a quadratic function (red, broken line); (b) illustrates an enlarged bottom part of (a). Enlargement ratios on the abscissa and ordinate are $7E2 (\approx 700)$ and $5E5 (= 500,000)$, respectively. Hence, the area of (b) is an extremely small area of (a). In getting the approximation, we employed the least-squares method (LSM): additional explanations can be found in Section 4.

*(C) In the close vicinity of $\theta_r$ [like the area in Fig. 3(b)], the quadratic approximation is quite effective to represent the RC precisely (as if the principal term of the RC near $\theta_r$ is a quadratic function of $\theta$ or its equivalence).
(P1) How to get the blue RC in Fig. 3(a): (i) Get the rough index $n_{1,\text{rough}}$; (ii) Prepare the minimal efficiency value $\min\rho_{\text{TM}}(\theta_r)$ of one’s desire; (iii) Follow the procedure in APPENDIX A, and get the azimuth $\phi$; (iv) Solve Prob. 1 with $\phi$ by the YM, and draw the blue RC in Fig. 3(a).

(P2) How to find the red approximations in Fig. 3(a): Find the parabola part of the blue RC in Fig. 3(a); represent this result in a closed form. How to do in Fig. 3(b): Draw the red curve in Fig. 3(a); set the scales for Fig. 3(b); put the blue and red curves in Fig. 3(b).

Now, we observe Fig. 3 again. We notice in Fig. 3(a) that the two curves are close to each other in the bottom area, i.e., near the resonance angle $\theta_r$. Fig. 3(b) is an expanded version of the bottom area and illustrates the area in detail. Although there are two curves (blue and red) also in Fig. 3(b), we observe only one purple curve there. This is because the two curves overlap each other, and we find almost perfect agreement near the bottom (i.e., $\theta_r$) of the RC. Please confirm that blue curve is obtained by the YM, and the red curve is a parabola approximation. According to our trial computations, the approximation power of quadratic functions seems valid in every conceivable case. Of course, the range of validity or the strength of this phenomenon depends on the existing conditions. However, if we are looking for the resonance angle $\theta_r$ and if the quadratic approximation works well, let us employ the method with the conjectured approximation because it is an easy way to determine the location of $\theta_r$ with high accuracy.

3.3. Workspace and Estimation of Resolution

When measuring the index $n_1$ of a sample by our method, we first need to find the rough index value $n_{1,\text{rough}}$ (a four-digit approximation). It is easy by using a refractometer. Then, we make some computation to find a recommended azimuth angle $\phi$. Before making this, we should have a candidate of $\min\rho_{\text{TM}}(\theta_r)$, which we should choose when getting the angle $\phi$. Having made these preparations we can visit APPENDIX A to learn how to obtain the azimuth angle.

Getting the recommended angle, we can start the main part of computation (solving Prob. 1) under the constraint in the azimuth. However, we usually record the azimuth and a range of the rough $n_1$ for which the newly obtained azimuth is available. The Workspace (WS) is made in that way, which extends the area coverage of our sensor. The structure of each WS is a combination of [an index range] and [a recommended azimuth], which takes a form like $[1.320, 1.322]|\phi = 29.53^\circ = (1.320 \leq n_{1,\text{rough}} < 1.322)|\phi = 29.53^\circ)$. If we make an accumulation of WS, we have something like Table 1. In addition, there is Table 2 below Table 1, which shows one-to-one correspondence between the resonance angle $\theta_r$ and the sample index $n_1$.

Table 1. An example of a table of WS’s. Each WS is a combination of index range ($1.320 \leq n_{1,\text{rough}} < 1.322$) and recommended azimuth ($\phi = 29.53^\circ$). The width of a WS range is between 2E($-3$) and 3E($-3$).

<table>
<thead>
<tr>
<th>Index range</th>
<th>Azimuth $\phi$ (deg.)</th>
<th>Index range</th>
<th>Azimuth $\phi$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1.313, 1.315]</td>
<td>29.60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. One-to-one correspondence of $\theta_r$ and $n_1(n_{1,\text{rough}} = 1.306, \phi = 29.73^\circ)$.

<table>
<thead>
<tr>
<th>$n_1$</th>
<th>$\theta_r$ (deg.)</th>
<th>$n_1$</th>
<th>$\theta_r$ (deg.)</th>
<th>$n_1$</th>
<th>$\theta_r$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.306000</td>
<td>14.7893818</td>
<td>1.306003</td>
<td>14.7896304</td>
<td>1.306006</td>
<td>14.7898790</td>
</tr>
</tbody>
</table>
Using the information to draw a figure, we have Fig. 4, which shows the one-to-one relationship, again. This is an extremely enlarged picture, and we see the linear relation between $\theta_r$ and $n_1$ clearly, the relation which contributes the high resolution. The proportional constant plays an important role in examining the resolution. Because that is the most important target, we would like to discuss this subject again in the next occasion. Here, we briefly guess the resolution of our system. We employ two methods of estimation (EM) and explain both of them briefly.

**EM1** Method using the relation between $\Delta \theta_r$ and $\Delta n_1$. In this method form is important. From the proportional connection in the caption of Fig. 4, we have $\Delta n_1 = \Delta \theta_r / 82.9$. If we employ a rotating stage with 4,000 pulses per degree IR, then $\Delta \theta_r \approx 2.5 \times 10^{-4}$. Hence, we obtain $\Delta n_1 \approx 3 \times 10^{-6}$. Roughly speaking, the 7-digit resolution means $\Delta n_1 = 1 \times 10^{-6}$. Thus, the resolution of our method is a little less than 7-digit.

**EM2** Method using Fig. 4. This is a simple estimation. The distance between neighboring two small dots on the abscissa is $4 \times 10^{-7} = 0.0000004$. On the other hand, an example of a seven-digit liquid index usually takes the form like $n_1 = 1.234567$. In these situations, we can say that: provided that the conjecture works, we can use Fig. 4 to find $n_1$ through $\theta_r$, and the resolution might be no less than 7-digit. We think that an artificial intelligence should be introduced in determination of the index.

**4. NUMERICAL EXAMPLES AND DISCUSSION (II): WITH ARTIFICIAL NOISE**

So far we did not care much about noise because inclusion of the noise issues would make our problem complicated. The noise, however, is inevitable in experimental works, and we would like to try our elementary Nanotech equipment to reduce the noise. If it works, we will try to improve the equipment. To know the noise effect and to understand the influence of noise, we first prepare a set of data by adding an artificial noise to the solution by the YM. That is, we make a new set of data by adding noises to the original data shown in, for example, Fig. 3(a) or 3(b). As an artificial noise, we employ an additive white Gaussian noise with zero average. We assume three levels of noise classified by their magnitudes: [s] small, SD = min/5; [m] middle, SD = min; and [b] big, SD = 2 × min. Here, min = min($\rho_{TM}^0$) stands for the minimum $TM_0$ reflection efficiency at $\theta = \theta_r$. To simulate the data obtained through a noisy experiment, we use the superposition of original data by the YM and one of the three levels of noise. We use [s], [m], or [b] in nominating a noise level. To know the influence of noise, we show three pictures in Fig. 5 showing the distortions of the YM result by the superposition of noises [s], [m], and [b].

In the presence of noise, we should consider how to choose the area from which we sample the RC data. Although the points in the close vicinity of $\theta_r$ must be important, the efficiency values at these points might have been altered by the noise. Therefore, we employ three kinds of filters (boxes) shown in Fig. 6. This figure displays the same phenomenon as the one in Fig. 3(b). The only difference is the ranges of the coordinates: specifically speaking, the ordinate is from 1.7E($-6$) to 3.7E($-6$) in Fig. 3(b), while it is from (about) $-0$ to 2E($-4$) in this figure (Fig. 6). The three types of filters are shown as
Figure 5. Examples of distortion caused by noises: (a) (left, [\text{s}]), (b) (center, [\text{m}]), and (c) (right, [\text{b}]). The simulation result (red curve) is from a grating mounted similarly as one in Fig. 3.

Figure 6. Two RC’s and three kinds of filters (boxes). \( n_1 = 1.306000, \phi = 29.73^\circ, \) and \( \theta_r = 14.789382^\circ. \) The curves [an original data (blue) and a quadratic approximation (red)] show almost the same overlapping as we saw in Fig. 3(b); although the ordinate range is extended.

rectangular boxes with rounded vertexes. Each box is colored by blue, green, or yellow, and the common function of the filters is to choose a necessary number of sampling points from the RC inside the box.*

Let us see the concrete definitions of the filters and the number of sampling points. In our discussions concerning the filters, we call them by their colors like in the following definitions.

- blue box: This box focuses on the RC near \( \theta_r, \) and the definition of the set of sampling-point candidates is given by \( \{\theta : |\rho_{0\text{TM}}^n(\theta)| \leq 2 \times \text{min}\}, \) where \( \text{min} \) is the minimum TM\(_0\) efficiency.

- green box: The definition of this box values the symmetry of the RC near \( \theta_r \) and hopes to escape from the possible rewriting by noises. Hence, we should neglect the small efficiency part of the RC, and we have chosen the efficiency range \([1.0, 2.0] \times \text{E}(-4)\) after some trial. Note that the data passing through this box include almost nothing from the close vicinity of \( \theta_r. \)

- yellow box: The idea of this box is a union of blue and green: the acceptable range of efficiency

* Please understand that this is no more than a trial. We use the raw data from the filters in data processing to find the shape of RC. We are planning to introduce a window function. Also, we are trying to find filters with good performance.
can be from a tiny negative number [roughly $-1E(-7)$] to $2E(-4)$.

In trial computations we used the data from several hundred sampling points. The points were located on the piece(s) of the RC in one of the boxes. In the following examples, we will find that the green and yellow boxes work well in general, but the blue box is difficult except for the noise \([s]\).

We show in Fig. 7 some results of simulation to compare the performance of the three filters (boxes). The number of sampling points is 600 independent of the box color or the noise level. Probably, we

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Illustrations of one-to-one correspondence between $\theta_r$ and $n_1$ in the presence of noise. Parameters are the same as those in Fig. 4: $n_{1,\text{rough}} = 1.306$, $\phi = 29.73^\circ$. The noise classes are as follows. (a) := [s], (b) := [m], and (c) := [b].}
\end{figure}
can decrease the number of points. The three illustrations correspond to the three levels of noise: Figs. 7(a), 7(b), and 7(c) display the result of [s], [m], and [b], respectively. First, we observe 7(a), which summarizes the results obtained in the presence of noise [s]. The red straight line is the original data, and the colored points are obtained by our data processing system, which is composed of filter boxes and curve fitting procedure (CFP). This means that the information passed through the filter is the input to the CFP and that the CFP reconstructs an approximation of the RC using the least-squares method (LSM).\(^5\) Note that the input of the CFP is a part of the RC data and that the reconstructed RC is an analytic curve. In the [s] case, all the boxes work well except for small fluctuation in the blue result and for the tiny upward shifts of the green and yellow marks.\(^6\)

In Figs. 7(b) and 7(c), we find that the green and yellow boxes work well in the environments with [m] or [b] noise. Checking the details, however, we find the green is better than the yellow in the tiny fluctuation. The blue box suffers by comparison with green or yellow as we predicted. As for the possible resolution of our method, we can anticipate that the longest possible significant figures will be seven, but the seventh digit may have a small uncertainty factor depending on the situation.\(^#\)

5. CONCLUSION

As we mentioned in Section 1, we have been working in the development of a grating-coupled plasmon index sensor and have got a few research findings on the sensor structure and directions for using it. This article explained the present situation together with the knowledge. Also, this article examined the achievable resolution of the sensor a few times. In Subsection 3.3, we tried two times and got inconsistent results: (EM1) used the proportional relation \(\Delta n_1 = \Delta \theta_r / 82.9\) and concluded that our result was a little below the target (7-digit); on the other hand, (EM2) examined the enlarged figure and recognized that our method could determine no less than 7-digit. We are discussing to introduce the AI technology for this issue.

We have been preparing this report of our research work, which is focused mainly on the theoretical part of the sensor development. If we call our past study the first step, then we move into the second step after submission of this manuscript according to our schedule. The second step will continue a few years where we hope to deal mainly with the practical issues. At the same time, we have to consider the problem above. Hence, the probable subjects of our second step will be as follows:

1. **Another article:** We are writing another article on a selectable range of the min, the TM\(_0\) reflection efficiency at \(\theta = \theta_r\). This allows to employ a bigger value, e.g., \(1E(-3)(=0.1\%)\) or further as the minimum efficiency. This makes experiment and data processing easier than before.

2. **A carry over problem:** We continue to consider how to maintain the stability of the sensor.

3. **Investigation and examination of the variety of components for verification by experiment:** Specifically, the components include: grating profile and material, light source, artificial noise, geometry of the noise filter, refractive index standard, etc.

4. **Improvement of environment for precision measurement:** This will be a tough work, and we need joint research for this subject.

**APPENDIX A. A METHOD TO GET THE RECOMMENDED AZIMUTH ANGLE**

To get recommendable or proper azimuth angles, we should make a systematic survey of the \(n_1, \phi\)-dependence of the resonance absorption strength. As widely known, strong absorption spends a greater part of incident power to excite the plasmon surface wave and to generate TE modes for the boundary conditions. Consequently, the return-signal (the reflection efficiency) is usually small, and sometimes, photo-detectors fail to catch the signal. Hence, our target is to find the return-signal with proper strength for data processing.

\(^5\) Although we usually look for parabola (quadratic) approximations, inclusion of higher-order terms in the approximation may be useful to deal with small asymmetry of the RC. This will be one of our future subjects.

\(^6\) We know the root cause of this problem, and we can make compensation if we have sufficient data for comparison.

\(^#\) Assuming a tiny error of \(\theta_r\) in Fig. 7, we predicted the error in \(n_1\). Using this result, we can conclude that an acceptable error of \(\theta_r\) is \(5E(-5)\) to endorse the 7-digit resolution.
Figure A1. The minimum efficiency $\rho_0^{TM}(\theta_r)$ as a function of $\phi$. There are four curves corresponds to four values of $n_1$. Note that the $n_1$ values should have additional three zeros: $1.306 \rightarrow 1.306000$.

Figure A1 shows the strength of the minimum return-signal as a function of the azimuth angle $\phi$ (abscissa) and refractive index $n_1$ (parameter). When we use this figure, we first draw a horizontal line like two broken lines that we see at $1E(-3)$ and $1E(-6)$. Finding the intersection points with the $n_1$ curve, we get the candidates. We add some comments:

1. Usually, the first-time try is not enough, and we should repeat the same procedure (with narrower $n_1$ ranges) a few times.

2. There are two $\phi$'s ($\phi_1$ and $\phi_2$) for a given value of efficiency. Therefore, we should examine which combination ($n_1, \phi$) ($\phi = \phi_1$, $\phi_2$) works better.

3. When Fig. A1 is not clear to read in detail, we use a similar figure whose ordinate shows the reciprocal of the minimum efficiency $1/\rho_0^{TM}(\theta_r)$. This modified version is easier to read sometimes.

4. Although we define $n_{1,\text{rough}} = 1.235$, we use $n_1 = 1.234567$ in YM numerical computations. Usually, the results using 1.235000 and the one using 1.234567 are not so different. Hence, this rule is for uniformity.

REFERENCES


