Method to determine the coupling state of the microwave resonator

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Abstract: A simple and real-time method to determine the coupling state of resonators has been proposed. The coupling states are determined by the graphical relationship between the resonance circles and the match point in Smith chart. The extraction of the coupling coefficient and unloaded quality factor can be performed directly and timely. The procedure is validated with experimental measurements of manufactured cavities. The proposed method could provide an effective way to solve parameters extraction problem for resonators.

1. Introduction

The high quality microwave resonator is an important component in the microwave systems, such as filters, antennas, oscillators, dielectric resonators and dielectric measurement, etc. The precise characterization of resonators is crucial to many microwave applications. The basic parameters of the resonator are the resonance frequency, unloaded quality factor and coupling coefficient. The coupling coefficient representing the ratio of the power dissipated in the external circuit to the power dissipated in the unloaded resonator can be defined as

$$\beta = \frac{Q_0}{Q_e}$$  \hspace{1cm} (1)

where $Q_0$ and $Q_e$ represent the unloaded and the external quality factors. If $\beta < 1$, the resonator is said to be undercoupled, whereas if $\beta > 1$ the resonator is overcoupled. The match condition is referred to as critical coupled when $\beta = 1$ [1]. Fig.1 describes the $S_{11}$ circles in Smith chart for three coupling conditions.

The key to get the quality factor is the coupling state determination of the resonance system. Hence, we proposed a method to determine the coupling state of the system, which is adapted to VNA (vector network analyzer) measurement techniques. The procedure described here uses the data measured directly from the VNA. A one-port cavity for quality factor estimation of a metallic resonator is presented. The method is validated by fitting it to a one-port cavity measurement data of dielectric constants, permittivity determination of materials. The proposed method provides an effective way of determining the coupling state situation of resonators.
2. Method and Results

The measurement of quality factor is important for applications in which the resonator is used. The measurement and the influence of the quality factor in the presence of coupling are well described in [2]-[4]. The coupling coefficient is a measure of the degree of the coupling between the cavity and the input feed line. In order to obtain the quality factor, the coupling state of the system should be determined first.

\[
Q_0 = (1 + \beta)Q_l \begin{cases} 
(1 + 1/\rho)Q_l, & \text{undercoupled; } \\
2Q_l, & \text{critical coupled; } \\
(1 + \rho)Q_l, & \text{overcoupled. }
\end{cases}
\]  

(2)

\(Q_l\) and \(\rho\) represent the loaded quality factor and VSWR (voltage standing wave ratio) both directly measured by the VNA. Hence \(Q_0\) can be calculated from (2), as long as the coupling state of the system has been determined. There are several methods to determine the coupling state of the resonator [1]. Ginzton describes in detail the analysis of measured resonant circles on the Smith chart in order to deduce the values of quality factor and coupling coefficient [5].

Several techniques for obtaining unloaded quality factor are described in [6]-[12]. These methods require the creation of an equivalent circuit to accurately extract quality factor. The traditional processing method is relatively more complex. In the reference [7], to obtain the \(Q_0\) value, the following steps are needed: 1. rotate the resonance circle into the standard series circuit position; 2. draw the \(Q\) curve in the graph; 3. find the special point, then the quality factor can be calculated. Even taking no account of the coupling loss, finishing these steps is complicated in Smith chart.

From the measurements point of view, the measurement should be real-time and simple. Once the coupling state is known, the coupling coefficient can be acquired by the VSWR; by observing the power decay profile of \(S_{11}\) the loaded quality factor can be computed. Then the unloaded quality factor can be calculated through the VSWR and loaded quality factor. A graphical method to accurately determine the coupling state is presented here. It is inspired by the phase relationship between the closed contour and the origin point in complex variable function theory [13]. We can use the relationship between the resonance circles and match point \(O\) to determine the coupling state.

As shown in Fig.1, in the overcoupled state, notice that the matching point \(O\) is always on the left side, if one walks along the curve anti-clockwise. The \(S_{11}\) resonance curve enclose the match point \(O\). The phase changes by \(2\pi\) across the point \(O\). On the other hand, for under coupled condition
the position relationship between the matching point position and the walker will be changed. The undercoupled resonance demonstrates a relatively small phase variation. The resonator circle locates itself in the sector zone from point O, and the change of the phase is less than $\pi$.

As shown in Fig.2(a) we can directly obtain the maximum and minimum of the phase array of the resonance curves. The phase array can be directly get from the VNA, then the phase variation is

$$angle = max - min$$ (3)

However it will be more complex in practical operation. As shown in Fig.2(b), the actual phase difference should be the angle. But the maximum and minimum of the phase array are $\pi$ and $-\pi$ respectively, because the phase range of the VNA is $(-\pi, \pi)$. It will lead to wrong judgments. On the other hand, the other two cases are relatively simple as shown in Fig.2(c) and (d). When the resonator is overcoupled, the angle is $2\pi$, no matter what the resonance circle locates in any position in Smith chart.

According to the above problem, the following method based on graphical relationship and real-time measurement data from VNA, has been obtained as shown in Fig.3. If angle $< \pi$, we directly judge that the coupling state of the resonator is undercoupled; If angle $> \pi$, the phase array will be transformed as follows: the positive phases remain the same; the negative phases are added by $2\pi$, in order to transform the phase interval from $(-\pi, \pi)$ to $(0, 2\pi)$. Then we search the maximum and minimum again, get the new angle. At this time, if angle $< \pi$, the coupling state is undercoupled; if angle $> \pi$, the coupling state is overcoupled. The described measurement procedure is automatically completed by the computer, and needn’t any human judgment. The
Fig. 3. Flow chart for determining the coupling state.

most important thing is that it avoids the complex operation in Smith chart, compared with the traditional methods.

In order to verify this method, a one-port cavity system [14][15] has been realized, which is based on the reflected power technique. The method has been applied in the permittivity measurement system as shown in Fig.4. The measurement is based on perturbation theory. The system consists of a coupler, an upper end plate, a cavity and a VNA, etc. A coaxial line is connected between the type-N connector and the VNA. First of all the coupling state of the cavity should been determined by the above method as shown in Fig.3. Then the unloaded $Q$ factor can be obtained by (2). Hence, the real part of complex permittivity $\epsilon'$ is calculated from the change of the resonant frequency, and the imaginary part $\epsilon''$ is done from $Q$ factor[16].

\[
\epsilon' = \frac{2 f_c - f_s}{\eta f_s} \quad (4)
\]

\[
\epsilon'' = \frac{1}{\eta} \left( \frac{1}{Q_s} - \frac{1}{Q_c} \right) \quad (5)
\]

where $f_c$ and $Q_c$ are the resonant frequency and unloaded quality factor of the empty cavity, respectively, $f_s$ and $Q_s$ are the corresponding quantities in the presence of the dielectric sample. $\eta$ is the filling factor. Teflon Measurements are repeated for times, and results are averaged to reduce random errors due to uncertainty in the determination of the resonant parameters. To avoid signal aliasing, the VNA was set to its maximum number of points. Measurements results are in good agreement with the published works [17]-[19], as shown in Fig.5.
3. Conclusion

A method to determine the coupling state of the resonator has been proposed. Based on the geometric relationship between the resonance circles and the match point in the Smith chart, the coupling state can be determined. This method has the advantages of real-time and complete automation. The procedure is validated with experimental measurements of manufactured cavities. Measurements results indicate that this method is consistent with the published works, and it is commonly used for cavities or other kinds of resonators. The proposed method offered an effective way to obtain quality factor of resonators.

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5. References


